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Abstract
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Creating Successful Calculus Writing Assignments

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ABSTRACT: I discuss three different writing assignments that I have used in my calculus courses. These assignments are introduced with a discussion of purpose and audience. Defining these qualities of an assignment will ensure that your writing assignments are more successful. The assignments discussed and explored here represent three different purposes: personal, informational and a blend of the two. The audiences for these assignments are diverse and force the students to incorporate particular modes of writing that demonstrate much of their thinking. Assessment of student learning as a result of these assignments is discussed. A fourth writing assignment is developed from a basic idea in order to illustrate the use of purpose and audience to create meaningful assignments.

KEYWORDS: Writing, calculus, audience, purpose, meaningful assessment

1. Introduction

> "We have been compiling evidence on The Exponential family for years. They [the members of this family] have contributed to many events, from drug overdoses, increases in debt, the polar ice caps melting, and even the hole in the ozone layer. These all depend on an initial amount, a rate, and an amount of time."

This excerpt was part of one student's response to the second writing assignment below. In the excerpt, the student has described a wide variety of applications for exponential functions. We did not discuss these applications in class, and they are not explicitly stated in the text. She has either made connections for herself or gone to other resources for these applications. More likely, she has combined these two approaches. The student has furthermore generalized from these separate examples to identify the qualities of an exponential function – initial amount, rate, and time – using her own language. While the excerpt does not demonstrate an ability to solve problems that involve exponential functions, the student has built a framework in her mind so that future problems involving exponential functions can be more readily assimilated. In fact, since this assignment was completed before rates of change and

derivatives were discussed, the student has demonstrated that her mind is ripe for a new understanding of "rate" that will apply to more general settings. At the same time, the writing demonstrates that there is a critical "missed connection" for this student. She has stated that all exponential functions depend on three quantities, but she did not explain *how* these functions depend on those quantities. In responding to her work, I can call her attention to this missed connection by asking, "How do exponentials depend on those quantities?" Thus, her writing gives a thorough peek into the way she thinks about these functions. This type of assignment clearly exhibits the conceptual understanding of the students.

This is in stark contrast to the more in-depth, application-type problems that require several computational skills to be used in order to determine a specific answer. The problem with these assignments as a measure of conceptual understanding is that it difficult to distinguish between the students who understand the concepts but cannot perform all of the separate skills in the problem, and those students who have all the skills but not a "big picture" of the problem to tie the skills together or to aid them in deciding which of the many possible skills are appropriate. It can be argued that writing assignments muddy the waters of assessment further since the ability to write is essential to express one's understanding. While I agree that this is a potential danger, Assignment III below and the concluding remarks discuss a multi-step, shared (between student and instructor) model for writing assignments that can mitigate this difficulty while helping students to improve their writing skills.

Writing assignments provide students with opportunities to demonstrate conceptual understanding at various levels. First of all, as the excerpt above shows, a good writing assignment provides an unparalleled way to look at what the student knows and does not know. Writing assignments provide an alternative learning style through the verbal/linguistic intelligence that allows a broader range of students to participate in the course to a higher degree. By forcing our students to write more often, we are forcing them to develop stronger reading skills for collecting, analyzing and synthesizing information, and for proofreading their own work. The process of writing helps students to scaffold their learning by forcing them to collect and organize their thoughts on a subject. This last reason deals with *metacognition*: thinking about the way you think in order to create a single framework in which to place new ideas and skills.

Below I describe three writing assignments that have been successful in my calculus courses at St. John Fisher College. A brief overview of two important writing characteristics of the assignments, purpose and audience,

is given. Discussion of each assignment concludes with notes on how I grade the assignments and how the assignment can be used to assess the learning of the students.

2. Two Important Aspects of Writing

There are many different components of writing. The two aspects that seem to make the biggest difference in the success of my assignments are purpose and audience. Purpose impacts the definition of the assignment and the grading of the assignment. Audience plays a large role in the quality of the work that my students create. Effective use of these two components can lead to more effective writing assignments.

