Increasing Motivation in the Mathematics Classroom: An Epistemological Approach Through High Quality Learning Opportunities

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Increasing Motivation in the Mathematics Classroom: An Epistemological Approach Through High Quality Learning Opportunities

Abstract
This research consists of details pertaining to the epistemological beliefs of both students and teachers in the domain of mathematics. The findings indicate that the knowledge beliefs of a student directly impact their motivation in the classroom. The research further says that the classroom environment and the teacher are the most influential factors in generating and changing a student's beliefs. High quality teaching will be outlined as defined by the literature. This will be looked at from the perspective of generating availing epistemological beliefs and generating motivation. Further, research will be reported that details the value of learning opportunities inside and outside of the classroom and their potential to increase motivation.

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Increasing Motivation in the Mathematics Classroom: 
An Epistemological Approach Through High Quality Learning Opportunities

James Tiffin Jr. 
GMST 640: Research I 
Fall 2006 
GMST 641: Research II 
Spring 2007
Abstract

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Dedication

I would like to thank three main people for their direct and indirect support of my efforts to complete this research.

First, my mother-in-law who graciously invited us into her home while we awaited the purchase of my family’s new home. Her help with the grandchildren, the chores, the cooking of meals, and especially her mathematical teaching talents have been indispensable. The extra support, and sometimes stress, she provided helped keep everyone together as we all got through this demanding endeavor.

I would like to thank my father for providing me with all of the “high quality learning opportunities” that helped shape my values and beliefs into what they are today. I could never fill his shoes, but I can still hope to walk in his footsteps.

And last of all, I must thank my loving wife. She knows first hand what it is like to complete a research project of this size, and still juggle a teaching career. Her love and caring have been a blessing upon me during these most difficult of times. I cannot thank her enough for her understanding and tolerance, all while delivering our third child into this world. I can only hope to be as strong when she needs me. Thank you and I love you.
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Increasing Motivation in the Mathematics Classroom

Success in the mathematics classroom has always been a priority for both teachers and students. Students want to succeed on tests and quizzes, while teachers want to the same types of success. Additionally, teachers want to be successful at not only getting their students to pass the assessments given to them, but to also come away with an appreciation for mathematics, mathematical thinking, problem solving. Teachers also want students to recognize the value and importance of these elements in their current and future experiences. The inclusion of these extra measures of success does not detract from the other measures. It is possible for them to actually positively impact the student measures of success.

A variety of researchers concur that higher levels of interest, motivation, self-efficacy and engagement can produce higher levels of achievement (Koller, Baumart & Schnabel, 2004; Schwartz 2006). Therefore, the focus of this research project will be on monitoring ways of increasing these factors in students, particularly motivation. The focus on motivation is based on a belief that this is what generates higher amounts of interest and self-efficacy. The expected result is that the increased motivation levels will cause higher amounts of positive engagement in the class and its content, which will in turn produce high levels of achievement.

The main variables being manipulated are the mathematical epistemological beliefs held by students, and the opportunity to participate in a real-world application of the content being studied by the students. Through discussion, evaluation and reflection, students will become more cognizant of their own beliefs about mathematics and mathematical knowledge. The results of these typically negative beliefs can be viewed as
explanations for the low motivation levels and personality traits typically demonstrated by students toward the study of mathematics (Muis, 2004). These negative characteristics are in stark contrast to those possessed by students generally recognized as gifted and talented in mathematics (Hong & Aqui, 2004). By making students aware of their own beliefs and recognizing their actions (or inactions) are the results of those beliefs, the expectation is that they will then change their behaviors and attitudes. With more positive beliefs should come more positive levels of motivation.

The second variable under consideration is the presence of opportunities for students to apply their mathematical skills in areas of interest. Students in a New York State Regents class are typically exposed to application problems that are entirely applicable to the real-world, but are not often presented in such a manner that models the real-world. Unlike the questions typically posed on a Regents exam, a real-world problem does not immediately provide all of the necessary information in order to utilize one of perhaps many mathematical formulas. It could even be considered that the hardest part of a real-world problem is getting the information necessary to use a formula. It is experiences like this that should develop student appreciation of mathematics, and motivate them to successfully learn and apply their mathematical skills. Opportunities for students to apply their mathematics can inspire them to learn more mathematics (Menon, 2004).

In summary, the effects of this research can have profound effects on a typical mathematics classroom. Through a relatively simple exchange of ideas and beliefs, and a few chances to actually utilize the mathematics being taught, a student can change their
Attitude from one with declares 'I want to succeed, but cannot,' to one of 'I am going to success because I can.'
Literature Review

For the purpose of this review, the nomenclature and language of Muis (2004) was adopted. She took the definition of epistemology from the Cambridge Dictionary of Philosophy, given as “the study of the nature of knowledge and justification: specifically, the study of (a) the defining features, (b) the substantive conditions or sources, and (c) the limits of knowledge and justification” (p.324). The purpose of this was to unify the thirty-three research articles she reviewed with a common jargon. When referring to examining personal epistemology, this definition allows for inclusion of the exploration of the nature of knowledge, justification of knowledge, sources of knowledge, and development of knowledge acquisition.

A second adoption put forth by Muis (2004) was the classifications of availing and nonavailing beliefs. Availing beliefs will be beliefs held by a person that relate to quality learning and achievement. Nonavailing beliefs will be beliefs that do not affect learning or achievement, or affect them in a negative way.

The review begins with an analysis of the qualities that are present in students generally recognized as gifted in mathematics, with an emphasis on the beliefs held by those students. In an effort to know more about the effects of these beliefs on students, the review looked at the types of mathematical epistemological beliefs present in students. Following this information, this research then focused on how to change student beliefs from nonavailing to availing beliefs. Finally, research findings were presented regarding the types of high quality classroom environments and activities necessary to generate greater levels of student motivation.
belief to the classroom. In their research, they found a strong correlation between mathematical interest levels in students and their achievement levels. Students that achieved highly in mathematics were more interested in the content than those student classified as low achievers. Further, those higher achieving students also generally chose to take more advanced math courses. It is these advanced courses that Sriraman (2005) says will advance students through the eight levels of mathematical talent.

**Mathematical Epistemological Beliefs**

This review now seeks to find evidence regarding influencing the interest and motivation to achieve in mathematics. Buehl and Alexander (2005) are two of the few researchers to look at the interaction between motivation and epistemological beliefs held by students. Despite the lack of research supporting or refuting their position, these authors said students' beliefs about knowledge influences their levels of motivation. Further, they believed that it is possible for a student to have both general-learning beliefs and domain-specific beliefs. Bong (2004) offers her research in support of this conclusion.

The available research detailed how a student's own perception and beliefs about knowledge can either hinder or foster motivation. Buehl and Alexander (2005) made use of the Expectancy-Value Theory of Motivation as outlined by Eccles and Wigfield. This theory stated that motivation is a function of an individual's expectancy of success and their achievement values (the reasons one would engage in the task). If a student perceives a task or problem as difficult, the student will either become more or less engaged. The direction taken hinges on the student's epistemological beliefs. Students who believe more in the isolation and certainty of knowledge tended to have lower
motivation levels. Additionally, if a student sees knowledge as emanating from a specific source such as a book or a teacher, their engagement level will again be lowered upon confrontation with a difficult problem (Buehl & Alexander).

These situations arise frequently in the mathematical domain. Epistemological beliefs here are frequently nonavailing. Mathematics and mathematical knowledge is often perceived as something that only matters in the mathematics classroom; its relevance outside of the classroom is often unrecognized (Berry, 2002).

Muis (2004) offered the most comprehensive look at the nonavailing nature of epistemological beliefs in the mathematics classroom setting.

In general, when asked about the certainty of mathematical knowledge, students believe that knowledge is unchanging. The use and existence of mathematics proofs support this notion, and students believe the goal in mathematics problem solving is to find the right answer. Students also believe mathematics knowledge is passively handed to them by some authority figure, typically the teacher or the textbook author, and that they are incapable of learning mathematics through logic and reason. Moreover, they believe those who are capable of doing mathematics were born with a 'mathematical gene.'

Another common belief is that various components of mathematical knowledge are unrelated; the structure consists of isolated bits and pieces of information. Students do not typically perceive relationships among concepts and thus rely on the teacher and textbook to tell them what they need to know for each type of problem they encounter. Students do not believe they are
Increasing Classroom Motivation 17

capable of constructing mathematical knowledge and solving problems on their own. Finally, students typically believe that learning of mathematics should occur quickly, within 5 to 10 minutes. (p. 330)

The ramifications of such beliefs are natural and obvious, and studies confirm them. Student’s nonavailing beliefs negatively impact the amount of time they spend on problems, the strategies used to solve problems, and their justification as to what constitutes a correct answer. Epistemological beliefs even go so far as to undermine a student’s self-efficacy (Muis). It is interesting to note that the gifted and high achieving students in Hong and Aqui’s work (2004) perceived their own math ability and math self-efficacy to be high, and at the same time recognized the value in learning math.

Confronted with such evidence, the next logical step is to begin searching for ways to change these beliefs. Fortunately, research has shown that it is possible to influence such changes (Buehl & Alexander, 2005; Muis, 2004). Even more encouraging is Muis’ observation that beliefs about mathematics become more availing over time.

