Educational Technology in Mathematics & Its Impact on Student Understanding

Stacie Murphy
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Dedication

This manuscript is dedicated to my mom and dad who have made many sacrifices over the years to provide the best for us kids. Thank you for always believing in me.
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First, I would like to thank my fiancé, Cory, for his constant love and support. You are my rock and I thank you from the bottom of my heart for helping and encouraging me when I needed it most. You are a very patient man. Also, my family for being so loving and always being behind me in everything that I do. Thank you for all your love and guidance throughout the years, you have taught me so much about life and helped me to become the person I am today. Lastly, thank you to the best puppy in the world, Cooper, for giving me a warm greeting every day when I come home.
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Educational Technology in Mathematics and Its Impact on Student Understanding

Over the years teachers have been faced with many new challenges in education. One of the largest challenges is the widespread use of educational technology in the classroom. Educators all over the world are trying to keep up with the technological revolution by increasing the use of technology in their curriculum. In mathematics alone there are numerous ways that technology applications could enhance the curriculum as well as increase students' understanding. Technology such as the graphing calculator, Calculator Based Laboratory, Microcomputer Based Laboratory, Carnegie Learning Tutorial and Geometer's Sketchpad are all programs used to help students' understanding of mathematics.

The implementation of technology in the classroom poses a small number of problems for some teachers. While issues of cost, availability, teacher training and technology support may contribute to why some teachers do not incorporate technology in their classrooms. However, this should not keep teachers from integrating some form of technology into their curriculum. The benefits of solely using the graphing calculator in mathematics could greatly increase students' understanding of functions and graphical representations as well as provides alternate ways of looking at mathematics. Technology is the future for the students. It is essential that they are provided access to it in order to prepare them for their future. Exposing students to technology in the mathematics classroom allows students opportunities to engage in real world experiences and provides meaningful learning opportunities. Technology helps students to make connections between mathematics and the world around them. Learning with technology creates a constructive environment where students explore, engage and interact with mathematics.
Studies have shown that students' understanding of mathematical concepts are increased when technology is used in the classroom. The purpose of this research is to examine and reflect on how students' understanding increases when technology is used in the mathematics classroom. Through a review of the literature, lesson study and action research; this manuscript will explore possible correlations between educational technology and students' understanding. Likewise, the research will examine potential problems associated with technology in the classroom as well.
Literature Review

The following review of literature examines many facets of educational technology and its importance in the classroom. The literature defines educational technology and discusses the different types of technology that could be used in the classroom. Likewise, it provides examples on how technology could be implemented into the classroom. Throughout the review, many of the authors have conducted studies in the areas of constructivism and technology, educational technology and students understanding as well as advantages of using technology in the classroom. This review of literature will formulate the foundation for the lesson studies and action research that will be conducted on educational technology in mathematics and its impact on students understanding.

What is Educational Technology?

Educational technology has been around for many years. From the abacus to the slide rule, calculators to graphing calculators, teachers have seen an array of technology in their lifetimes. “According to Kuhnian; instructional technology has undergone several paradigmatic shifts in its brief history. These shifts have occurred because they were driven by shifts in underlying psychological theories of learning and instruction. Some of the first recent technology that was implemented to enhance instruction was television, film and radio. Today computers of any kind have revolutionized the field of instructional technology” (Koschmann, 1996, p. 1).

In the literature Hooper and Rieber (1995) discussed that there are two types of technology, product technology and idea technology. Product technology includes anything that is tangible, for example: videos, computers, software, books, worksheets
and overheads. Idea technologies are those that are not tangible. Idea technology is oriented through some form of product technology for example; simulations and computer based activities. In the past, most attempts at educational technology have been product based. However, over the years it has been made known that both product and idea technologies are needed to improve instruction.

Moreover, Heid (1997) identified educational technology as a cognitive technology. Cognitive technology is media that help transcend the limitations of the mind; in thinking, learning and problem solving. Examples of cognitive technologies are Computer Algebra Systems (CAS), Microworlds, Dynamic Geometry, technology based laboratory devices (such as Calculator-Based Laboratories [CBL's], and Microcomputer-Based Laboratory devices [MBL's]), and graphing calculators.

Graphing calculators are the most widely used cognitive technology in mathematics classrooms today. According to Dueer and Zangor (2000) the low cost, portability and ease of use of graphing calculators have resulted in its widespread use for teaching about functions and graphs in secondary schools in the United States. In addition, the National Council of Teachers of Mathematics (NCTM) curriculum standards (2000) recommended using the graphing calculator to provide students with new approaches, multiple representations and investigate mathematical ideas.

Some uses of graphing calculators are described in Dueer and Zangor's (2000) literature. They described the graphing calculator as a computational, transformational, data collection, visualizing and checking tool. The graphing calculator's ability to perform numerous functions such as evaluating numerical expressions, gathering data to control phenomena and find patterns, finding symbolic functions, solving equations,
confirming conjectures and understanding multiple symbolic forms makes the graphing calculator very attractive to mathematics and science educators. Heid (1997) added that graphing calculators provide easy access to computational and graphical results.

Graphing calculators also allow teachers to focus on student understanding of functions and encourage students to use real time graphs to reflect and develop conclusions (Hooper & Rieber, 1995). Furthermore, Heid (1997) discussed that “Using the graphing calculator would not result in the atrophy of students computational skills, the use of the graphing calculator actually provides the impetus and opportunity for mathematics teachers and students to focus on more conceptual learning” (p. 16).

In conjunction with graphing calculators, data collection devices such as Calculator Based Laboratory (CBL), Calculator Based Ranger (CBR), and Microcomputer Based Laboratory (MBL) are used to collect data and store real life phenomena into a computer or calculator to be analyzed and displayed. Over the years increased availability and low cost has made them more attractive for mathematics and science educators (Cyrus & Lapp, 2000). According to Linn, Kessel, Lee, Levenson, Spitulnik and Slotta (2000) graphing calculators and data collection devices help deal with messy questions in studying real life phenomena. Heid (1997) said that Calculator Based Laboratory provide students with easy access to collecting and analyzing real world data. In addition, Microcomputer Based Laboratory probes allow for real time acquisition which provides students with a unique power to explore, measure and learn from their natural environment. Using CBL’s, CBR’s and MBL’s help connect graphs with physical concepts. In Cyrus and Lapp’s (2000) literature they discussed that even a delay of twenty seconds between the conclusion of an experiment and the physical graph
it creates, makes a difference between student ability to connect the graph to physical concept.

Other important cognitive technologies are intelligent tutors. McGuire and Ritter (2006) discussed that many school districts in America have a population of students who are either at risk of failing or have already failed and interventions are necessary in order to help students succeed. Carnegie learning’s Cognitive Tutor Math program was designed as an intervention for at risk students as well as to increase performance in other students as well. Carnegie Learning’s Cognitive Tutors were developed at Carnegie Mellon University as part of a research project by world-renowned scientists who were testing a theory on how people learn. After numerous field tests in many schools across the United States, Carnegie Mellon took over twenty years of research and created the Cognitive Tutor. This program provides a cost effective and easily implemented curriculum to help struggling math students prepare for the future.

The Cognitive tutor integrates formative assessment and differentiated instruction into every lesson. It constantly monitors students’ actions and each action the student makes is tied to a set of skills. While students are using the tutor it displays skills for them to see on the top of the screen. That display is called the Skillometer, when they demonstrate skills the bar increases, when they make mistakes the skill bar decreases. Students move to the next lesson only after sufficiently demonstrating all required skills. Meanwhile, the tutor checks every action performed by the student against the cognitive model (answer key). If the student makes a mistake the tutor will flash an error and provide hints to keep the student from falling further behind. The program is very helpful and guides students on the correct path. If the cognitive model recognizes multiple ways
to solve any particular problem, it only restricts work when it notices the student is on the wrong path.