2.1 Writing with a Purpose

Meier and Rishel [11, p. 43] describe the three main purposes for writing as *personal*, *informational* or *argumentative*. These three purposes represent different outcomes from the students' interactions with the text they are creating. Personal statements require no evidence or chain of reasoning. They are strictly personal statements of belief or interest. These types of assignments make good starter assignments in courses. Assignment I below is a good example of this type of assignment. Informational assignments are basically collections of facts that have been chained together. They require some supporting materials and require students to write coherently on a topic that they are learning, such as the development of imaginary numbers. Assignment II is an example of an informational assignment. The third category of writing assignments requires the most work for the students. Argumentative papers require students to collect, organize and analyze facts in support of or against a particular mathematical issue. At the end of this section, I will provide an example of taking an idea for a writing assignment and molding it into an effective argumentative paper.

Assignment III represents a different category of writing that falls between the personal and the informational. The bridge for this connection is the audience. The purpose of the paper is for the students to inform someone else of their personal and informational connections. In this case, students inform a relative about what calculus is all about. To accomplish this goal, the student must create meaning out of the material in a new way that also serves as reflection. That is, this assignment informs the writer as much as it informs the audience. This aspect of the assignment is made even more explicit since the writing takes the form of a personal letter from the student to

his or her relative. This combination of purposes – writing to inform someone else and writing to make personal meaning out of a broad subject – results in a unique learning experience.

Before creating a writing assignment I find it helpful to define the purpose of the assignment. This sets the stage for how I will grade the assignment in the end. Many statements that are appropriate for a personal type of assignment are not appropriate for the other two kinds of assignments. Likewise, many aspects of an informational assignment might appear in a personal assignment, but these are "icing on the cake" so to speak: If one student includes certain information in a personal assignment while another does not, it is not appropriate to lower the second student's grade.

To select the purpose of your assignment, ask yourself about the outcomes of the assignment: What do you want the students to learn or gain from the writing assignment? The answer to this question will help to define the scope of the assignment. The following sub-questions will help to further identify the purpose of your assignments.

- Do you want the students to explain their reactions and feelings with regard to a topic? This type of assignment is of the personal variety. You must be prepared for a wide variety of responses to such assignments. Your grading scheme must reflect this by including categories that are more process-oriented than content-oriented. The final grade must also remain independent of the particular perspective espoused in the papers; it is the ability to express that perspective that you are assessing. The "What is mathematics?" essay described below is one such assignment.
- Do you want the students to research some ideas that you will not present in class? Then you are looking to have the students create an informational piece. Be sure that you think carefully about what you want to see in the writing and let them know. An example of this type of assignment is Assignment II below. Grading this type of assignment depends on carefully laying out the criteria to the students. Explain what you want them to do and how you want them to do it. Tell the students what level of mathematical expertise will be considered acceptable. Then grade them on how well they meet the goals you have outlined.
- Do you want an answer to a very specific question, or writing about a narrowly focused topic, such as how to use second-degree polynomials to approximate other functions? This will probably turn out to be a very short informational piece or an argumentative piece, depending on the question. Short proofs fall into this category, especially for less-mathematically mature students who are not comfortable with the idea of what

a “proof” is. For example, I ask my calculus classes to write a one-page proof of the statement “The definite integral of an odd function over a symmetric region is zero.”

- Do you want the students to search their own understanding of a particular topic and try to put together a coherent explanation of it? This type of assignment can reveal a lot. In my opinion, this type of writing should involve a personal perspective but should be informational. Assignment III is an example of this. In the description of that assignment, I provide some ideas for grading this category of writing.

2.2 Writing with an Audience in Mind

Consideration of the audience of a piece of writing is an important step for writers. Phrases, concepts, terminology and examples must be chosen to reach that audience. Unfortunately, this part of an assignment is often neglected in mathematics and science assignments. The audience that I will discuss here refers to the second meaning of the audience defined by Meier and Rishel (see chapter eight in [11, p. 45]): to whom the students should write.