Origin of Epistemological Beliefs

In an effort to find ways to change beliefs, it is logical to target the causes for the development of these ideas, remembering Muis’ last observation regarding the changes in beliefs as a student progresses through school. The most often cited source for the direction of belief development is the classroom environment (Muis, 2004).

Formal education settings promote the apparent disconnectedness of mathematical concepts. With short classroom periods and contact time, students begin to believe learning should be quick. With limited time, teachers are forced to create teacher-centered forms of instruction where they merely demonstrate use of formulas and explain
how to solve problems. This leads to students viewing the teacher as the source of
knowledge. Finally, the teachers own epistemological beliefs can be linked to student
epistemological beliefs. The teacher’s own attitudes lead directly to student attitudes
(Muis, 2004).

It is important to recognize the flaws in this summary of research as pointed out
by Muis. Few researchers she summarized directly measured student beliefs. Most are
conclusions drawn from observation. Despite the inability of researchers to conclusively
define a cause-and-effect relationship between classroom experiences and student beliefs,
the empirical evidence did support such a conclusion.

Encouraged by Buehl, Alexander (2005) and Muis (2004) that student beliefs are
malleable (Muis) and can be changed, this review of the literature now begins to focus on
how to affect such changes in the classroom.

Changing Epistemological Beliefs

Muis (2004) offered three interrelated components on which to focus. The first
component she mentioned is time. In order to change beliefs from nonavailing to
availing, it is not necessary to enact a life long alteration in mathematics classrooms.
Interventions lasting for two months to a year are enough to induce change.

In order to help those changes become more permanent, she made her second
recommendation: discussing student beliefs with students. This “important catalyst”
(Muis, 2004, p. 362) revolves around making students aware of their own beliefs as the
much needed factor in making the new beliefs permanent. Not only should the students
be aware of their beliefs, but the teacher will benefit to. By making the teacher aware of
his or her own student's beliefs, classroom decisions can be made and individualized
attention given to generate the highest possible amount of motivation (Middleton, 1995).

The third interrelated component is the context in which the students learn. She
suggested activities and teaching that focused on justifying mathematics in meaningful
context, engaging student in collaboration and group activities to construct knowledge,
and providing them with time to learn (Muis, 2004).

These types of instructional designs are associated with beliefs that
mathematics is a way of thinking and that mathematical knowledge is
interrelated and related to other disciplines and other facets of life, is learned
over time with effort, is not innate, and can be constructed individually rather
than passively received from the authority—the teacher. (p. 363)

In order to create a learning environment such as the one described by Muis, a
different instruction approach must be employed. Winstead (2004) advocated a learner-
centered approach. This constructivist style of instruction seeks to emphasize the student
as the central figure in the classroom by focusing on their abilities to process information
through metacognition and the construction of their own knowledge. Such a teaching
style would directly combat the nonavailing belief that mathematical knowledge is
authority based as opposed to student constructed. Plus it would encourage the student to
create and develop comprehension of their own epistemological beliefs.

With more learning put in the students hands, the likelihood of those motivating
and enlightening 'Eureka' moments Sriraman (2005) mentioned have a higher probability
of occurrence. Through the introduction of surprise and inquiry via an open-ended
Increasing Classroom Motivation

approach to teaching, an aesthetic mathematical learning experience can be achieved. The results are increased motivation and interest (Gananidis, 2004).

The changes necessary to create the environment advocated through Muis' (2004) research need not even be so drastic. Turner and Patrick (2004) detailed very simple measures that are effective not only in changing student participation, but interest level as well. By monitoring the effect of teacher expectations, calling patterns, instructional support and motivational support via the participation of two students and the instructional techniques of different teachers, their research made the following conclusion: “how [teachers] communicate with students can have measurable effects on student work habits” (p. 1783). Indirectly the researchers were referencing the increasing of interest levels, the interest levels which Hong and Aqui (2004) referred to earlier when discussing the qualities possessed by gifted students.

Not only is the interaction between teacher and student important, but between students and students as well. Once again supporting Muis’ (2004) findings about shifting the epistemological belief about the source of knowledge from the teacher to the students, Blair (2004) called for greater amount of peer interaction. A classroom environment should provide opportunities for students to express their solutions and reasons to one another. With such a system in place, the learning dialogues moved from teacher-regulated to student-regulated.

One situation that is recommended to remain absent from the classroom is competition (Belcastro, 2004). The research suggested replacing this competition environment with a challenge environment. Such an environment consists of posing problems and creating contexts in which students can readily see the relationships
between mathematical ideas and raise their levels of self-efficacy (Turner & Meyer, 2004). The similarities to Vygotsky’s Zone of Proximal Development are readily seen (Muis, 2004). Based on Vygotsky’s ZPD theory, a student learns best when presented with problems which are challenging, but not so challenging that the student cannot successfully complete the task. At the same time, the problem cannot be so easy that the student is not challenged at all. The best problems are those which cause the student to develop and exercise their content skills and knowledge which supplements these as a result of the problem. A student that initially viewed the problem as ‘too difficult’, but then realized that the problem is solvable after ‘doing a little bit of hard work’ is one method for building that student’s self-efficacy.

**High Quality Learning Opportunities**

Despite all of the efforts made by a teacher to create the ideal learning environment for producing availing epistemological beliefs, the student must also come to the classroom prepared to take advantage of the changes. While the teacher can again directly or indirectly influence the tendency of the student to benefit from these changes, they cannot do it all. Some of the burden falls upon the students. Jones and Byrnes (2006) pointed out that while some evidence for improved achievement exist when the student was merely in a classroom taught with high-quality instruction; other evidence exists to the contrary. This research stated that students must actively take advantage of any high-quality learning opportunities. Regardless of the osmosis-like or osmosis-unlike nature of learning, one common thread passes through both camps: high quality instruction and learning opportunities.
The classroom environment is just one area on which to focus change for the betterment of learning beliefs. But as Marshall (2006) points out, it is also important to focus on the instruction of mathematical content as well. He references a study in which the percentage of lessons containing high-quality mathematical content from American teachers was zero. He endorses a fundamental change in teaching methods that focuses on understanding, while also improving mathematical skills. He closes with the point that despite the hardships and retooling of the typical mathematical teacher, “the rewards will be tremendous, and at last math class will be a class worth going to” (p. 363).

Summary

It appears from this research that beliefs and opportunities directly impact the motivational levels of students in the mathematics classroom. The primary individual who has the most impact on these items is the classroom teacher. The teacher has the ability to shape student beliefs in such a way that they positively influence the motivational levels, and therefore the engagement levels, of his or her students. This increase in motivation can be further supplemented with activities and programs that allow the student to practice their mathematical skills. The one most responsible for creating such a classroom environment is the teacher, but it is the students who have the most to gain from that environment.
Methodology

This methodology represents the initial research procedure planned upon in the design phase of the project. Alterations and modifications in the procedure are noted in the Results section of this paper.

Participants

The participants in this research study consisted of 50 students enrolled in the Math 3 Honors program at James A. Beneway High School in Ontario Center, New York. The demographics of these 50 students included 49 sophomores and one junior, with a total of 26 males and 24 females.

These students were subdivided into three classes that met for 80 minutes on the B- and D-days of Beneway High School’s four day block schedule. The second block class had 11 students, the third block had 17 students, and the fourth block had 22 students.

The classroom was primarily arranged into three rows of paired seating. This arrangement allowed for quick informal discussions, collaborative work and peer reinforcement of daily classroom lessons and practice items.

When larger student groups were required, student moved themselves into pre-assigned heterogeneous groups. The students rearranged their desks to form a table with their desktops. The assignment of students into these groups was done by the classroom teacher. Groups consisted of four to five students, with a mixture of ability levels. Based on academic scores, the students were divided into three levels. The top 25% of the class was designated as high achievers, the middle 50% were designated as average achievers, and the bottom 25% were designated as low achievers. Each of the three blocks was
separated in this fashion. A group was then formed by assigned one high achiever and
one low achiever with either two or three average achievers. Deviations from this
method only occurred when the groups contained a student matching that suggested there
would be behavioral or management issues. Under such circumstances, the teacher
altered the composition of the group accordingly.

A third classroom arrangement utilized was a pair of concentric circles. This
setup was used for the purpose of formal class discussions involving the entire class. The
inner circle initial consisted of five to eight students based upon class size. The
remaining students formed the outer circle. The decision to sit in either the inner or outer
circle was made by the students. Students sitting in the inner circle were expected to
openly contribute thoughts and comments on the discussion topic introduced for that day.
The teacher facilitated the discussions while students were the primary participants.
Students could offer their comments at any time providing they were polite and did not
interrupt; there was no need to raise their hand and wait to be called upon. Students in
the outer circle were expected to be active listeners and secondary participants. If they
had any thoughts or comments to contribute, they were instructed to raise their hand and
wait to be called upon. If their comment was singular and brief, they could choose to
stay in the outer circle. If they wanted to more actively participate in the discussion, they
were encouraged to join the inner circle by moving their desk inward. Students made the
ultimate decision of where they wanted to sit based on their own level of comfort and
aptitude to participate.
Instruments and Materials

There were two major factors that the classroom teacher enhanced as part of the research project. The first involved the epistemological beliefs held by the students, while the second involved student participation in a non-classroom problem solving experience.