Another intelligent tutor created by Carnegie Melon group is the GPTutor Program; this program provides students with the ability to generate proofs of geometry theorems. Just like the Cognitive Tutor, the GPTutor identifies when students lines of reasoning is off and helps guide them in the correct direction. In the literature Fey (1989) commented on intelligent tutors, “The system would present information to the student, the student would work practice problems, the system could speed the student along when her work was going well, but could also diagnose the students mistakes and help when things went wrong, and it could answer the students questions on a wide range of related issues” (p. 264).

The goals of intelligent tutors are to increase time on task and to provide a different approach to learning. It is important to provide students with real-world problems. McGuire and Ritter (2006) claimed that “Connections between new information and prior knowledge will be more easily established when the new material fits with the student’s prior knowledge and when connections with prior knowledge are highlighted” (p. 12).

Hooper and Rieber (1995) and Heid (1997) discussed a few more cognitive technologies in their literature such as Computer Algebra Systems (CAS), Spreadsheets, Microworlds, Hypermedia and Dynamic Geometry. Computer Algebra Systems allow users to generate symbolic, graphical and numerical representations and to reason within and among them. Microworlds and Dynamic Geometry provides computer worlds in which student can express, develop and investigate mathematical ideas (Heid, 1997).
Two types of dynamic geometry programs are Geometer Supposer and Geometers' Sketchpad. The two programs are basically the same. Geometer Supposer is a computer based geometry tool, which teaches deductive reasoning by allowing kids to experiment with geometry, measure and create tools with straight edge and compasses (Hooper & Rieber, 1995). Likewise, Geometers' Sketchpad is a dynamic geometry program that turns classrooms into laboratories for the generation and discovery of geometric relationships (Heid, 1997). Hooper and Rieber (1995) added that hypermedia also lets users browse and build relationships as well as make conjectures for geometric concepts.

In the literature Heid (1997) discussed how Computer Intensive Algebra (CIA) courses focus on the development of algebraic concepts such as function families, equivalence and systems. They are another great way to use technology to provide students with easy access and help with many algebra topics. In addition to CIA courses, Heid (1997) suggested using spreadsheets to help students improve their understandings of functions. Furthermore, spreadsheets allow the user to manipulate entire related sets of data at once.

According to Hooper and Rieber (1995) and Kozma (1994) the Jasper Woodbury television series provides students with a realistic environment which features real world mathematics. The series encourages students to explore and solve real-life mathematics problems. Students collect information after episodes and solve smaller problems leading to a larger problem. The literature stated that problem-based learning creates active meaningful learning while keeping the students engaged. In addition to the television
velocity, trajectory as well as the mathematical topics of multiplication and division. These tutorials all guide learners and provide hands-on experience.

Just like the other programs, they are also aligned with the state standards. Because the program is web-based, feedback is immediate. The assessment program allows teachers to build their own assessments or use the ones they have already created.

The last program Reilly (2004) talked about was the Compass Learning Odyssey. It is a standards based curriculum that can be used in many content areas. This program is self-paced; project based and promotes model based reasoning. It meets a wide variety of learning styles such as constructivist, inquiry, and multiple intelligences. Compass Learning Odyssey integrates assessment and management tools to provide immediate feedback that allows teachers to assess and monitor in real time. The Compass Learning Odyssey web site says “It's engaging, interactive and stimulating, capturing the attention of today's technology literate students and motivating them to learn” (p. 3).

Using educational technologies such as graphing calculators, data collection devices, computer tutorials and computer software can greatly benefit teaching and learning. The use of educational technology in the classroom helps to prepare students for a world that is immersed in technology, mathematics and science. A recent PISA study found that the United States ranked twenty fourth out of twenty nine countries on the ability of fifteen year olds to solve real-life math problems. This study has widened the learning gap between the United States and its competitors in Europe and Asia even more. McGuire and Ritter stated that, “Mathematics is at the foundation of a science or engineering degree. A solid understanding of mathematical concepts and principles lies at
the heart of bridging this learning gap and providing our students with the tools to achieve and succeed, and compete in the global market" (McGuire & Ritter, 2006, p. 16).

**Barriers of Educational Technology**

The literature suggested that educational technology has not been widely adopted by faculty, nor has it been deeply integrated into the curriculum. Many people view it as merely a high tech substitute for blackboard and chalk. Despite all the technology expenditures, educational technology has not being integrated into teaching and learning. Glenn (1997) described the organizational structure of a classroom today as looking much like it did in the 1970's. The teacher is standing in front of the students lecturing, asking questions and keeping order. Many teachers are reluctant to make changes because they are comfortable where they are. The 1970’s looking classroom is all they have known and time is limited to learn about technology and to implement it.

Geoghegan (1994) claimed that there are only isolated pockets of success. Educational technology is being integrated in no more than five percent of courses being taught today. The problem is that only a very small proportion of faculty are actively developing or using such applications in their classrooms. The teachers that develop the activities using technology are the only ones using it. Geoghegan (1994) suggested that there are many different factors such as academic and professional goals, interests, needs, patterns of work, sources of support and social networks that keep faculty from being willing to adopt and use technology in the classroom. Heid (1997) added that other factors such as finance, access, equity, nature of technology use, learning, curriculum balance, implementation, teacher preparation, and public perception are all problems associated with technology.
A big issue that Simmt (1997) discussed was that computers are not readily available in most classrooms on a daily basis. Even if you have the technology, Heid (1997) added that the cost of maintaining it was another issue. Teachers sometimes struggle with finding and using appropriate software for instruction. In addition, it could be difficult to develop creative innovative learning opportunities. Furthermore, the varied levels of technological skills can make it hard to differentiate instruction.

Most teachers are more concerned with teaching and administrative work rather than implementing technology. Teachers only use technology for word processing and preparing active notes and handouts. Although teachers are using technology, many believe that this does little to exploit real value of educational technology. Teachers are also uncomfortable and lack confidence with implementing technology. Heid (1997) suggested that technology is misused and people develop a sense of false security. Heid also added that technology requires more time in and out of class which many teachers were not willing to give up. With unrealistic expectations and the realities of time, money and skills for implementing technology, it is hard for teachers to accept the technological change (Geoghegan, 1994).

Damarin (1998) claimed that people are worried that students would become dependent on technology and will not understand basic concepts in mathematics. Likewise, Heid argued that calculators are becoming a crutch. Many people struggle with what students will not be learning because of the technology. Some people also believe that technology takes over before students have a chance to fully explore the routes they should follow to problem solve. Heid (1997) added that “Technology tools may reveal or hide the mathematics underlying them, and they make it easier or harder for the students
to portray their individual mathematics conceptualizations” (p. 7). Likewise using technology to carry out tasks that are just as easily done without technology may actually have a hindrance to learning (Garofolo, Drier, Harper, Timmerman & Shockey, 2000).

**Integrating Educational Technology into the Curriculum**

Glenn (1997) described in his literature that over last twenty years there have been large efforts directed to enhance teacher’s abilities to use technology as part of instruction. In efforts to increase technology in the classroom, school districts have invested significant amounts of money and resources in new technology each year. The National Council of Teachers of Mathematics has published updated and elaborated standards in 2000 that incorporated new research on teaching and learning. They called for the use of technology in inquiry based learning (Damarin 1998). “There is an expectation that no school can prepare students for tomorrow’s society if new technologies are not available for students” (Glenn, 1997, p. 123).