When I fail to identify the audience for the writing, my students assume that I am the audience. They assume that I already know the details of and the reason for the assignment. Thus, they find no need for their writing to be complete. This results in papers that are unfocused, since the primary thesis is left out; the students assume that you understand the assignment, so they do not need to restate it. This failure to restate the assignment means that the students have not fully come to terms with the assignment; restating the assignment forces the students to make meaning for themselves. If this meaning is lacking, the entire paper will be less focused. This attitude persists despite all the formulaic writing templates that they have learned in non-mathematics classes with regard to restating the question as part of the answer. It is well known that skills from other subjects, perceived as distinct realms of knowledge by students, do not transfer easily from one subject area to another [4, p. viii-ix and 8]. Defining the audience can help them transfer some of their writing skills, learned in non-mathematics contexts, into the mathematics classroom. By making the audience a part of the assignment, you give your students a lot of information about how they should be writing and why they should be writing in that way: They must decide what is important to the audience, and they must decide how to get this across to the audience. My students always write better when I have chosen the audience for them.

Instructors have two options with regard to audience when grading a writing assignment. The first option is the simplest – leave it out of the grading scheme. Simply defining the audience leads to better quality work, so the audience has played its part behind the scenes. The second option is more interesting – grade the students on how well their writing reaches the target audience. The second option is best used when students and instructors are comfortable with writing assignments. As an example of this, in Assignment III below, I could have chosen to penalize students who relied on formulas and book definitions since it is unlikely that the audience would pay much attention or understand them. On the other hand, I could have simply chosen to reward those students that chose to report only concepts and explained how these related to every day living. In Appendix C, you will find a rubric for grading writing assignments that is based on the "6+1 Traits" approach to writing [12]. The category of "voice" on this rubric addresses how well the audience is addressed in the writing.

The following is a list of several alternative audiences that could be used in mathematics writing and some comments on how each choice of audience may impact the writing. This list is not meant to be exhaustive, and the options are listed in order of increasing technical sophistication.

A non-technical family member: Be prepared for less formal, more familiar writing. The language may be "looser" and less appropriate in some cases, but this choice of audience provides a high level of comfort to most students. Students should leave out many details and focus on larger issues and concepts.

Non-technical members of the general public (public-service announcement type of writing): This will be similar to the non-technical family member type of audience, but not as comfortable for most students.

A company president/boss: The writing should be focused and supported with details of calculations available, but not as part of the main text. The results/final answer should be stated directly and the reasons for this answer given. The writing should be loosely technical, but not packed with jargon.

Students in lower-level courses: Writing focuses on an overview of the current topic, written so that someone with a little math background could understand it. There will be some technical language and ideas, but overall, the paper will have a less formal feel.

Government agencies: Writing should be a report on a particular topic (see Assignment II). The writing will be formal, and should be succint. Overall, this is similar to using the company president/boss as the audience, except that the language will be more technical.

Mathematicians of the past: This option would allow students to play with the history of mathematics in an interesting way. For example, students could consider the effects of introducing functional notation to Euclid. This would require the students to understand Euclid's perspective on mathematics in order for them to explain the current notation and uses, as well as the background that Euclid missed.

2.3 Example of Creating a Meaningful Argumentative Assignment

In the second semester of a traditional calculus sequence, students encounter the idea of using polynomials of different orders to approximate different functions. These Taylor polynomials can serve as fertile ground for growing new writing assignments. Consider the question, "How can we use second-degree polynomials to approximate any function?" By re-wording this question, we will convert it into a detailed argumentative writing assignment:

> *The government has decided to implement standards to simplify mathematics teaching. Rather than using all of the possible functions – exponentials, logarithms, polynomials, trigonometric functions and so forth – they have decided that we will use only quadratic approximations to these functions. Choose one of the following positions:*
>
> 1. *As a supporter of this legislation, you need to defend the idea and explain how this can be done and how it will simplify the learning of mathematics. Explain why we need to have simpler functions to deal with and what benefits this will provide.*
> 2. *As someone completely opposed to this legislation, you need to explain why you do not agree with this proposal. You need to address both a mathematical perspective and the issue of whether or not it will simplify mathematics education.*
> 3. *You represent a group that is not completely opposed to the idea, but would like a less radical change. Suggest alternatives to using only quadratic approximations and explain how your compromise will meet the needs of the mathematical public.*
>
> *Regardless of your position, you should consider the other sides; try to anticipate and address their concerns as much as you can.*