Based on the findings of Muis (2004) students' beliefs are more likely to go from nonavailing to availing if they are made aware of their own beliefs. Consequently, class-wide discussions were held in class revolving around the following mathematical epistemological beliefs: (1) perspectives on the nature of mathematical knowledge, (2) justification of mathematical knowledge, (3) sources of mathematical knowledge, and (4) acquisition of mathematical knowledge.

Seaton and Carr (2005) concluded that ancillary educational programs designed to increase student engagement in the classroom were most effective when they aligned with the content being taught currently in the classroom. In order to comply with their research, the facilities at the Rochester Museum and Science Center (RMSC) were utilized.

The primary mathematical content area being taught was Conics and Conic Sections. This content area is part of the New York State Educational Curriculum under the Math, Science and Technology Learning Standards, specifically Learning Standard #3. It can be found in the 1999 Core Curriculum under MST #3 – Key Idea 4: Modeling and Multiple Representations, items 4D and 4L. It will not be a major component of the new 2005 New York State Core Curriculum, although elements can be found in the Geometry curriculum under the Geometry Content strands G.G.71-74 and in the New
York State Education Departments (NYSED) Suggested List of Mathematical Language for the Geometry curriculum. Regardless of the suggestions made by NYSED, the study of the conic sections should at minimum be a part of any school's pre-calculus program.

The emphasis of the unit of study for the Math 3 Honors course were the reflective properties of the conic sections and the ability to write the equations of the various conics in both standard and general forms.

Located at RMSC were various displays applying the reflective properties of the conic sections in both entertaining and practical ways. An exhibit at the museum called Creation Station contained most of these displays. Another display was located outside as part of a whispering gallery. The remaining displays could be found in the Strasenburgh Planetarium and in the Raceways exhibit, all located on the RMSC campus. See Appendix C for brochures from many of these exhibits.

In the classroom setting, models and manipulatives of the conic sections were used to allow hands-on interaction with the mathematical concepts being studied. These included cones that could be assembled and disassembled to show the effects of a plane slicing through at different angles. A string cage that could be twisted to produce a double cone that allowed a thin beam of light to pass through the strings simulated the same slicing plane. Sketches on Geometer's Sketchpad® were used to allow students to work with locus definitions of the conic sections. See Appendix D for screen shots from these labs.
Data Collection

Data was collected to determine if changes in the motivation level of the class lead to higher engagement levels of the students. Further analysis was performed to see if there was an impact on the achievement levels of the classes as a whole.

Engagement levels were measured throughout the two months of research. Additionally, achievement data and academic performance records were kept as part of the normal grading structure of the class.

Engagement data was collected from observations made in the classroom and also at RMSC. Classroom observations were made by the head of the math department at the high school and by a retired mathematics teacher and former lead mentor for the same high school. RMSC observations were made by the chaperones accompanying the students on the trip. Data collection detailed the frequency of participation, the amount of interaction between students and teacher, and between students themselves. The types of interactions were monitored and categorized as mathematical or non-mathematical in nature. Finally, records of homework completion were kept to track out of class engagement.

To measure any changes in achievement levels as a result of the instructional modifications, various grade comparisons were made. The three classes used had their grades analyzed and their class averages and class medians found. This was then compared to similar analyses done on the classes’ grades in previously completed units. To enable comparisons to classes that had not experienced the modification but still completed a unit on conic sections, the current classes’ grades were compared to two
prior Math 3 Honors classes that had completed the class at James A. Beneway High School.

**Procedure**

The opening day of the research period began with a discussion about epistemological beliefs. Details including what they are and how they affect a student’s behaviors were shared by the teacher. Students were also encouraged to comment on their own observations regarding their own beliefs. The discussion then turned to specifically mathematical epistemological beliefs. Students were given an opportunity to share their personal beliefs.

While studying the locus definitions of the conic sections, class-wide discussions were held regarding their beliefs pertaining to the justifications of mathematical knowledge. Students were also shown a variety of sketches produced with Key Curriculum Press’ Geometer’s Sketchpad software. Sample screen captures of some of these demonstrations can be found in Appendix D.

The creation of these sketches relies upon the Locus feature of software along with the Parametric Coloring feature. By generating situations that model the locus definitions of the four conic sections, the screen can be painted with color that creates a dramatic portrait of each section.

The use of this software and the images produced were utilized to increase student motivation toward the study of the mathematical concepts and techniques involved in their creation. They also provided a foundation for further mathematical discussions.
With these discussions as a basis for further development of the concepts in the Conics unit, the course started to cover the algebraic representations of the conic sections. Constant references to the epistemological discussions on justifications were made.

Throughout the unit, the conic definitions given by Apollonius were studied. Students examined a variety of models simulating a mathematical plane passing through either a cone or double cone. Students constructed their own definitions of the conic sections based on their observations. A class-wide discussion was then held regarding their epistemological beliefs about the sources of mathematical knowledge. Immediately afterward, the historical nature of the conic sections was discussed. This included highlights from the works of Apollonius and Hypatia of Alexandria. This also served as an introduction to the reflective properties of the conic sections.

A final discussion was held to explore student beliefs about the acquisition of mathematical knowledge. The discussion centered upon the ideas of being naturally talented in mathematics and gaining talent through practicing mathematical skills. This segued into practicing the application of conics. Given various situations, students derived equations of the conics described. This is also when the field trip to RMSC was introduced.

In order to better prepare for the RMSC problem solving tasks, and to boost student self-efficacy, a Geometer’s Sketchpad lab activity was done to practice writing equations based on measurements made by the students. The prior practice problems were modeled after questions from the New York State Math B Exam in which all of the necessary information is already provided to you. Students utilized the measurement
capabilities of Geometer's Sketchpad and the slider components to find an equation that could model both the arches and the driving surface itself for the bridge.

As part of the preparation for the trip, students participated in a mathematical modeling lab activity in the classroom. Using the laptops to access the Internet, The Geometer's Sketchpad, Microsoft® Word, and Microsoft® Equation Editor, the students were instructed to research the Troup Howell Bridge in Rochester, New York. The primary site used was www.trouphowellbridge.com, maintained by the New York State Department of Transportation. At this site, students were able to view pictures of the bridge and find information relevant for creating a mathematical model of the arches of the bridge.

After viewing the pictures of the bridge, a class discussion was held to determine which type of was best represented by the arches. The decision made in each of the three classes was that a parabola best modeled the arches. The students were then given two Geometer's Sketchpad files. These files each contained an image of the bridge.

On the first image, students were asked to generate two modeling equations. One equation was to be created using numerical data found on the website to generate a quadratic equation in standard form, \( y = ax^2 + bx + c \). The information used and the steps involved in the computation were to be electronically documented on Word with the utilization of Equation Editor. The graph was then to be plotted on the Sketchpad file. The second equation was to make use of the Sketchpad sliders incorporated into the file. Similar to a lab done in an earlier unit of study, students were to incorporate the measurements from the sliders into a function that could then be plotted. For this model, students were told to use the vertex form of a parabola, \( y = a(x-h)^2 + k \); this was the
primary form used during the Conic Sections unit. Finally, a comment was to be left on the sketch discussing the student's opinion about which model best fit the arch of the bridge.

The second part of the lab was a chance to explore the support cables placed on the bridge. As part of the discussion about which type of conic to use for modeling purposes, observation were made that the cables all seemed to converge in at a single point. Students were reminded about the reflective properties of the conic sections, and they were encouraged to continue their debate using more mathematical terminology. It should be expected that the students will conclude that a parabola would best model the bridge.

The image on the pre-created file was of another photo of the bridge showing a more detailed view of the cables. Students where instructed to use the Parabola – by Point and Directrix in the Custom Tools folder of Geometer's Sketchpad to create a curve that best matched this view of the Troup Howell Bridge. Then using the Ray Tool, create rays on top of the cables to see if they had a single point of concurrency. If so, was it at the focus created earlier? Multiple approaches to creating the model were encouraged. Students were instructed to save their work electronically so that future students would be able to see samples of their work. See Appendix E for sample student work from this preparation lab.

Prior to arriving at the RMSC, the students were divided up in to heterogeneous groups of five based on each class using a method similar to the one described earlier. Adjustments were made due to class sizes. Each group was assigned two displays in
which to explore and examine for the purpose of writing an equation to model the behavior of the display.

Appendix C contains reproductions of exhibit brochures from The Rochester Museum and Science Center (RMSC). They are provided to allow the reader of this paper a glimpse of the models used for this research paper. Not all of all the exhibits found in these brochures were studied by the students, nor were these the only exhibits included in the activity. Of those shown, students worked with “Parabolas” in the Creation Station – Mirrors & Illusions exhibit; “Catenary Arch” and “Round Arch Bridge” in the Creation Station – Forces & Structures exhibit; “Motion Dish” in the Raceways – Newton’s Laws of Motion exhibit.

Each group had a ninety minute time limit in which to generate the modeling equation. After the ninety minutes was up, the groups moved to a second display to complete the same equation writing task. After the that time, the half of the groups were given a break for lunch while the other half went to the RMSC computer lab to prepare presentations. After an hour, the halves switched allowing the first lunch groups to go to the lab and the others to go to the cafeteria.