In the literature Hooper and Rieber (1995) proposed that applying technology in the classroom has five phases; familiarization, utilization, integration, reorientation and evolution. After all five phases are attained full potential is achieved. Familiarization is the initial exposure to the technology. This could be done at a workshop or collaboration with other teachers. The second phase is utilization. That is when the teacher tries to teach with the technology in the classroom. The next phase is integration, which is when teachers consciously decide to designate certain tasks and responsibilities to the technology. This is when the technology and curriculum are intertwined. The fourth phase is reorientation. This is when the technology is student centered. Students are actively using the technology and the teacher acts as a facilitator. The last phase is
discussing functions. Similarly, graphing calculators provide ways to make students interact, check answers, discover concepts, and provide students with unlimited computational power-drill and practice.

In a Pre-Calculus class, a study was done observing how technology was used to enhance curriculum, the teacher used graphing calculator, calculator based measurement probes for motion, temperature and pressure and computer software. All students used Texas Instruments graphing calculators. Much like Simmt (1997) said, the Pre-Calculus students used graphing calculators to investigate the rate of change of a function, and the transformations of exponential and trigonometric functions. During the students work time the teacher encouraged student to use calculator freely in their work. In addition, the teacher allowed students opportunities to share their work on the overhead projector when the students were finished with the activity. Furthermore, the students used Calculator Based Laboratory devices such as a pressure belt to gather data that represented the pattern of normal breathing and to find a function that could be used to describe that pattern. The graphing calculator and pressure belt became a tool to create graphical results to physical phenomena of breathing (Duerr & Zangor, 2000).

Simmt (1997) discussed how many teachers teach quadratics with graphing calculators and use them to check answers and plot graphs. They are also used to understand the minimum and maximum of a quadratic function as well as understand word problems related to quadratic functions. Graphing calculators also facilitate exploration beyond the concept taught. A few reasons why teachers use the graphing calculator are because it varies instruction, saves time, generate more examples, and is a great motivation for students (Simmt, 1997).
to make interdisciplinary connections, as well as generate multiple representations so they can understand mathematical concepts more deeply. Sometimes learners have difficulty providing representations and operations that are sufficient for learning due to their physical limitations. These students are likely to benefit from using technology to provide and model these representations (Kozma, 1994). Also, students that may be uncomfortable interacting with groups or who may not be physically able to review numbers and display symbols can use technology to display the concepts and use computer manipulatives without being judged (Damarin, 1998).

Technology enhances technical skills and explores mathematical worlds, real worlds and computer worlds. Tools such as graphing calculators display simultaneous changes in graphical, algebraic and tabular representations and provide a mathematically rich environment for learning about functions (Heid, 1997). Fey (1989) claimed that calculator usage shifts emphasis from computational procedures to problem solving. Furthermore, calculators enhance students' conceptual understanding, problem solving skills, and attitudes towards mathematics with no apparent harm to traditional skills. Duerr and Zangor (2000) suggested that graphing calculators provide more visual examples and they are a helpful tool for students to use in finding meaningful responses to mathematical tasks. Likewise, computers allow students to work with interesting and realistic collections of numerical data. Computers also provide instant feedback by speaking to you when you make mistakes (Heid, 1997).

Educational Technology's Impact on Student Understanding

Garofolo et. al. (2000) stated that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students'
Technology and Students’ Understanding

learning. Technology enables users to explore topics in more depth and more interactive ways” (p. 71). The National Council of Teachers of Mathematics updated report paid greater attention to the role of instructional technology and the advantage it has on student’s understanding. “Students can learn more mathematics more deeply with the appropriate use of technology” (Damarin, 1998, p. 3). Technology is not a replacement to teaching, but it should foster better understanding.

Damarin (1998) claimed that American students are at a disadvantage because they are not required to use the mathematics that they have learned in the classroom and apply it to their lives. How students learn in class may prevent them from developing their own mathematics understanding. Technology enhanced curriculum can address their shortcomings and encourage students to think more mathematically. Cyrus and Lapp (2000) believed that activities which emphasize qualitative understanding, requiring written explanations, cooperative learning, and addressed students’ prior knowledge are more effective for endearing conceptual change. It is also essential to develop learners’ curiosity and exploration skills as well as developing problem solving skills. The best way to develop students’ skills is through real world simulations. When technology is used students are willing to challenge themselves, and they invest more effort in a task that they view as attainable versus one that they see as challenging (Reilly, 2004).

In the literature, Schacter (1999) investigated how technology impacted students understanding by looking at large scale state and national studies to provide new visions for new uses of technology. The first study analyzed a statistical technique called meta-analysis to compile 500 individual studies to draw a single conclusion. This study found that students who used computer based instruction scored at the sixty fourth percentile on
achievement tests compared to no computers at the fiftieth percentile. Also, Students learned more in less time when receiving computer-based instruction. Furthermore, students liked classes more and developed more positive attitudes when their classes included computer-based instruction.

The second study reviewed hundreds of individual studies where authors shed light on consistent patterns that emerged across studies. This study concluded that students in a technology rich environment experienced positive effects on achievement in all major subject areas. Likewise, students showed increased achievement in preschool all the way through higher education. In addition students’ attitudes toward learning and their own self-concept improved consistently when computers were used for instruction.

The third study was a partnership with Apple and five schools across the nation. In the Apple Classrooms of Tomorrow (ACOT) study, the experience resulted in new learning experiences which required higher level reasoning and problem solving skills. Apple Classrooms of Tomorrow had a positive impact on student attitudes. It addition, it had an impact on changing teachers teaching practices toward more cooperative group work and less teacher lecturing.

The fourth study was the result of a West Virginia ten year statewide educational technology initiative. Students who participated in this study improved test scores on the Stanford 9. With the consistent access to technology, the students and teachers both developed positive attitudes towards the technology. Likewise, the teacher training in the technology led to the greatest student achievement gains. All students test scores rose on the Stanford 9, but the lower achieving students’ scores showed the most significant gains. Also, half of the teachers thought that technology had helped with West Virginia’s
instructional goals and objectives. The teachers became more excited about the technology and their jobs.

The last study was a national sample of fourth and eighth grade classes using newer simulation and higher order thinking technology. This study demonstrated that eighth grade students who used simulation and higher order thinking software showed gains in math scores of up to fifteen weeks above grade level. Another outcome was that eighth graders whose teachers received professional development on computers showed gains in math scores up to thirteen weeks above grade level. Lastly, higher order uses of computers and professional development were positively related to students' academic achievement in math for both fourth and eighth grade students.

In addition to Schacter's study, Bitter and Hatfield (1994) found that using graphing calculators in the classroom led to higher levels of graphical understanding among students. Likewise, students demonstrated deeper understandings of functions through interpreting graphs. Simmt (1997) concluded that students exhibited more confidence in their accuracy of graphing when using a graphing calculator. Also, students became more independent and highly motivated. Guided discovery led students to find out for themselves through investigating, empowering and internalizing the concepts that were being taught through calculator based activities.

Cyrus and Lapp (2000) suggested how using Calculator Based Laboratory systems in conjunction with the graphing calculator helps students to connect graphs with physical concepts. Real time data collection seems to be the most effective way to connect a graph with real world experiences of the student. Calculator Based Ranger and Microcomputer Based Laboratory activities can progress students from physically
modeling to diagramming or graphing. Using technology helps put problems into an abstract mathematical form. Likewise, Microcomputer Based Laboratory helps students understand the relationship between the real world phenomena and the graphical representations. Also, the graphing technology provided immediate feedback that students could interpret almost instantly. Using Data collection devices such as CBL's, MBL's & CBR's allowed students to make more connections among a variety of representations. Students developed greater flexibility in their approach to problem solving as well as increased willingness to work at a problem for a longer period of time (Cyrus & Lapp, 2000).