The second version will result in more student-engagement and more creative responses than the original question. Furthermore, this second version could be followed up with an in-class activity where the students from each perspective gather, pool their ideas from their papers, and then lobby for their plan. Drafting a group of students to serve as the "jury" would get all sides of the issue into a discussion of the topic and would allow the ideas of each student to be voiced. The final decision would be in the hands of the students, giving them ownership of this material in a way that the first version of the assignment would not.

We note that, as of yet, there is no audience defined for this assignment. What audience would be appropriate? Students could choose to write their responses as factual summaries intended for science advisors to various members of Congress. They could choose to write a public-service type of announcement for the general public. They could write a call for assistance from the mathematics community or a cry for action from those who want an easier mathematics to deal with. All of these options, and I am sure there are more, would also require the students to learn a little about how the government works, leading to an more interdisciplinary assignment combining mathematics and political science. Each of these options could be presented to the class, resulting in a possibility of twelve different assignments combining the different points of view and audiences. The writing assignment and in-class debate could then be followed by another writing assignment. Assume the legislation is enacted – the students could write a sort of "teacher's manual" listing important mathematical topics in the new setting and describing lessons to teach them. (This last idea would probably work best in a course for pre-service secondary teachers.)

As a result of this assignment and the follow-up activities, you may think that too much class time would be lost for a topic as narrow as this. However, this assignment could be used as a problem to kick off a unit on Taylor series (or series in general). The Problem-Based Learning (PBL) model (discussed in [6], for example) and used successfully in many medical schools and pre-college settings could guide this approach. After completing the activities, students would have had an opportunity to learn about any or all of the following topics: Taylor series, splines, piecewise continuous functions, computer graphics (related to splines), errors in measurements, errors in series approximations, convergence, other series representations (Chebychev polynomials, Laguerre polynomials, Fourier Series, etc.) One question that students often ask during the study of Taylor series is "how do I go backwards from a Taylor series to a transcendental function?" This topic could easily arise and be investigated in PBL approach that uses the writing assignment above as a starting point.

3. Assignment I: What is Mathematics?

The first writing assignment in my calculus class is of the personal variety. It provides an opportunity for reflection and expression and takes the form of a one-page essay entitled "What is mathematics?" I ask the students to tell me their personal experiences with mathematics and their ideas about what mathematics really is. By emphasizing that this will let me know each student better and understand their background more, I convince the students to take the assignment seriously. Many of them report that they have written little in previous math classes. Most of my calculus students report that they "like math" and see it as a "way of getting definite answers". Since these are mostly science-oriented students, the first response does not surprise me. As a professional mathematician, the second response is disappointing, but not surprising. I have yet to accumulate a significant number of responses that describe mathematics as the science of looking for patterns or a way of asking questions about the world or as a process. It is clear that my students perceive mathematics almost as an oracle: when you have a quantitative question, mathematics will give you the answer. Many of my calculus students have expressed their enjoyment of mathematics as being based upon its certainty, as contrasted with English or history or psychology. This is a useful assignment to give students at the beginning and again at the end of the semester. In the final essay, ask the students to identify their new thoughts about mathematics (if any) and to explain where/when they arrived at these new thoughts. In this way, a simple personal statement from your students becomes a valuable research tool for assessing your own teaching!

For this assignment, the instructor serves as the explicitly stated audience. Students are aware that I do not know them at this point in the semester. Many of them are new to college and are possibly trying to find a way to express their personalities in a healthy way. This assignment gives them an opportunity to explain their point of view – which is neither correct nor incorrect – in a setting that has little or no pressure. They understand implicitly that it is their responsibility to give me information. At the same time, this assignment has an implicit audience – the students themselves. Through the medium of writing they have an opportunity to reflect on their personal situation and to decide for themselves what they think.