After that time, the groups presented their findings to one another. Since every display was explored by two different groups, students were encouraged to discuss any disparities in their results. A summarizing discussion was then held by the teacher to explore the nature of the activities and allow for student commentary on the days events.
Results

Details regarding data collected and adjustments made to the originally documented methodology are given in the Results section.

Data Collected

As part of this research project, data was collected for the purpose of measuring motivation and engagement levels of students. Two sets of data were collected for this endeavor: classroom observations made by teaching professionals and homework grades from before and after the start of this research. Additionally, to measure any possible performance and achievement effects as a result from this research, unit test grades for the unit of study were compared.

The first data set presented represents qualitative observational data made by teaching professionals regarding the motivation and interest levels of students. Using a simple ‘+’, ‘0’, ‘-’ scoring approach, the teacher watched student’s behavior during various parts of a lesson. A ‘+’ signified that the student was actively participating in the current classroom activity. A ‘0’ score represented that the student was mildly engaged in the lesson, whereas a ‘-’ score meant the student was disinterested in the lesson and not focused or motivated toward the activity. During the lesson, scores were taken during the following four lesson time periods: 1) warm-up; 2) going over homework; 3) lesson material and notes; 4) class practice problems. Observational data was collected during two lessons. The first lesson data was collected from took place prior to the epistemological beliefs discussion. The second set was collected from a lesson during the Conics Sections unit; the unit of study immediately following the beliefs discussion.

Refer to Tables 1 and 2 for the results for each class and the entire Math 3H population.
### Table 1 - Motivation Scores

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The second data set presented represents quantitative observational data made by teaching professionals regarding the engagement of the students by counting responses. Responses included instances where students answer teacher prompts, and instances that were student initiated and mathematically relevant. Examples would include students asking their own questions, or sharing an observation. The collection method was a simple tally method. Average, median and quartile number of responses per student has been calculated. Refer to Tables 3 and 4 for the results for each class and the entire Math 3H population.

As a secondary measure of engagement, homework grades are analyzed. The Daily Homework grade for each student is measured during the first two semesters, which occur prior to the start of the study. These are compared to Daily Homework grades for the third semester during which the research took place. The Daily Homework grade is an assessment made by the teacher pertaining to how complete a student’s homework is. Accuracy and correctness is not assessed. A score from 0 to 10 points is awarded for each assignment and then a percentage is calculated to represent the semester grade. Refer to Tables 5, 6 and 7 for the results for each class and the entire Math 3H population.

To measure any changes in performance or achievement, unit test scores from the 2006-2007 Math 3H classes were compared to the 2004-2005, and 2005-2006 Math 3H classes. The Conic Sections unit test was modeled after the New York State Math B exam. It consisted of 12 Part I multiple choice questions, 4 Part II short-answer questions, 3 Part III short-answer questions, and 1 Part IV longer-response question. This format is consistent with the other unit tests given in the Math 3H program.
### Table 3 - Responses

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The grades for the entire Math 3H population for each year were analyzed as a whole. There is no break out by class periods for any of the years. Table 8 shows the results for each year.

The final piece of data collected is quantitative data analyzing the 2006-2007 Math 3H Conic Sections unit test grades as compared to all other unit test grades covered thus far in the curriculum. Additionally, an analysis of each of the other four units has been included (Unit 1 – Analytic Geometry; Unit 2 – Functions; Unit 3 – Algebra Topics; Unit 4 – Radicals and Complex Numbers; Unit 5 – Conic Sections). Tables 9 and 10 refer to the comparison of Units 1 – 4 with Unit 5, while Tables 11 through 14 represents the results for each unit of study.
### Table 8 - Historical Math 3H Grades

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<td>76</td>
</tr>
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<td>72</td>
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<td>Maximum</td>
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<td>Mean</td>
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### Table 11 - Math 3H Unit Test Grades: 2006-2007

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<td>Total</td>
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<td>71</td>
<td>57</td>
<td>57</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>72</td>
<td>77</td>
<td>77.25</td>
<td>77</td>
</tr>
<tr>
<td>Median</td>
<td>80</td>
<td>80</td>
<td>81</td>
<td>80</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>86</td>
<td>86</td>
<td>90.5</td>
<td>87</td>
</tr>
<tr>
<td>Maximum</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mean</td>
<td>79.00</td>
<td>82.65</td>
<td>83.05</td>
<td>82.02</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>11.45</td>
<td>7.89</td>
<td>9.55</td>
<td>9.43</td>
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</table>

### Table 13 - Math 3H Unit Test Grades: 2006-2007

<table>
<thead>
<tr>
<th>Unit 3</th>
<th>2BD</th>
<th>3BD</th>
<th>4BD</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Students</td>
<td>11</td>
<td>17</td>
<td>22</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>900</td>
<td>1362</td>
<td>1732</td>
<td>3994</td>
</tr>
<tr>
<td>Minimum</td>
<td>73</td>
<td>50</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>75</td>
<td>75</td>
<td>75.75</td>
<td>75</td>
</tr>
<tr>
<td>Median</td>
<td>82</td>
<td>85</td>
<td>81.5</td>
<td>82</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>85.5</td>
<td>91</td>
<td>85</td>
<td>87</td>
</tr>
<tr>
<td>Maximum</td>
<td>95</td>
<td>96</td>
<td>94</td>
<td>96</td>
</tr>
<tr>
<td>Mean</td>
<td>81.82</td>
<td>80.12</td>
<td>78.73</td>
<td>79.88</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>7.36</td>
<td>13.90</td>
<td>11.72</td>
<td>11.60</td>
</tr>
</tbody>
</table>

### Table 14 - Math 3H Unit Test Grades: 2006-2007

<table>
<thead>
<tr>
<th>Unit 4</th>
<th>2BD</th>
<th>3BD</th>
<th>4BD</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Students</td>
<td>11</td>
<td>17</td>
<td>21</td>
<td>49</td>
</tr>
<tr>
<td>Total</td>
<td>842</td>
<td>1246</td>
<td>1629</td>
<td>3717</td>
</tr>
<tr>
<td>Minimum</td>
<td>63</td>
<td>0</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>72</td>
<td>67</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Median</td>
<td>76</td>
<td>80</td>
<td>80</td>
<td>78</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>81</td>
<td>86</td>
<td>86</td>
<td>86</td>
</tr>
<tr>
<td>Maximum</td>
<td>92</td>
<td>92</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Mean</td>
<td>76.55</td>
<td>73.29</td>
<td>77.57</td>
<td>75.86</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>8.94</td>
<td>21.32</td>
<td>10.93</td>
<td>14.89</td>
</tr>
</tbody>
</table>
Adjustments to Research

Over the course of the research, two additional students were dropped from the Math 3H program. Both occurred in the 4BD class. This changed the number of participants in that class to 20, leaving the final number of Math 3H students at 48. The demographics of this new set of students included 48 sophomores with a total of 25 females and 23 males. The changes are reflected in the Unit test grades reported in the Data Collection subsection above.

A second adjustment made involved the reporting of data from the Motivation scores and Response scores. One the day of the second lesson observation, seven Math 3H students were absent from their afternoon classes due to a field trip. This included four students from the 3BD class and three students from the 4BD class. Due to this alteration, Motivation and Response scores were removed from the set of data collected from observations taken in the first lesson.

The third adjustment involved the trip to the Rochester Science and Museum Center. School was cancelled on four separate days during the course of the research project, three of which involved the Math 3H classes meeting on B- and D- days. This drastically impacted the timeline for the entire course and would strain the ability of the course to cover all the material necessary for the June Math B Regents Exam. Due to these events and delays occurring in the first half of the year, the trip to RMSC was cancelled. Replacing it was group project involving the modeling methods and techniques similar to those done with the Troup-Howell Bridge lab. At the time of this writing, the projects have not been completed.
Discussion

The Discussion section is a reflection back on the study and its results. Interpretation of the results has been made in light of the literature reviewed and the current theory base regarding epistemological changes in the mathematics classroom.

Summary

References to class size will be made throughout this section. Figure 1 graphically depicts the relative class sizes of the Math 3H – 2BD, 3BD, and 4BD classes. As previously noted, a number of students in the 3BD class and 4BD class had to miss the second lesson due to other school activities. The population of students involved in the response data is shown in Figure 2.