A study by Cyrus and Lapp (2000) found that data collection devices can help correct students misconceptions. They discovered that students were able to correct their misconception of distance versus time using a CBR and attempted to replicate a given distance versus time graph. Data collection devices can be used for the difficulties students have with connecting graphs with physical concepts, connecting graphs to the real world, transitioning between graphs and physical events and building graphical concepts through students' discussion. Repeated activities with data collection devices can improve student understanding about physical phenomena. In addition, on screen graphs allow MBL students to focus more on what was happening, the graph created a constant reference from their experiment.

Other technology that promotes better understanding among students includes hypermedia, video, and dynamic geometry programs. Hooper and Rieber (1995) claimed that hypermedia encourages students to browse through the information and construct their own relationships and personal experiences to the lesson therefore making it more
meaningful. Bitter and Hatfield (1994) agreed that students who used hypermedia based interactive instructional program exhibited higher cognitive skills development, effective problem solving skills, good management skills and more positive attitudes towards mathematics.

In the literature, Hannafin (2004) discussed how some students have difficulty drawing on knowledge they have of real world situations. Likewise, knowledge they learn in school is not stored to be used outside school. The Jasper Woodbury video series helps bridge the gap from analytical knowledge to connected knowledge. The series provides rich stories embedded with problems to be solved and data to be collected. A study by Vaultanenganhan, about Jasper Woodbury video based stories, showed that students who used this series had better math scores and the students who did not use the series were unable to apply procedures to real world problems. Hooper and Rieber (1995) claimed that learning with media can be thought of as a complimentary process within which representations are constructed and procedures are performed. Students genuinely liked the use of interactive video as math instruction and the video series provided real life experiences.

Moreover, Hannafin (2004) discussed how a Geometer’s Sketchpad study demonstrated that low ability students scored higher in less structured geometry activity versus medium and high ability learners perform better in a structured geometry activity. The study found that there was a lower margin between high and low students in a less structured program versus more structured program. Dynamic geometry such as sketchpad provides learners with powerful learning opportunities. Using Geometer’s Sketchpad in the classroom allows students to develop their understanding at their own
rate. Likewise, the knowledge that is being obtained is more meaningful because the students are constructing it individually.

Overall, all the studies demonstrated that if students have access to computer assisted instruction, integrated learning systems technology, simulations and software that teaches higher order thinking, the students demonstrated positive gains in achievement. Through understanding educational technology and learning how to implement it into the classroom, teachers can promote better understanding among students. Although there are barriers associated with integrating technology in the classroom, the benefits greatly outweigh the risks. It is essential that students construct their own knowledge. Educational technology provides students with the opportunity to think and act like mathematicians. Students are able to emulate real life experiences with technology, therefore, making learning more meaningful.
Methodology

Over a two week period high school students from three different mathematics courses were studied to determine if there was a correlation between the use of educational technology and students understanding of mathematical concepts. Using a research design based on lesson study and action research, students were observed in their natural classroom setting. Their performance was monitored by collecting assignments, making daily observations and assessing the students using authentic and traditional methods.

One hundred and seventy two subjects were selected to participate in the study. The subjects were students selected from the mathematics classrooms of two, fourth year, tenured teachers at Fairport High School. The courses the subjects were enrolled in consisted of Math I, Math III and Math III Investigations. Math I is a course where students develop a strong base in algebra as the key to operational skills in higher math courses. Students use the Carnegie Learning Tutorial twice a week in the computer lab. The other three days are spent in the classroom. Through using the Carnegie Learning Tutorial, students develop a better understanding of mathematical concepts through exploration and guidance. The tutorial allows students to work at their own pace. Areas of study include probability, statistics, coordinate geometry, and trigonometry which are all integrated into the study of algebra. There were thirteen students; nine boys and four girls in the Math I class observed. In this classroom there were two paraprofessionals who work in conjunction with the teacher and students due to the high concentration of students with special needs.
In Math III, students extend their study of algebra to include complex numbers and their study of geometry to the theory of circles and transformations. They also expand on the study of trigonometry, probability, the binomial theorem, statistics, and logarithms. Students in Math III will take the Math B Regents Examination in June. The subjects observed in the Math III classes consisted of ninety nine students; fifty boys and forty nine girls within four sections of Math III.

Math III Investigations students develop their math skills to a higher level and apply these skills to the study of intermediate algebra, general trigonometry, statistics, and probability. The subjects studied in the Math III Investigations classes consisted of sixty students; twenty eight boys and thirty two girls within three sections of Math III Investigations.

In all of the Math classrooms, the desks were aligned in rows. The teacher’s desk was positioned at the front of the classroom. In addition, the teachers used the overhead in the front of the classroom for most of the lecturing portions of the lessons and activities. Most importantly, there was a television with a Texas Instruments Presenter that both teachers utilized frequently.

The materials and instruments used in this study included everyday materials used in a typical mathematics classroom. Students received daily handouts for notes as well as handouts for their daily homework assignments. Furthermore, students in the Math III and Math III Investigations also had access to graphing technology such as the Texas Instruments 83/84 graphing calculator. In addition, students in the Math I classroom had access to computers for the days spent in the computer lab working on the Carnegie Learning Tutorial. They also used workbooks developed by Carnegie Learning for the
days they spent in the traditional classroom. Students in Math I always had access to scientific calculators. All of the materials that were used to determine the results for this study were materials that were collected on a day to day basis.

The design of the research was based on lesson study and action research. Data was collected by observing the lessons taught in Math I, Math III and Math III Investigations. In addition, more data was collected on a daily basis by analyzing homework assignments and making daily observations of students work and performance in the classroom. Furthermore, authentic assessments such as presentations and a ticket out the door were given to provide additional feedback on whether the subjects understanding of mathematical concepts were enhanced by utilizing technology. After all of the lessons were observed, the data was collected and compiled to establish if there was in fact a correlation between educational technology and students understanding of mathematical concepts.

In Math III and Math III Investigations the unit of lesson study consisted of studying real world quadratic functions. The first three days students were introduced to finding the roots, y-intercept, vertex and axis of symmetry of a quadratic function algebraically. Instruction included methods such as direct instruction and cooperative learning groups. The following three days students were exposed to finding the roots, y-intercept, vertex and axis of symmetry graphically with the use of the graphing calculators. Instruction included discovery learning, guided notes and cooperative learning groups as well. As a culminating assessment students were placed into heterogeneous groups of four assigned by the teacher to solve a real world quadratic word problems extracted from various Math B Regents Exams. Students were required to solve
their assigned problem using both algebraic and graphic methods. The following day students presented their assigned problem in their groups. Once the presentations were finished, as a ticket out the door, students were required to solve one quadratic application problem using the method they preferred. This provided insight on which method helped students to develop a better understanding of quadratics.

The Math I lesson study focused on solving linear equations and basic algebra skills. This unit was a two week unit where students were in the regular classroom and the computer lab on alternating days. The days they were in the classroom, the students reinforced their algebra skills such as combining like terms, distributing and solving multiple step linear equations. Instruction varied from teacher directed activities and lessons to cooperative group work. Assignments were given each day they were in the classroom and collected the following day. When the students were in the computer lab students worked individually to solve linear equation on the tutorial. The tutorial provided guidance as well as hints to help the students succeed. The teachers acted as facilitators and monitored students’ progress. As a culminating assessment students were given a formal examination on solving linear equations algebraically.
Results

Once the study was completed, the data gathered provided great insight on how technology impacted the students understanding in each of the courses that were observed. There were both advantages and disadvantages to using technology in the classroom. This section will discuss how technology impacted the teaching and learning as well as some challenges that were faced when conducting this study.

In the Math III course the unit of study was quadratics. The first three days were spent teaching how to find the roots, y-intercept, vertex and axis of symmetry of a quadratic function algebraically. After observing the classes, the study showed that students were able to calculate the roots, y-intercept, vertex and the axis of symmetry. However, students were not really sure what they were finding, they were just plugging the numbers and getting answers. After realizing that they had no connection to what the numbers meant, the next lesson the teacher discussed the critical values and had the students graph them to get the visual representation of the concept.