When it comes to grading this assignment, I give the students full credit for turning it in if they have followed the guidelines (typed, etc.). Since this assignment is personal, I feel that it would do harm to assess the writing, either for content or mechanics. These are freshmen in college, struggling to find their own voices, and this assignment is one way to help them. Also, one must keep in mind the purpose of the assignment: expression and

reflection. By expressing their situation to me I get to learn more about them. This gives me information that I can use to make adjustments to my plans for the semester. By writing this reflection, the students learn more about themselves. A simple rubric that avoids criticizing the voice of the papers could include the following questions: Did the student answer the question (i.e. did the student define mathematics) from his/her own perspective? Did the student explain his/her history that led to this idea of mathematics? Did the student explain his/her opinions of mathematics and their origins? Are the mechanics acceptable? If the answer to any of these questions is "No" then consider allowing the student to re-write the assignment.

I also use this assignment in my course for Elementary Education students. The results for the two groups are strikingly different: the elementary education students report more personal situations that have led to "math anxiety" and most express a dislike/distrust of mathematics. One of the more creative descriptions related mathematics to stumbling around in a dark maze full of snakes. These attitudes are prevalent in about 90% of the elementary education students that I have worked with. However, when I use the end of the semester version of this essay, I discover that many of the students have some new perspectives. Most students indicate at least a willingness to try mathematics, with several students coming around to actually expressing pleasure at doing mathematics!

4. Assignment II: Organized Families of Functions

This second writing assignment was created with several different intentions in my mind. I wanted to access the creative side of my students. I wanted to force them into an in-depth study of one family of functions (in the sense of chapter one of our calculus text [8]), and I wanted them to express their knowledge through writing. Student responses would be focused on providing information about a particular family of functions, so this assignment falls into the *informational* category with respect to purpose.

The assignment is entitled "(Organized) Families of Functions" and is based on a play on the use of the word "family" to describe a set of functions that differ by a parameter, as well as to describe the mafia. Students produce a "dossier" describing one family of functions in detail, showing its relation to the real world and to the other families. Students are allowed to choose one organized family of functions, but I prohibit the use of the family of linear functions so that I can provide examples. The assignment is given in the form of a single-page handout. This is provided in Appendix A. I hand this assignment out during the review portion at the beginning of the calculus course (chapter one), and students typically have two to three weeks to complete the assignment.

The audience for this assignment is a government agency. This agency needs information that is clear and concise in order to speed the process of justice. Certain types of information are required to assist the agency in accomplishing a goal. However, students are not restricted to this level of information since they can pursue each part of the assignment in as much depth as desired. This allows the students to be more creative and to make individual judgments as to what is important.

The first time I assigned this project was in the fall of 1999. The work my students submitted took me by surprise. They had each spent a great deal more time on the assignment that I had anticipated and had produced rather lengthy documents. I was so impressed by the work as a whole that I had it bound with a cover and an introduction so that it could take its place on my shelf alongside rows of more "serious" mathematical texts. Sharing my reactions with the students resulted in greater student engagement: the students began to understand that I valued them and their perspectives.

In grading this assignment, I look at five categories: completeness, quality, mechanics, correctness, and creativity. I put 30% of the grade on completeness and 30% of the grade on quality. This ensures that a student who works hard on the assignment will earn something for their work, but not necessarily do well. In checking for completeness, I usually take off 5% for each piece of the assignment that is not completed. For example, a student that does not provide a glossary will receive 25 out of 30 points for completeness if the rest of the assignment is handed in. To check for quality, I look for whether the paper is typed, whether it has a cover sheet, the research (number of sources) they have consulted and so forth. Ten percent of the grade is based on the mechanics of the paper – grammar and style conventions. Rather than a detailed assessment of this, I simply give student 0, 5 or 10 points based on the overall mechanics. This is enough to potentially change a grade by a full letter while not requiring that I perform a detailed check of the mechanics on every paper. The remaining 30% of the grade is based on the correctness of the paper; I am checking their mathematical and historical facts. Typically, these are located in a few sections of the paper so that I can concentrate my attention on the "family tree" for the relations between different functions of this family and the section on "connections" for the relationships among families. The last category, creativity, is not an actual requirement of the assignment; however, sudents that write a particularly creative paper can receive up to 10 extra points, resulting in possible grades up to 110%. A sample of a scoring sheet is provided in Appendix B.