The Motivation Scores from the first and second lessons are shown in Figures 3 and 4. These results point to a greater amount of engagement by students in the second lesson after the epistemological discussion. Other observations from this data include a roughly equal change by all classes in each scoring category; each class had about seven more ‘+’ scores, about seven less ‘0’ scores and a net change of zero ‘−’ scores. The ‘0’ scores were the most affected. This subset of students could be those that were initial too unsure of their own skills and not wanting to take a personal risk by offering a chance to be called upon. After the discussion, it could be hypothesized that these students no longer felt that sense of risk or perhaps felt more confident in their answers. The least engaged students, those getting a ‘−’ score, were not affected by the discussion.
Number of Students in Math 3H at the End of Research Project

<table>
<thead>
<tr>
<th>Class Period</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>2BD</td>
<td>11</td>
</tr>
<tr>
<td>3BD</td>
<td>17</td>
</tr>
<tr>
<td>4BD</td>
<td>21</td>
</tr>
</tbody>
</table>

Figure 1 - Number of Students in Math 3H at the End of Research Project
Increasing Classroom Motivation

Figure 2 - Number of Students Involved in the Collection of Response Data
Figure 3 - Motivation Scores from 1st Lesson
Motivation Scores from 2nd Lesson

<table>
<thead>
<tr>
<th>Population</th>
<th>2BD</th>
<th>3BD</th>
<th>4BD</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>5</td>
<td>13</td>
<td>27</td>
</tr>
<tr>
<td>+</td>
<td>34</td>
<td>43</td>
<td>50</td>
<td>127</td>
</tr>
</tbody>
</table>

Figure 4 - Motivation Scores from 2nd Lesson
Figure 5 summarizes the Response data collected. The smallest class, 2BD, showed a much greater change. 3BD and 4BD classes were affected by the field trip; some of the stronger and more vocal students where among those missing. However, of particular concern is the observed down trend in the number of responses given as the school day progresses from 2nd block to 4th block. Figures 6 and 7 help to make this observation more clear.

This trend could be attributed to three things. The first being the time of day, the second the instructor, and/or the third being the composition of the two classes. It is reasonable to assume that it is a combination of the three items that contributed to the decreasing number of responses.

The time of day issue refers to the natural tendency for students and teachers to lose their energy as the day progresses. At the start of the day, everyone is fully charged and attentive. At the end of the day, students and teachers can both become fatigued and as a result may put forth less effort.

The instructor as the cause could be due to the experience gained after each time the lesson is taught. Since the same lessons were taught three 80 minute blocks in a row, it is natural to expect that the instructor would modify and improve the lesson. The instructor would anticipate problems and correct difficulties in the learning process of the students based on events in earlier classes. Basically the teacher could have become more efficient in their instruction and eliminated the chance for students to be confused by the material. Without the confusion in the students, the necessity to interact is lessened.
Figure 5 - Response Data
Figure 6 - Response Data for the First Lesson
Figure 7 - Response Data for the Second Lesson
The third and most likely the largest contributor to the problem is the composition of the classes. Looking at the unit test scores for the first three units of study helped to clarify the observations to follow; the unit test scores data can be found in Table 9.

The 2BD class had the lowest class average and class median, but the smallest standard deviation. This implied that while they were weaker students as compared to the other classes, at least they were all in the same boat, ability wise. There were no standout students that took over the class, nor were there extremely weak students that bogged the class down. Due to this homogenous composition, the effects of the changes in epistemological beliefs could be seen as impacting the entire class as opposed to just individuals. Evidence for this conclusion came from the tremendous growth in the number of responses by this class. A jump of 39 total responses, an increase of four in the median number of responses and a 3.5 gain in the average number of responses shows the increased engagement level of these students. Figure 8 illustrates the gains by the 2BD class.

The 3BD class also showed some improvement through their response data, although not as much as the 2BD class. Based on the unit grades for this block of students, this is the mathematically strongest group, but also the most diverse in regards to ability level. Based on the higher standard deviation of test scores, the class is comprised of a very heterogeneous mixture of high and low ability levels. Based on the amount of growth in the minimum number of responses and the 1st quartile number of responses, as compared to the median, 3rd quartile and maximum number of responses, it can be assumed that the weaker level students were more impacted by the epistemological discussions than the upper ability level students. This assumption is
based on the belief that the weaker students participate in class less, and stronger students do the opposite. The discussions on epistemological beliefs and the emphasis on real-world applications throughout the Conic Sections unit of study seemed to hit home with this set of students. Figure 9 summarizes the response data for the 3BD class.

The 4BD class showed a decrease in the amount of responses when comparing response data from prior to the initial epistemological discussion versus after. Academically, this class is the middle ability level based on their unit test scores. Their standard deviation indicates that this is a more homogenous mix of students than 3BD, but more heterogeneous mix than 2BD. The reason for this classes overall lack of participation could be attributed to a number of factors. The biggest being that fact that this class has had the greatest number of students dropped from the class; two students dropped during the course of the research. At the beginning of the year, there were 32 students in the class and at the time of the writing of this paper, there were only 20. This undoubtedly has had a psychological impact on the class and has hurt the overall mood of the class. There is almost a fear in the classroom coupled with a tremendous amount of stress and dread. It is possible that the introduction of a new stimulus in the classroom (i.e. the research project) negatively affected the motivation and engagement levels of the class. If any sense of comfort had been created in the class due to familiarity and expectations with the class structure and teaching style, this change in procedure could have upset this fragile balance. Figure 10 shows the response data for the 4BD class.
Figure 8 - Response Data for Period 2BD
Figure 9 - Response Data for Period 3BD
Figure 10 - Response Data for Period 4BD
Overall, the collected data indicates that the epistemological discussion can best change motivation and engagement in smaller class with a more homogeneous mix of ability levels. The largest set of students most affected seemed to be in the population of students that were limitedly involved in the classroom ('0' motivation scores) and the mathematically weaker students.

Looking at the academic data to see the effects of the motivational changes is not as encouraging as expected. The homework data was collected to see if the class discussions translated into motivation to complete more homework. Referring to Tables 5 – 7, the data indicates that there was a decrease in the homework averages for all classes. The homework averages reinforce the conclusions drawn from the unit test data (Table 9) about the ability levels of the classes. 2BD is the most homogeneous group and 3BD is the most heterogeneous group. Heartening evidence can be gleaned from the 2BD class’ standard deviation. The statistic decreased indicating more consistency in homework scores. This would confirm the conclusion drawn earlier based on the Motivation and Response data. Further, in the 3BD class, the median and 3rd quartile statistics increased from the second marking period to the third. The average and standard deviation were greatly affected by a few students on the lower end of the scores; hence the decrease in minimum and 1st quartile scores. This indicates that some students were affected by the epistemological discussions for the positive, while others saw the discussions as an opportunity to not complete their homework. Perhaps some of this may be attributed to the fact that there was an increased amount of extra teacher and student-lead classroom discussion time that was not directly related to the curriculum. Students
may have gambled that class would be a talking session instead of a working session, so they could postpone their work.

The comparison data for the Conic Section's Unit Test Grades does not provide evidence of increase achievement from increased motivation. Figure 11 illustrates the historical data for the Math 3H program at James A. Beneway High School based upon Table 8.

The trend for this particular unit exam has been downward for the three years that the Math 3H program has been in existence at Wayne. The most relevant data here could be that the population of honors students has grown. Keeping in mind that the program started the 2006-2007 school year with a total enrollment of 63 students, the program has become abnormally large for a school of Wayne's size as compared to previous years. The effects of the larger population can only be implied to have negatively impacted the initial pool of talent by including a more heterogeneous group of students. With student placed in a course that they are improperly prepared for, or a class that they have not maintained the requisite level of achievement, they are less likely to succeed. In order to have the best chance of success, a student needs to be properly placed. Such a statement is the observation of the researcher; it is outside of the scope of this research paper and not backed by any researched literature.

When comparing the unit test grades of the Conics Sections unit exam to the previous four units of study, it is readily observed that the grades were lower in the unit utilized in the research than in those units outside of the research time frame, except for the 2BD class. The Figure 12 summarizes the average scores from Tables 9 - 14.
Figure 11 - Historical Math 3H Grades for the Conic Sections Unit Test
Figure 12 - Unit Test Averages for Math 3H 2006-2007
While each class had a lower average for the fifth unit than for the combined average of the previous four units, it is interesting to see the statistics for the 2BD class. The 3BD class had their lowest exam grade in Unit 5, while 4BD had nearly their lowest. If the amount of grade drop from the average of Units 1 – 4 versus the Unit 5 grade, the 2BD class had the smallest change in score as compared to the other classes.

This result is not the unbelievable when it is pointed out that this set of students saw the greatest increase in motivation and response scores. This is the most significant result that supports the conclusion that epistemological beliefs can generate improved academic performance.

It should also be noted that the Conic Sections unit is one of the hardest units studied in the NYS Math B curriculum. Further, based on scoring data in the Conics category form the Monroe County Math League (a series of mathematics competitions held between schools throughout the Rochester area that hosted over fourteen hundred competing math students), this is one of the harder subjects studied in high school math. The MCML scores for this topic are the lowest of all thirty-six topics competed in. Even the best math students in all the competing schools struggle with this mathematical topic.

It may be possible to argue that due the increased focus on epistemology, this unit’s test grades for this particular group of weaker Math 3H students were not as bad as they could have been. However, no data was collected to support this conclusion. But to summarize this discussion, there was some evidence to support the conclusion that student performance was increased by higher levels of student motivation brought about by epistemological discussion revolving around mathematical knowledge.
Hong and Aqui’s (2004) research pertaining to the difference between gifted students and their non-gifted peers.