The next problem students faced was finding the roots. Many of the students lacked the skills to factor. Likewise, when they had to use the quadratic formula to find the roots, there was an abundant amount of students who forgot the formula. After checking the homework in the Math III classes, students were making algebraic errors everywhere. Homework was another issue in itself; some students were not getting it done. On average seventy five percent of the students turned their homework assignments in during these three days. Even though the students knew it would be graded they still did not hand it in.
The concept of finding the y-intercept was easy for them. Each student demonstrated mastery based on the homework that was collected. On the other hand, finding the axis of symmetry and vertex was hard for all of the classes. Students had trouble remembering the axis of symmetry formula and then could not remember what to do to get the vertex. The next time this lesson was taught more modeling was done and students seemed to grasp it better. Also, instead of using the term vertex, maximum and minimum were used which helped the students to be able to visualize what they were finding. The terms maximum and minimum were familiar terms, so they were able to connect more with what they were actually finding.

After finishing the three days of finding the critical values of quadratics algebraically, the students were introduced to the graphing calculator. A major problem arose right at the beginning; some students had never graphed using a graphing calculator. This was a shock and caused some delay in the learning process. Time was lost having to teach about putting the function into the calculator, what to press to graph the function, how to get a table, etcetera, all of which were expected knowledge prior to beginning Math III. Once the students learned how to graph they really enjoyed the ease of use and the lack of algebra they had to do.

Students commented on how easy it was and why they had to do it by hand first. Of course that was a teaching moment on the importance of algebra as well as the benefits of technology. After the excitement of the graphing technology, students discovered how to find the critical values of the quadratics. The first thing that they learned how to calculate was the maximum or minimum (vertex) of the parabola as well as the axis of symmetry. It was easy at first when the graph was nice and fit in the
window, however, things got a little frightening when they could not find the graph.

Following the realization that the students would not always see the entire quadratic function on their graphing calculator, a mini lesson was taught on how to get a *nice* window so the students could see all the critical values. Still, students had problems with plugging in values for their window. Some of them would receive “error window range” on their calculator screen because when they typed in the x minimum and the y minimum they forgot to put negatives in front of the number. It was definitely a learning process for them as well as the teachers. The next time this lesson was taught, it was spread over the course of three days since students lacked the background knowledge of the graphing technology.

The last concept that was taught was finding the roots. Being able to visualize what the students were calculating made this lesson more meaningful for the students. The only problem that the students had with finding the roots was not being able to figure out which was left bound or right bound of the root. So, a mini lesson was taught on how to determine what left bound and right bound meant when calculating the roots. After the mini lesson, the students really understood what to do. Also, the teachers explained how they will receive an error message from the calculator if they try to calculate the wrong bounds.

After grading the students’ homework for the calculator portion of the quadratics unit, there were more correct responses than the algebraic homework. Likewise, more people completed the homework. This could be partially due to the fact that they completed some of it in class. However, it was evident that students enjoyed using the graphing technology, mainly because they are immersed in a technological world.
Students were given a quadratics quiz to determine how they understood the concepts taught thus far. The Quiz consisted of multiple choice and short answer (Appendix A). After grading the quizzes the average of all the Math III classes was a ninety percent, which was great! Therefore, the majority of the students had achieved mastery. This was an important landmark; from here the students learned how to apply their knowledge to real world situations.

After teaching the basics of quadratics, students were introduced to quadratic word problems. This was taught first by modeling a couple problems and then having the students work in groups of three to four of their choice. After walking around the room, it was obvious that students had an immense problem decoding the problems for what they were being asked to find. Therefore, the next time the lesson was taught, more time was spent showing the students how to underline the key words and dissect the problem for what to find. Students were reminded that all of the problems needed to be solved algebraically and graphically. Their homework was collected the next day and graded. The average for all of the classes was an eighty nine percent, which demonstrated that the students were grasping the concept. Unfortunately, the first class did not do as well because they did not receive the extra time that discussed how to find what they are looking for.

As a culminating assessment students were placed into heterogeneous groups of four assigned by the teacher to solve a real world quadratic word problems extracted from various Math B Regents Exams. Students were required to solve their assigned problem using both algebraic and graphic methods. After assigning the students into their groups, the teacher walked around and prompted the groups that needed help. For the most part
students were on target and were ready for their presentation the next day. The students had access to the graphing calculator presenter as well as the overhead.

The groups presented their word problems the next day (Appendix B). The students did a great job. However, the next time the students presented, each group had to evaluate their presentation instead of the teacher. This way more students were paying attention instead of just sitting there the whole time. After putting all the scores together from their peers and the teacher, the students received approximately ninety five percent on average for their presentations.

Once the presentations were finished, as a ticket out the door, students were required to solve one Math B quadratic application problem using the method they preferred (Appendix C). This provided insight on which method helped students to develop a better understanding of quadratics. The data that was collected was analyzed and put into groups based on their knowledge and understanding. Thirty three percent of the students solved the entire problem correctly using the graphing calculator. Twenty two percent of the students had the wrong answer because they used the table to find the maximum value. Thirty one percent of the students had answered two out of the three questions correct and left the last question blank. Thirteen percent of the students claimed they had no idea what to do and one percent of the students solved the problem incorrectly using the algebraic method.

Based upon this data, students apparently lacked some of the skills to complete quadratic application problems. Most of the problems lie in the fact that the students need to take more time to dissect the word problems and really think about what the problem is asking. Some students felt that they were rushed, and others claimed that they did not
read the entire question. Next time, the teachers will devote more time to word problems and problem solving skills. Also, it is essential to teach students throughout the unit using word problems, not just at the end.

In the Math III Investigations classroom students were taught how to solve quadratic equations and applications of quadratic equations problems both algebraically and graphically. The first three days of quadratics was taught algebraically followed by a quiz, and finishing with three days of solving quadratics graphically and another quiz.

During the first day the students were taught how to find roots to quadratic equations by factoring. The teacher observed that students grasped the concept of how to find a root algebraically, but were weak the mathematical skill of factoring. Factoring was a skill that students had previously learned. Subsequently, the class spent only one day on basic trinomial factoring. There was an assigned homework assignment of which seventy six percent completed for full credit. Eleven percent of the students completed the assignment for half credit, and thirteen percent of the students did not receive credit for the assignment.

During day two students were required to find the y-intercept of equations and find the axis of symmetry and vertex of quadratic functions. Students also reinforced and practiced the skill of factoring once again. The teacher observed that students did not have a problem with finding the y-intercept, but had a tougher time with recalling how to find the axis of symmetry. As a result, students struggled greatly with the concept of what a vertex was and why the axis of symmetry is helpful to finding the vertex. Most students were able to follow with the teacher and understand the concept of the vertex, but without the visual cue many students fell behind. Similar to the first day there was a homework
assignment for the second day. For this assignment sixty nine percent of students completed it for full credit, three percent of the students completed the assignment for half credit, and twenty seven percent of students did not complete the assignment.

The third day of solving quadratics algebraically involved real-life applications where students needed to find roots, determine a vertex, and identify a y-intercept. The teacher observed that students had great difficulty in realizing that the real-life applications were the same skills and same type of problems the students had been working on the previous days. On this day sixty nine percent of students completed the homework assignment for full credit and thirty one percent of students did not complete the assignment.

Day four of quadratic functions was a quiz on solving quadratics algebraically. Eighty seven percent of students passed and thirteen percent of students failed. The teacher noticed that the more difficulties the students had with the concept, the lower the percentage of students that completed the assignment. However, the pass/fail ratio is not directly correlated to the completion of homework assignments. If this were so the teacher would have expected roughly seventy percent passing rate and a thirty percent failure rate.