When attempting to put forth plans for the purpose of changing student held beliefs, this research points to mixed results. Muis (2004) maintained that the greatest chance of enacting change is to make sure the students themselves are aware of their beliefs. When this happens, here research says that the changes have a greater chance of becoming permanent. The 2BD class had the best self-directed conversations about beliefs. The data shows that this class had the highest Motivation and Response scores, confirming Muis’ research. However, Turner and Patrick (2004) published research that said changes in calling patterns and instructional support would increase motivation and interest levels. The data pertaining to the 4BD class does not support this. No amount of changes in student selection, volunteering or implicating could get these students to respond; the poor Motivation and Response scores strengthen the contradiction of Turner and Patrick’s research.

This same point made about the 4BD class supported conclusions made by Jones and Byrnes (2006). Their research referenced the need for students to take advantage of high quality learning opportunities and opportunities to participate in order to benefit from them. Since the 4BD class had such low Motivation and Response scores, Jones and Byrnes would not be surprised to see that they had such poor results on their unit tests. As a bright spot, they would point out that the exam results for 2BD were predictable based on the Motivation data and Response data collected on that class. The conclusion from this research for this expectation is a confirmation of Jones and Byrnes’ research.
Further references to the research from the Literature Review can be found throughout the Insight and Future Research subsections, and the Conclusion section of this paper.

*Insights*

While performing this research, there was both a sense of confirmation and a sense of confusion. Both of these stem from the results of the research and the review of the literature. The confirmation comes from supporting some of teacher hold beliefs while the other comes from evidence contradicting some of the expectations for the research.

It has been the experience of the researcher’s own personal practice of teaching, to always have understood the power of potential and the importance of motivation. A motivated student is more likely to reach their full potential. A wealth of personal and practical experience in and out of the classroom has backed my conclusion. Going into this research project, there was a motivation to find researched evidence to solidify these observations about the importance of motivation. Happening on an epistemological approach to altering motivation levels simply was good luck, and the most intriguing of all the approaches initially listed.

If people have a bad attitude about something, it seems obvious that they are unlikely to put forth their best effort. Note that this does not mean that they will not produce a superior product or necessarily do a bad job. To re-emphasize this point, the individual simply will not devote the full extent of their own talent and skills. Of course a good attitude and the use of all of one's talents does not necessarily guarantee a superior
product. This set of axioms holds for all aspect of life, including life in the mathematics classroom.

The idea of altering someone's attitude about math along with their perception of the purpose and utility of math class itself through self inspection and reflection on the beliefs about math and mathematical knowledge seemed like common sense. Reading about nonavailing beliefs regarding the purpose of math class being to 'get right answers' and 'to use the procedures taught by the teacher' were all backed up by personal experience. It could be seen in the students participating in this research and also seen it in the students of other teachers. This poor attitude was a detriment to their success. This research confirmed these initial observations.

The causes for these attitudes are often the stereotypes associated with mathematics. Not just the ones held by students, but the ones held by teachers. The extent to which teacher beliefs and attitudes impact and create student beliefs and attitudes was one of the biggest insights for me as a part of this research. Most math teachers believe it is their job to teach students to solve problems. This in turn feeds the student misconception of mathematics being a class in which 'getting the answer' and 'doing what my teacher told me to do' are the expectations of the class. Then when the end of the year exam comes out and the teacher says 'my students are in trouble because I didn't teach that problem,' the teacher should not act surprised when their students do poorly on that question. The nonavailing belief held by the teacher is that mathematics, and also their job as a math teacher, is to teach kids to solve problems. In reality, their job is to teach kids to problem solve.
The key difference is that the problem solving is coupled with the notion that the student can solve problems that they have never seen before. If the students hold the nonavailing belief that they are only capable of doing what their teacher told them to do (and sometimes their grades tell them that they cannot even do that) of course their motivation to tackle an unfamiliar problem will dwindle down to nothing, exactly as Muis (2004) described. By creating the availing belief that that mathematics is a tool for teaching problem solving skills, students will not generate the poor attitudes toward math.

With previous experience in Problem Based Learning and new research supporting the idea of beliefs pertaining to mathematics affecting student motivation, the researcher implemented a new practice in the classroom. As part of the attempts to change student belief patterns, the researcher introduced an Energizer question that students work on at the beginning of class. The question is a random problem that rarely pertains directly to the mathematical techniques being taught that day in class, but certainly pertains to some problem solving skill that will be utilized that day. It is not meant to be a competition, but more of the ‘challenge environment’ Belcastro (2004) and Turner & Meyer (2004) suggested be created that makes use of Vygotsky’s Zone of Proximal Development educational theory.

The motivation level in the classroom immediately rises when the students work on this problem, as evidenced by Motivation data collected. After an appropriate amount of time, the students are asked who has an answer to the question, but they are not asked what their answer is. The students are asked to share their problem solving strategy with the class. The students get reinforcement that their view of the problem and their technique for solving it are valid without fear that they didn’t do it the way the instructor
did it. Further, as more and more strategies are shared, students begin to realize that there are always multiple ways to solve problems. Eventually a student hears that someone else tried to solve it the same way they did; yet another reinforcement. Even the instructor benefits because they hear new and original ways that perhaps they had not thought of, and sometimes they are better.

Additionally, the Energizer question promotes mathematically questioning and dialogue. Students begin to believe that it is okay to talk about mathematics, and to ask mathematical questions. It was this type of questioning which lead to an exciting teachable moment during one of the lessons on ellipses in the Conic unit. A student discovered the concept of eccentricity and even the formula for describing it as the ratio of the distance from the center to the focus over the distance from the center to the vertex. The questions asked lead other students to ask more questions and make their own observations. Students created their own mathematics, which the instructor promptly told them they were doing. The awareness of the beliefs of these enabled me to take advantage of this moment in for the purpose of strengthening my student’s mathematical knowledge base and the motivational level of the class, just as Muis (2004) described.

Not surprisingly, this conversation took place in the 2BD class.

To further the creation of availing beliefs about mathematics and mathematical knowledge, the instructor utilized a highly application rich approach to the Conic Sections unit. To see mathematics as something outside of the classroom walls, students worked on modeling the behavior of natural phenomenon and man-made structures with conic sections. These were the high quality learning opportunities. Students looked at planetary orbits and the paths of comets, desalination machines, GPS and LORAN
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systems, no-lose pool tables, ultrasonic lithotripsy for treating kidney stones, bridges and
tunnels, 360-degree camera lenses, sonic booms, sideline microphones from the Super
Bowl, and even the light given off by table lamps in their own homes. Another student in
the 2BD class remarked that he was using conics to do some of his CAD work in
Technology class. The students had a wonderful experience modeling the Troup Howell
Bridge with Geometer’s Sketch pad. The utility of mathematics is a tremendous availing
belief to have. It is a shame that they were unable to travel to the Rochester Museum and
Science Center to practice some real-live modeling scenarios; it would have been exciting
to see what the students would have done. It will be on the list of activities for next year.

All of this fit the preconceived expectations and results the researcher should get
at the start of the project. At the beginning of this insight section, it was mentioned that
there was also surprise by some contradictions. Initially, there was a hypothesis that the
students who would benefit the most from these discussions would be the upper ability
level students. It was envisioned that they would become the students that Sriraman
(2005) talked about. But in fact it was the lower ability students that thrived in this
epistemological environment, as Schwartz (2006) talked about. The mathematically
weakest class 2BD produced the most significant results, particularly in their self-
efficacy. The instructor put in place an after school study session for the upcoming Math
B exam. It was exclusively populated by the twelve weakest students; another
homogenous group of students. It has become the instructor’s favorite ‘class’ due to the
motivation these students bring with them and their engagement level with the material
they are covering. It is believed that these elevated levels are due to the discussions,
Energizers, and applications done in class. The students have said so.
In retrospect, perhaps there should not be so much surprise. The same thing happened in the researcher's Project Based Learning experience. The weaker students were much stronger problem solvers than the students with the top GPAs. It would have been interesting to have found out the types of beliefs held by the participants in the PBL experience. It would not be too unbelievable to find that those top students held a number of nonavailing beliefs about mathematics, particularly about the purpose and point of mathematical studies.

**Future Research**

At the conclusion of this research, there were a number of things that could have done differently for the purpose of collecting more types of data, and for the purpose of monitoring long term effects.

The first research change to consider is the unit of study to conduct the research in. The Conic Sections are difficult enough without the added dialogue about beliefs and knowledge. Any gains made in changing attitudes could be wiped out by discouraging grades. Consider doing such a study in an easier unit, like functions. It offers a lot of chances to generate discussions about the creation of mathematics and it's relatively new concepts. Functions are a utility that can be constantly revisited throughout the year in nearly every topic studied in the Math B curriculum. This opportunity to show the connectivity of mathematical ideas would help counter some of the nonavailing beliefs measured by Muis (2004) pertaining to the isolation of mathematical knowledge. An instructor could draw upon the reconstructed belief systems of their students to help get through some of the more difficult topics later in the year. For the students, it could become something akin to the conditional training Pavlov's dogs went through.
This leads directly to the second research change to implement. This study should be done at the beginning of the school year. Students are more perceptive to new things at this time since it is the start of a new school year; everything is new! Coupled with a different unit of study, this could allow for a lasting impact on the students. It also plays to Muis (2004) research that says interventions of two months to a year are necessary for enacting lasting changes in student beliefs.

A third change in the research would be to collect data specifically addressing the beliefs of the students. Most of the data was collected for the purpose of measuring motivation, but nothing was collected for the purpose of measuring beliefs. All that was able to be gathered was observational data from interactions with students as individuals and as a class. From which to conclusions about student beliefs were drawn. This research suffers from the same lack of evidence that many of the research studies Muis (2004) summarized also suffered from. If quantitative data could be collected measuring the availing and nonavailing beliefs student held both before and after the discussions, this would enable stronger conclusions to be drawn regarding the relationship between epistemological mindsets and motivation in students.
Conclusion

The benefits from this research and the conclusions drawn are immediately applicable to any classroom instructor teaching any subject. The basis of this research was that students enter into a course of study with preconceived notions about the material being covered, the instructor teaching the material, and then authenticity of the material itself. Elements of these beliefs comprise a student’s epistemological mindset about the knowledge at hand.

It was not so hard to realize that not all of these beliefs are beneficial; many are detrimental to the student’s success in the course. If the student carries around too many of these nonavailing beliefs, the idea that the student cannot and will not be successful becomes a self-fulfilling prophecy.

When the beliefs are in fact beneficial to the student, these availing beliefs create motivational impulses that cause the student to engage in the learning process more effectively and efficiently. This in turn could lead to increase achievement and performance levels.

Recognition that the beliefs of a student can be changed became a powerful stimulus for the teacher. In the never-ending effort to improve student learning, a teacher must call upon their ability to change the epistemology of their students. But the teacher should also recognize that their own belief system is in fact the greatest influence on the beliefs of the students.

Nonavailing beliefs can be grown within the students just as readily as availing. The perceptions and attitudes espoused by teacher are assimilated by the students, both
good and bad. Teachers may be the primary cause of student held epistemological beliefs, but they are also the primary catalyst for changing those beliefs.
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classroom learning activities. Teachers College Record, 106(9), 1759-85.

Appendix A: Administration Letter

Dear Mr. Siracuse,

This letter is to formally notify you of my intention to conduct graduate research in my Math 3H classes during the second and third marking period.

The activity is part of the GMST 640 course I am taking at St. John Fisher College under Dr. Diane Barrett. This course is part of the year-long research component of the Graduate Math Science and Technology program at the college.

To revisit the preliminary conversation we have already had, the research will consist of monitoring the motivation levels of the students in the classes through observations. Further, a field trip to the Rochester Museum and Science Center (RMSC) will be conducted to give the students a chance to apply their mathematical skills in a non-classroom setting.

As a brief elaboration, I hope to impact the motivation levels of my students through an awareness of student’s previously held beliefs regarding mathematics and mathematical knowledge. By making them aware of these beliefs, the students can begin to see the effects of these subconscious notions on their conscious actions. By increasing their motivation, I hope to increase their engagement. The increased in engagement should lead to an exciting learning opportunity at RMSC. Finally, the higher engagement level should lead to higher achievement on the part of the student.

If you have any questions for me, please contact me. I will be sure to inform you of any additional research requirements that I will need to fulfill. I would also like to keep you informed of the progress that our class is making during my research experience, hoping that you might drop by to see what we are up to.

James Tiffin Jr.
HS Math Teacher
James A. Beneway HS
Appendix B: Parent Letter

To the parents of ___________,

This letter is to formally notify you of my intention to conduct graduate research in my Math 3H classes during the second and third marking period. The activity is part of the GMST 640 course I am taking at St. John Fisher College under Dr. Diane Barrett. This course is part of the year-long research component of the Graduate Math Science and Technology program at the college. An educator named David Perkins is quoted as having said, “If students do not learn to think with the knowledge they are stockpiling, they might as well not have it.” This research project is meant to investigate that knowledge they’ve been accumulating, and give them a chance to put it to use.

I am proposing that commonly held student beliefs about mathematics and mathematical knowledge can lead to lower levels of student motivation in mathematics. By making the students aware of their own beliefs and working to transform those beliefs into more constructive and positive ones, I hope to impact student’s attitudes and perceptions about mathematics. With these new beliefs, the students should become more motivated to improve their math and problem solving skills. With more motivation comes more engagement. And with more engagement comes more achievement.

The research that I am conducting will involve the measurement of motivation levels in the students as they proceed through the units of study in their current Math 3H class. The measurements will be taken through observational data gathered over the course of the research. It will ultimately culminate with a trip to the Rochester Museum and Science Center where your child will have a unique learning opportunity in which to apply the math skills they have been working hard to acquire in the classroom.

The students will be presented with this research near the end of the second marking period; up until that time they will not have any prior knowledge of the research activity. But once they have been introduced to it, please encourage your child to talk about mathematics and mathematical knowledge in an arena other than high school. Perhaps you could even share some instances when math found its way into an unexpected personal experience!

As part of the college requirement, I will be making a presentation to my class to show what your child’s class has been up to. For part of that presentation, I intend to show photographs of the students engaged in the research process. If you do not wish for me to use your child’s photo, please contact me.

If you have any other questions for me, please email me at school. I would very much appreciate to hear what you have to say. Wish us luck in our research adventure!

James Tiffin Jr.
HS Math Teacher
James A. Beneway HS
Work: jtiffin@wayne.k12.ny.us
Appendix C: Exhibit Descriptions from Rochester Museum and Science Center

Due to necessary image formatting, this first page of this appendix is left blank.

The exhibit brochures begin on the next page.
Mirrors & Illusions

What do you see when you look in a mirror? Often we can make predictions based on our experience, but sometimes things are not what they seem. The exhibits in this pathway may surprise you.

Think About It

- What do you notice about the shape or pattern of the mirror(s)?
- What path does light follow as it travels between the object, the mirror, and your eyes?

CORNER REFLECTOR
- If you put a pen or a key on top of the mirror, how many reflections do you think you'll see? Try it.
- Why do you think this happens?

YOUR FATHER'S NOSE
- What happens if you move up and down in your seat while your friend stays still?
- What do you see if you move closer or farther away?
- How would you describe a mirror and a glass? What are the differences?

DUCK INTO KALEIDOSCOPE
- Do all of your reflections look the same, or are there differences?
- How do you think the patterns would change if there was a forth mirror?

TOUCH THE SPRING
- Try touching the spring. What happened?
- Can you find the real spring?

MIRRORLY A WINDOW
- Do you feel something happening to your body as you try to do this experiment?
- Can you describe what you feel?
- What do you think is going on?

PARABOLAS
- Try to touch the object. How is what happens different than what you expected?
- Look closely at the mirrors inside: What do you think they have in common with a satellite dish?

What's Going On?

The image formed when light is reflected from a mirror depends on the shape of the mirror. Sometimes, mirrors can create images that are distorted and trick our eyes into believing things that are not there. We can't always believe our eyes!

When light hits a flat mirror, it is reflected straight back to produce an image that is the same size, shape, and direction. Light bounces off curved mirrors at an angle. By using convex or concave mirrors, images can appear to look different sizes, distances, or even shapes.

Illusions like some of the ones created in this pathway show us how important it is for a scientist to question ideas, even their own, about the world around them. Do we truly understand and know how things work, or do we just think that we know?
Vocabulary

**Concave Mirror** - A mirror that curves inward and forms an image that can make the object appear to be larger or upside down.

**Convex Mirror** - A mirror that curves outward and forms an image that can make the object appear to be smaller and farther away.

**Illusion** - An experience that tricks your senses into believing something that is different than reality.

**Image** - A view of an object formed by the reflection or refraction of rays of light.

**Kaleidoscope** - An instrument that uses reflected light from several mirrors to produce symmetrical patterns.

**Light** - The electromagnetic waves that make all things visible.

**Mirror** - A polished surface that forms images by reflecting light.

**Parabola** - A type of curve made from a set of points that are all the same distance from a fixed line and a fixed point.

**Prediction** - A statement that suggests likely future events or outcomes.

**Reflection** - The change in direction of a wave due to its bouncing off a boundary between two materials (like air and glass).

**Refraction** - The change in direction of a wave due to its change in speed when moving through a material (like water).

**NYS Learning Standards**

**ELA1**: Language for Information and Understanding

**MST1**: Analysis, Inquiry, and Design (1.2)

**MST3**: Mathematics (1.4,7)

**MST4**: The Physical Environment (1.2,3,4,5)

**MST5**: Technology (2.3,4)

**LOTE**: Communication Skills (1)
Forces & Structures

What makes some structures strong and others fall down? This pathway looks at the forces that help hold things together.

Think About It

- What are the forces that hold the structure together, and what are the ones that might make it fall apart?
- What are the different factors that can effect how strong a structure is?

Path

Catenary Arch
- Try building an arch. What are the most important steps for making the arch stand?
- Why do you think the hanging chain and the arch are the same shape?

Stress Analysis
- What do you think the changing colors tell you when you bend the plastic?
- Which parts of each bridge model are most likely to break? Are there any similarities among the models?

Braceable Bridge
- Try bouncing on the bridge with the sides up. Now try it with the sides down. What is the difference?
- What do you think causes the difference?

Round Arch Bridge
- Build the bridge and slowly walk to the center. Does it feel stronger on the outside blocks or the center?
- What forces do you think hold the bridge together?

Fluttering Bridge
- Try experimenting with different wind conditions? How does the motion of the bridge change?
- How does the motion change if you pinch the bridge at different positions?