The fifth day of the unit began with the first day of solving quadratic equations graphically with the assistance of graphing calculators. Students were able to recognize roots very easily with the graphs of the quadratics. Students were able to explain why the roots were when the y-value was at zero with the assistance of graphing calculators. This was a concept that students had a more difficult time grasping when solving quadratics algebraically. For this homework assignment eighty four percent of the students
that more students were able to understand the said concepts for quadratic functions with
the use of the graphing calculator. Overall, the use of the graphing calculator was a
beneficial tool to student achievement and student understanding of the mathematical
content.

On the ninth day of the quadratics unit students were placed in cooperative
learning groups of four with only two groups containing three members. The results
concluded that nineteen percent of students solved the assigned real-life Math B question
correctly both graphically and algebraically. Eighty one percent of students did not
successfully complete both parts of the Math B assessment question. All students made
an attempt at solving the quadratic equation graphically. Overall, seventy six percent of
students correctly solved the given quadratic equation graphically, and twenty four
percent of students made a computational or conceptual error in solving the equation
graphically. The teacher observed that all students who approached the graphic solution
of the quadratic equation with the use of the graphing calculator. Based on the teachers’
observations, most of the students using the graphing calculators were using them
appropriately and in a way which allowed for relatively successful completion of the
given question.

The teacher also observed that students struggled more solving the real-life
quadratic equation algebraically. Overall, twenty six percent of students correctly solved
the question algebraically. Of the seventy four percent of students that incorrectly solved
the quadratic equation algebraically fifty six percent of the students had already found a
correct graphic solution. Only five percent of the students failed to attempt the algebraic
solution. From the Math B assessment question it can be concluded that students prefer
the graphic method. Students had a higher success rate of solving a real-life quadratic equation graphically versus solving a real-life quadratic equation algebraically.

Student presentations varied in length, depth, and content. There was one group that did not fully complete their question and as a result was not allowed to participate in presentations. Another group had work that was unclear and irrelevant and for both the graphic and algebraic solutions. The groups that obtained a correct response on both parts presented clear concise solutions to the problem whereas the groups with incorrect problems tended to have group members in disagreement and typically found their error during their presentation. A majority of groups found their error during their presentation. However, the groups had difficulties finishing their presentations from the incorrect solution.

The last course that was observed was Math I. In the Math I class students were focused on solving linear equations and improving their basic algebra skills. This unit was a two week unit where students were in the regular classroom and the computer lab on alternating days. There is only one section of this course so all of the data is based on one class. The first week the students practiced combining like terms and solving one step equations. Mostly, the instruction was direct and students were given time to work with partners at the end of the period. When observing the students work, it was evident that some students really understood the concept while others could not understand what to do. All of the students have trouble adding and subtracting positive and negative numbers so calculators were readily available and used.

The data that was collected that week was daily observations and homework. As far as the daily observations, students participated in class and were able to follow along
with their notes. When given time to work the teacher and two aides observed that most of the students knew what they were doing. Some students said it was so easy, while others had trouble with the concept of adding the inverse to isolate the variable.

The first night their homework was collected only fifty one percent of the students completed their homework. In this course, students prefer to get their work done in class and do nothing outside of class. Of the homework that was handed in, the students did very well and demonstrated mastery of the concept learned that day. By the second time the homework was collected eighty percent of the students turned in their homework. It was really rewarding to see that more students not only turned in their homework, but they did a nice job on it as well. However, the next time the homework was collected only thirty two percent of the students did it. So, they were back to where they started.

When the students were in the computer lab, observations were made as well. Students really enjoyed the computer program because it prompts them and helps them to add and subtract. It provided them individual guidance and support that the three teachers in the classroom could not physically do. Also, after the first week of solving one step equations ninety seven percent of the students obtained mastery.

The second week students solved two step linear equations. They spent three days that week in the classroom. The teacher spent the beginning of the class modeling problems and then put them into groups to complete their notes. Students did well with removing the constants by multiplying and dividing, however, they had a lot of trouble with the fractional coefficients. So, the second day they spent more time practicing that skill, which seemed to help. This week students completed their homework in class. This
constructivist environments should create situations that should stimulate students to make the maximum use of their own cognitive potential (Tam, 2000).

The research done in the Math I classroom could also be expanded upon. It would be interesting to look into other methods of solving linear equations using technology and how it impacts students understanding. Through using the tutorial students were just receiving more practice on solving equations rather than really understanding the concepts underlying linear equations. The students knew they were solving for a variable, but they could better understand it if they saw a visual representation of what they were doing.

Further research could be done by having students solve linear equations by the intersect method on their graphing calculator in addition to solving them algebraically. This would allow students to see that linear equations really represent two lines that intersect and the variable that they are solving for is the x value of the point of intersection. Using graphing calculators could help Math I students better understand linear equations and make learning more meaningful.
Conclusion

Overall, it is essential that educators are aware that technology is an essential tool for teaching and learning. Students should be provided every opportunity to utilize technology to help prepare them for their future. As educators, it is imperative that students are provided with a sound education that promotes life long learning and skills that can be carried with them in their every day lives. This research study demonstrates the benefits of using educational technology in mathematics. The technology greatly increased students’ understanding of mathematics as well as provided alternate ways of looking at mathematical concepts.

Through using technology to enhance the learning process students were able to make more connections to mathematics and their lives. In addition, students were engaged in activities where students constructed their own knowledge made mathematics more fun. Students felt more successful and actually understood what was being taught to them. Furthermore, students were engaged in real world experiences which they could carry with them in the future.
References


Appendix A

Quadratics Quiz

Part I
Each question is worth 2 points. Please place the answer on the space provided.

1) Which is the axis of symmetry of the graph of the equation \( y = -x^2 - 2x - 1 \)?
   A) \( x = 1 \)  
   B) \( y = 1 \)  
   C) \( x = -1 \)  
   D) \( y = -1 \)

2) Which is an equation of the axis of symmetry of the parabola whose equation is \( y = 2x^2 - 5x + 4 \)?
   A) \( x = \frac{3}{4} \)  
   B) \( y = \frac{3}{4} \)  
   C) \( y = -\frac{3}{4} \)  
   D) \( x = -\frac{3}{4} \)

3) Which is an equation of the parabola that intersects the x-axis at the points (-2,0) and (-3,0)?
   A) \( y = x^2 + 6x - 5 \)  
   B) \( y = x^2 + 5x + 6 \)  
   C) \( y = x^2 - 6x + 5 \)  
   D) \( y = x^2 - 5x + 6 \)

4) Which is true of the graph of the parabola whose equation is \( y = x^2 - 2x - 8 \)?
   A) The only x-intercept is at \( x = 4 \)  
   B) There are no x-intercepts  
   C) The x-intercepts are at \( x = 4 \) and \( x = -2 \)  
   D) The x-intercepts are at \( x = 2 \) and \( x = -4 \)

5) The turning point of the graph of the function of \( y = 2x^2 + 4x + 3 \) is
   A) \((-1, 1)\)  
   B) \((1, 1)\)  
   C) \((-1, -1)\)  
   D) \((1, -1)\)

6) The parabola \( y = -5x^2 + 20x + 14 \) will have
   A) a maximum at \((2, 34)\)  
   B) a maximum at \((-2, 6)\)  
   C) a maximum at \((-2, 34)\)  
   D) a minimum at \((2, 6)\)

7) What is the y-intercept of the graph of the equation \( y = 2x^2 - 5x + 7 \)?
   A) 7  
   B) 2  
   C) -5  
   D) -7

8) A young girl standing on a cliff is throwing stones up into the air so that they land in the ocean below. The height (h, in meters) of the stones above the ocean is related to the time (t, in seconds) after it has been thrown by the function \( h = -2t^2 + 24t + 40 \). What is the maximum height reached by the stones?
   A) 20 m  
   B) 40.5 m  
   C) 40 m  
   D) 56.5 m
9) Which is an equation of the axis of symmetry of the graph of the equation \( y = 2x^2 - 5x + 3 \)?