Beam Bridge
- How does each beam's shape affect its ability to support your weight?
- Try rearranging the beams and try again. What happened?
- Try standing in the middle and then on the ends of the beam. Did anything change?

What's Going On?

There are three major types of bridges: the beam bridge, the arch bridge, and the suspension bridge. Each bridge has certain strengths and weaknesses that are related to how they withstand the forces against them. The two most important forces that act on a bridge are called compression and tension.

Compression and tension are present in all bridges, and each bridge needs to be engineered to handle these forces without buckling or snapping. The best way to deal with these forces is to either dissipate them or transfer them.
Where to find more...

Exhibits
- Raceways
- K'NEX
- Creation Station
- Native Peoples

Other Experiences
- Carlson Inquiry Room

(check for booking availability)

Read More About It!

C.A. Johann, Elizabeth Reith, and Michael P. Kline
Bridges: Amazing Structures to Design, Build, & Test
Williamson Publishing, 1999
Etta Kaner & Pat Cupples
Bridges
Kids Can Press, 1995
Building Big (60 min., DVD)
WGBH, 2000
Building Big
http://www.pbs.org/wgbh/buildingbig/

Fun & Learning About Bridges
http://www.bridgesite.com/funand.htm

How Stuff Work
www.howstuffworks.com

Structures Around the World
http://www.exploratorium.edu/structures/index.html

J.E. Gordon
Structures: Or Why Things Don't Fall Down
De Capo Press, 2003

Structure & Shape Teacher Resources

FORCES & STRUCTURE...continued

Vocabulary

Analysis – A thorough examination.

Arch Bridge – A curved structure with supports on each end.

Beam Bridge – A rigid horizontal structure that is resting on two supports, one at each end.

Buckling – What happens when the force of compression overcomes an object’s ability to handle compression, causing it to collapse.

Catenary (kat-n-airy) Arch – The perfect shape of a curved structure so that it can support its own weight.

Compression – A force that acts to squeeze or shorten the thing it is acting on.

Dissipate – To spread a force out over a greater area, so that no one point carries all of the force.

Engineer – To use scientific knowledge to solve practical problems.

Force – A push or a pull.

Snapping – When the force of tension overcomes an object’s ability to handle tension, causing the object to break.

Stress – The force per unit area on a given point of a structure.

Suspension Bridge – A horizontal structure supported by cables that are anchored at both ends.

Tension – A force that acts to expand or lengthen the thing it is acting on.

Transfer – To move a force from one area to another.

Weight – The downward force exerted by a mass due to gravity.

NYS Learning Standards

ELA1: Language for Information and Understanding
MST1: Analysis, Inquiry, and Design (1.2)
MST3: Mathematics (1.4.7)
MST4: The Physical Environment (1.2.3.4.5)
MST5: Technology (2.3.4.)
LOTE: Communication Skills (1)
Newton's Laws of Motion

Experiment with these exhibits to learn about the three laws that explain how all objects move.

**Think About It**

- What are the different variables that affect how the balls start, stop, speed-up, or slow down?
- How are things like friction, gravity, momentum and inertia related to one another, and how do they affect the ball's motion?

**Path**

**LOOP-THE-LOOP**
- How close to each loop can you start the ball and still have it travel all the way around?
- What are the different forces that keep the ball on the track? In what directions is the ball being pushed or pulled?

**ROLLERCOASTER MODEL**
- Why do you think the first hill is the highest?
- Do you think the coaster could keep going forever? What are the different things that make it slow down?

**SKI JUMP**
- Can you get a ball into each of the buckets using each ramp? What changes about the ball's motion?
- How does the speed change as the ball moves down each ramp?

**MOTION DISH**
- Why do you think the ball accelerates as it rolls?
- What do you have to change to make the ball circle around longer before it falls in the hole?

**HIT THE BUCKET**
- Start the ball at the top of the ramp and try to get it in the bucket. What point in the circle is the bucket at when you let go of the ball?
- Try again with the ball only half-way up the ramp. What changes about when you need to let go?

**THE SPIRAL**
- Try to walk around and follow the ball as it rolls. How can you explain what happens?
- Does the ball ever stop before it reaches the bottom? Why? What do you need to do to start it rolling again?

**What's Going On?**

The study of how forces affect movement is called dynamics. In 1687, English scientist Isaac Newton described three laws of motion that explain the principals behind the movement of all objects. Scientists use terms like inertia and momentum to describe how easily objects both start and stop moving. As a ball rolls around "The Spiral", sometimes it may stop because an outside force called friction slows it down; this is Newton's first law. When a ball goes upside-down in "Loop-the-Loop", it must accelerate, causing it to exert a force towards the middle of the loop. Since there is a force down, there must also be an equal force up. These are Newton's 2nd and 3rd laws and the reason the ball doesn't fall down.

*How to use this guide*

To help guide your visit, we have developed this learning pathway to explore a specific topic using some of the exhibit components.

- Look up the words in bold in the vocabulary list on the back.
- Continue your investigations into other areas of the museum by checking out "Where To Learn More" on the back of this page.
- Follow this path as you explore the gallery, try a different path, or create your own path and follow where your curiosity takes you!
NEWTON'S LAWS OF MOTION
...continued

Vocabulary

**Acceleration** - The rate of change in speed or direction.

**Exert** - To have and use.

**Force** - A push or a pull.

**Friction** - The force that tends to slow down moving objects that are touching.

**Gravity** - A force that acts at a distance and attracts objects toward each other. The force that attracts objects toward the center of the Earth.

**Inertia** - The tendency of an object to resist a change in motion.

**Law** - A statement that summarizes the identical results observed in an experiment that is repeated many times by many different scientists. A scientific law is widely accepted as true or as a fact.

**Mass** - The amount of material an object contains.

**Momentum** - The mass of an object multiplied by its velocity.

**Motion** - Movement: a natural event that involves a change in the position or location of something.

**Newton's Laws of Motion** - The three laws of motion: #1) An object at rest remains at rest. An object in motion remains in motion. #2) A force is directly related to an object's mass and acceleration. #3) With every force there is an equal, but opposite, force.

**Variables** - Properties of an object or experiment that can change.

**Velocity** - How far something moves in a specific amount of time.

NYS Learning Standards

**ELA1:** Language for Information and Understanding

**MST1:** Analysis, Inquiry, and Design (1.2,3)

**MST3:** Mathematics (7)

**MST4:** The Physical Setting (3.4.5)

**MST5:** Technology (2.3.4)

**MST6:** Interconnectedness (5.6)
Appendix D: Screenshots of Locus Definitions with The Geometer’s Sketchpad

Due to necessary image formatting, this first page of this appendix is left blank.

The screen captures begin on the next page.
Circle – the set of points \( P \) in a plane that are equidistant from a fixed point, called the center, \( C \).
**Ellipse** – the set of points $P$ in a plane such that the sum of the distances from the point $P$ to fixed points $F_1$ and $F_2$, the foci, is constant.
Hyperbola — the set of points \( P \) in a plane such that the difference of the distances from the point \( P \) to fixed points \( F_1 \) and \( F_2 \), the foci, is constant.
Parabola – the set of points $P$ in a plane that equidistant from both a fixed point $F$, the focus, and a fixed line, the directrix.
Appendix E: Samples of Student Work

Due to necessary image formatting, this first page of this appendix is left blank.

The student work samples can be found starting on the next page.
Equation from my calculations using roots
\( (x-a)^3 + h = 0 \)

Equation using my parameters
\( y = f(x) \)

Observations:
Even though I think that the equation from my calculations from the bridge measurements is more precise, the parameter equation seems to fit the picture better. This is because the picture is not straight and more at an angle so lines are distorted.
Sample calculations from file: (lab) Data Collection and Work for Bridge Model.doc

**Data:** The existing Troup-Howell Bridge will be replaced with a three-member steel arch structure measuring 132 m (433 ft.) in length and 20 m (70 ft.) above the roadway at its highest point.

**Source:** https://www.nysdot.gov/portal/page/portal/regional-offices/region4/projects/troup-howell/i-frame

**How will you use this data?:** I will use this data to create a parabola. The maximum of the parabola will be at the point (0,70) because of the height of the arches. The ends of the bridge will be the x-intercepts of the parabola at (216.5,0) and (-216.5,0). The origin of the graph will be the middle of the bridge.

**Generation of Modeling Equation:**

\[
y = a(x + x_1)(x + x_2) \\
y = a(x + 216.5)(x - 216.5) \\
y = a(x^2 - 216.5x + 216.5x - 46872.25) \\
y = a(x^2 - 46872.25) \\
\]

Then use the point (0,70) to find the missing a value.

\[
y = a(x^2 - 46872.25) \\
70 = a(0^2 - 46872.25) \\
\frac{70}{-46872.25} = a \\
-0.0014934209 \approx a \\
\]

The model for the equation is then

\[
y = a(x^2 - 46872.25) \\
y = (-0.0014934209)(x^2 - 46872.25) \\
y = -0.0014934209x^2 + 70.0037336 \\
\]

Rounded to three significant digits, the equation is

\[
y = -0.00149x^2 + 70.0 \]
Image from the file: (lab) Intersection of Troup Howell Bridge Cables.gsp