A) \( x = \frac{5}{2} \)  
B) \( x = 5 \)  
C) \( x = -5 \)  
D) \( x = -\frac{5}{4} \)

10) Which is an equation of the axis of symmetry of the graph of the equation \( y = x^2 - 6x + 21 \)?

A) \( x = -3 \)  
B) \( y = -3 \)  
C) \( y = 3 \)  
D) \( x = 3 \)

11) For the graph of which equation is \( \frac{y}{2} \) an equation of the axis of symmetry?

A) \( 3x^2 + 6x - 8 = y \)  
B) \( 4x^2 - 2x + 10 = y \)  
C) \( x^2 - 4x - 6 = y \)  
D) \( x^2 + 2x - 3 = y \)

12) What are the coordinates of the turning point for the graph of the parabola whose equation is \( y = x^2 - 6t \)?

A) \((0, -4)\)  
B) \((0, 2)\)  
C) \((0, 4)\)  
D) \((1, 0)\)

Part II

Answer all questions in this part. Place your answers on the spaces provided. Each question is worth 2 points. You must show all work.

13) Find the equation of the axis of symmetry and the coordinates of the turning point for \( y = 2x^2 + 3x \).

Axis of symmetry: \( \underline{\phantom{0000}} \)  
Turning point: \( \underline{\phantom{0000}} \)

14) Find the equation of the axis of symmetry and the coordinates of the turning point for \( y = 2x^2 - x + 3 \).

Axis of symmetry: \( \underline{\phantom{0000}} \)  
Turning point: \( \underline{\phantom{0000}} \)

15) John throws a ball into the air so that its height at any time \( t \) is given by the function \( h = \frac{1}{2}t^2 + 6t + 3 \). If the maximum height of the ball occurs at time \( t = 3 \), what is the value of \( h \)?

Ans. \( \underline{\phantom{0000}} \)
Part III

Answer all questions in this part. Place your answer on the spaces provided. This question is worth 4 points.

16) (a) Sketch the graph of the equation
\[ y = x^2 + 4, \text{ including all values of } x \text{ in the interval } -3 \leq x \leq 3. \]
(b) Write the coordinates of the turning point of the graph drawn in part (a).
(c) Indicate whether the point in part (b) is a minimum or a maximum point.
(d) On the same set of axes, sketch the graph of the image of the graph drawn in part (a) after a reflection in the x-axis.
Appendix B

Group Presentations

Math 3
Name ___________________________ Date ________________________

Quadratic Word Problems

Directions: In your groups work on your assigned problem and write the answer and explanation on an overhead. Your group will present the problem tomorrow in class so be prepared.
All other problems are for homework.

ASSIGNED PROBLEM: __________________________

1. The graph of a projectile is given by the equation 
   \[ h = (t + 1)(t - 4) \] where \( t \) is measured in seconds and \( h \) is measured in feet. Find the maximum height, in feet, of the projectile and the number of seconds it takes to reach that height.

2. The height of a ball, in feet, is given by the equation 
   \[ h = -16t^2 + 64t + 256 \] where \( t \) is time in seconds and \( t \geq 0 \). For what positive values of \( t \) is the height of the ball more than 301 feet?

3. Evelyn bought a framed picture at a garage sale. The frame measured 24 cm by 20 cm and 230 square cm of the picture were visible. If the frame was of uniform width, find the width of the frame to the nearest hundredth of a centimeter.

4. As shown in the accompanying figure, a rectangle is circumscribed around the region enclosed by two parabolas \( y = 20 - x^2 \) and \( y = -x^2 - 78 \). What is the area of this rectangle?
5. A rectangular garden has dimensions of 8 feet by 12 feet. A gravel path of equal width is to be built around the garden. How wide, to the nearest tenth of a foot, can the path be if there is only enough gravel for 200 square feet?

6. When a vendor at the ballpark sells souvenirs, T-shirts for $x$ dollars each, he normally sells $25 - x$ shirts per day. The vendor's supplier charges him $3.00 per shirt.

a. If profit, $P(x)$, equals total revenue minus total cost, write a function to express the vendor's daily profit and determine the daily profit if the selling price is $15.00 per shirt.

b. Draw a graph of the function and use it to estimate the selling price that would make the vendor's profit a maximum.

c. Algebraically, find the answer to part b and give the maximum profit.

7. A graphic artist wants to place an advertisement with an area $A$, within an 8$\frac{1}{2}$ in. x 11 in. page so that there is a border of uniform width $x$ inches on all sides, as indicated by the shaded region below.

![Diagram of shaded region]

a. Write an algebraic expression for the area of the advertisement as shown in the shaded region $A$.

b. Find, to the nearest tenth of an inch, the width of the uniform border if the advertisement, indicated by the non-shaded region, is 50 square inches.
Problem #1

- find maximum on calculator using
  \[ h = -(t + 1)(t - 4) \]
  \[ \text{max} = x = 1 \frac{1}{2} \text{ seconds} \]

- find number of feet it takes to reach 1 1/2 seconds
  - put in 1 1/2 for t in the original equation
  \[ = 6 \frac{3}{4} \text{ feet} \]

\[ h = -(t^2 - 4t + 4) \]
\[ h = -(t^2 - 3t - 4) \]
\[ x = \frac{3}{2a} = \frac{3}{-2} = -\frac{3}{2} \]
\[ y = \left(\frac{3}{2}\right)^2 = \frac{9}{4} \]
\[ y = \frac{9}{4} + 6 \frac{3}{4} = \frac{21}{4} \]
\[ y = \frac{21}{4} - \frac{9}{4} = 3 \frac{3}{4} \]
\[ \text{vertex} = (1 1/2, 3 3/4) \]

highest point = 6 3/4 feet
time = 1 1/2 seconds

maximum height = 6 3/4 feet
in 1 1/2 seconds
We plugged $-16t^2 + 64t + 256$ into the graphing calculator (TI-83 Plus) equation and looked at the table (by pressing $\text{Graph}$) which brought us within to our estima-
tive table. We then examined the table to uncover the solution to our predicament which showed us......

<table>
<thead>
<tr>
<th>x</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>176</td>
</tr>
<tr>
<td>0</td>
<td>256</td>
</tr>
</tbody>
</table>
| 1  | 320| highest point aka maximum
| 2  | 304|
| 3  | 256|
| 4  | 176|

2 seconds

#2 $h = -16t^2 + 64t + 256$

1. Plug the equation into the calculator
2. Press 2nd graph
3. Find the points where $y > 204$
4. The positive values of $t$ are between 1 and 3
Evelyn bought a framed picture at a garage sale. The frame measured 24 cm by 20 cm and 230 square cm of the picture were visible. If the frame was of uniform width, find the width of the frame to the nearest hundredth of a centimeter.

\[ (24 - 2x) \times (20 - 2x) = 230 \text{ cm}^2 \]

Calculators required.

\[ x = 3.35 \text{ cm} \]

**Quadartic Word Problem**

**Question #2**

Evelyn bought a framed picture at a garage sale. The frame measured 24 cm by 20 cm and 230 square cm of the picture were visible. If the frame was of uniform width, find the width of the frame to the nearest hundredth of a centimeter.

Area of inside

\[ 230 = (24 - 2x) (20 - 2x) \]

\[ 230 = 480 - 48x - 40x + 4x^2 \]

\[ 4x^2 - 88x + 280 = 0 \]

\[ x^2 - 22x + 70 = 0 \]

\[ (x - 7)(x - 10) = 0 \]

Choose reasonable answer and check:

\[ 230 = (24 - 2(7))(20 - 2(7)) \]

\[ 230 = (24 - 14)(20 - 14) \]

\[ 230 = 10 \times 6 \]

\[ 230 = 120 \]

\[ 230 \checkmark \]
Technology and Students' Understanding

Equation 1: \[ k = -16t^2 + 64t - 256 \]

\[ a = \frac{64}{2} = \frac{64}{32} = 2 \cdot x \]

\[ y = -16t^2 + 64t + 256 \]

\[ y = -64 + 128 + 256 \]

\[ y = 256 \]

\[ y = 320 \]

Vertex: \((2, 320)\)

**AND SOOOO....**

\[ t = 2 \text{ Second} \]

Thank you! -

**PROBLEM NUMBER THREE!!!**

![](image)

\[(24 - 2x)(20 - 2x) = 230 \]

\[480 - 48x - 40x + 4x^2 = 230 \]

\[480 - 88x + 4x^2 = 230 \]

\[4x^2 - 88x + 250 = 0 \]

\[4(x - 3.35)(x - 18.65) = 0 \]

\[x - 3.35 = 0 \quad x - 18.65 = 0 \]

\[x = 3.35 \quad x = 18.65 \]

\[ \sqrt{(24 - 2(3.35))(20 - 2(3.35))} = 230 \]

\[\sqrt{(24 - 6.7)(20 - 6.7)} = 230 \]

\[17.3 \times 13.3 = 230 \]

230.09 = 230
Question 3

\[(x-2x)(2x-2x) = 23c\]

\[2x - 40x - 48x + 14x = 23c\]

\[4x^2 - 88x + 480 = 230\]

\[4x^2 - 88x + 250 = 0\]

\[2(2x^2 - 44x + 125) = 0\]

\[\frac{2(2x^2 - 44x + 125)}{2} = \frac{0}{2}\]

\[x^2 - 22x + 62.5 = 0\]

\[\text{Go to Calculator}\
\text{Guess 200 Calc Zero}\
\text{Left bound, Right bound, Middle}\
\]

\[\frac{44 \pm 616 - 1008}{4} \approx \frac{44 \pm 136}{4} = \frac{30.6}{4} \approx 7.65\]

\[\frac{44 - 30.6}{4} = 3.35\text{ cm} \leftarrow \text{Not correct}\]

\[\frac{44 - 20.6}{4} = 3.35\text{ cm} \]

\[x = 3.35\text{ cm}\]

1. Find the x-value of both equations.
   - \[y = 20 - x^2\]
   - \[y = x^2 - 78\]
     - \[x = 78\]
     - \[x = -78\]
   - \[x = 0\]

2. Find the y-values of both equations and figure out the max of \[y = 20 - x^2\] and the min of \[y = x^2 - 78\]
   - \[y = 20 - x^2\]
     - \[y = x^2 - 78\]

3. Add the number of units between 20 and 78 to get the height. \[20 + 78 = 98\] h = 98

4. Put \[y = 20 - x^2\] and \[y = x^2 - 78\] into the calculator, hit graph, hit trace, and hit intersect.
   - \[(-7, -79)\] and \[(7, -79)\]

5. Find the number of units between -7 and 7 to find area. \[7 + 7 = 14\] b = 14

6. Solve for the area of a rectangle.
   - \[A = bh\]
   - \[A = 14(98)\]
   - \[A = 1372\]
Technology and Students' Understanding

Y = x^2 - 78

Y = -x^2 + 208

Intersections:
(7, -29)
(-7, -29)

Calculator:
1. Go to Y=
2. Hit Graph
3. Go to 2nd Trace
4. Hit Intersect
5. Follow directions

7 + 20 + 98 = 1578
98 / 14 = A

A = 1372

Distance from y-axis = 7 on each side
7 + 7 = 14
98 / 14 = 7

Distance from y-axis = 14 on each side
7 + 7 = 14
98 / 14 = 7

\[ L \times W = A \]

\[ 98 \times 14 = 1,372 \]
5. \( (12 + 2x)(8 + 2x) = 200 \)

Use calculator
- Plug in formula to the \( y = \)
- Hit graph and go to window
- Change the \( x_{\text{min}} \) to -15
- Then hit 2nd calc, minimum 0
- Then go left bound then right bound and figure out the value.

The answer is \( x = 2.5 \) not -12.1 because it's negative so you reject it.
\[
(8 + 2x)(12 + 2x) = 200
\]

\[
96 + 24x + 16x + 4x^2 = 200
\]

\[
4x^2 + 40x - 104 = 0
\]

\[
x = 2.5
\]

\[
= 2.5
\]

\[
= -12.2
\]

Reject the negative root.

12 x 20 x 14 = 199.9 ft
Math B Exam – August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of \(2 + 24t - 4.9t^2\) after \(t\) seconds. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach, in meters? How many seconds after the rock is thrown will it hit the ground? Round your answers to the nearest hundredth. (Only an algebraic or graphic solution will be accepted.)
A rock is thrown vertically from the ground with a velocity of 24 meters per second. How many seconds after the rock is thrown will it reach maximum height and what is the maximum height the rock will reach? (Round your answer to the nearest hundredth. Only an algebraic or graphic solution will be accepted.)

Maximum Height: 31.39 meters
Maximum height = 31.39 meters
Hit the ground: 2.45 seconds

Math 8 Exam - August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2.45 seconds. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach? (Round your answer to the nearest hundredth. Only an algebraic or graphic solution will be accepted.)

Maximum Height = 31.39 meters
Maximum height = 31.39 meters
Hit the ground: 2.45 seconds

Math 8 Exam - August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2.45 seconds. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach? (Round your answer to the nearest hundredth. Only an algebraic or graphic solution will be accepted.)

Maximum Height: 31.39 meters
Maximum height = 31.39 meters
Hit the ground: 2.45 seconds

Math 8 Exam - August 2002
A rock is thrown vertically from the ground with a velocity of 24 meters per second. Find the maximum height the rock will reach. In exact form, how many seconds after the rock is thrown will it hit the ground? Round your answer to the nearest hundredth. (Note: algebraic or graphic solution will be accepted.)

Math Exam - August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2.25 m after 1 second. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach? In exact form, how many seconds after the rock is thrown will it hit the ground? Round your answer to the nearest hundredth. (Note: algebraic or graphic solution will be accepted.)

Math Exam - August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2.25 m after 1 second. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach? In exact form, how many seconds after the rock is thrown will it hit the ground? Round your answer to the nearest hundredth. (Note: algebraic or graphic solution will be accepted.)

Math Exam - August 2002

A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2.25 m after 1 second. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach? In exact form, how many seconds after the rock is thrown will it hit the ground? Round your answer to the nearest hundredth. (Note: algebraic or graphic solution will be accepted.)
A rock is thrown vertically from the ground with a velocity of 22 feet per second, and the equation is height = 20 + 22t - 16t^2 (seconds). How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height? The rock will reach its highest point. How many seconds after the rock is thrown will it hit the ground? Round your answer up to the nearest hundredth. (Only an algebraic or graphic solution will be accepted.)
## Appendix D

Carnegie Skills Report

### Class Skills Alert Report

<table>
<thead>
<tr>
<th>Skill</th>
<th>Description</th>
<th>Last Name, First Name</th>
<th>Skill Level</th>
<th>% Mastered</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Remove constant in two-step equations, integer.</td>
<td>Student 1</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Multiply/divide in two-step equations.</td>
<td>Student 2</td>
<td></td>
<td>91</td>
</tr>
</tbody>
</table>

**Legend**

* Student mastery is less than 50.