5. Assignment III: Letter to Granny

The third writing assignment that I would like to share grew from Sarah Greenwald's article on letter writing in geometry [7]. It is quite different from the others for several reasons. First, it requires a good deal more work on the part of the instructor. This work does pay off, however. Second, this assignment does not fit neatly into any single category described above. It involves some aspect of the personal, the informational and the argumentative. The assignment addresses several levels of Bloom's taxonomy (summarized in appendix B of [2],) touching on the levels of knowledge, comprehension, application, synthesis and evaluation. At the same time, it provides a unique method for students to review the course (any course, not just calculus!) in that they can explore multiple levels of interpretation of the subject in order to construct a deeper meaning for themselves. The final difference between this assignment and the others mentioned above is that this assignment is interactive. The writing is created in several stages with the student and the instructor both contributing to the final product. The assignment is part of the review activities for calculus, and I have assigned it during the last few weeks of the first-year calculus sequence.

5.1 The Pre-writing Assignment

My teaching philosophy is constructivist, so I wanted to design a more learner-centered (see [10, pp. 4-8] for a definition and guiding principles) review activity that would allow the students to demonstrate the meanings that they had constructed from the course material. Thus, I chose to have the students actively review the material through a large writing assignment, rather than passively review through a lecture.

The first part of the assignment was intended to "jump start" their thinking about the course. It is fairly common in other fields (such as English) and at other levels (primary through secondary) to use a "pre-writing" assignment for this purpose. For a more detailed description of pre-writing, see chapter nine of [11] and the references contained therein. Pre-writing assignments generally lead to better results in the final product since the students get a formal chance to get their very informal thoughts organized onto paper before attempting the full assignment. As a pre-write for the final assignment the students simply made an organized list of the topics we had discussed during the year (Calculus I and II). This let the students focus on organizing the concepts rather than worrying about the individual skills that comprised the course. I encouraged the students to consider using a graphical organizer, such as a concept map. Concept maps have received more attention in the mathematics

community lately (see [1] for examples) but have been used more extensively in science and other areas. For a good overview of graphical organizer formats you should consult dimension 2 of *Dimensions of Learning* [9, pp. 43-44]. Unfortunately, many of the students simply copied the table of contents from the text. In class discussions after the assignment was turned in, I emphasized that the "table of contents approach" was a start, but that this organization had two drawbacks:

1. It was inherently linear, showing only a flow of topics, but not the inter- and intra-connections. Graphical organizers allow for the connections to be made more explicit and thus play a major role in organizing information in a meaningful way [9, p. 62].
2. It was someone else's (the textbook author's) opinion of how to organize the material in order to make sense of it. Having students make their own lists or charts forces them to organize it *for themselves*, which is important for developing the habits of lifelong learners [4, p. 49].

Despite the drawbacks, this assignment did get the students thinking about the material that we had discussed during the semester. Many student comments indicated that the assignment at least showed them how much material we had covered and reminded them of topics long forgotten. This assignment was merely a prelude to the actual finale of the course, and so the "table of contents approach" was a legitimate response to the assignment. As far as grading was concerned, I did not put a lot of weight on this assignment and treated it as a daily homework assignment, worth about 7% of the overall homework grade, which constituted 20% of the course grade. This small amount of "grade impact" was another reason for allowing the students to use the table of contents as their organized list.

5.2 The Assignment

This assignment, more than any other, showed which students "got it" and which students needed help. I introduced the assignment by posing the following situation to the students.

> *Pretend that your grandmother is paying for your entire college education. She does not want you to turn out the same way that your "hippie" father did, so she wants to monitor what you are learning. In order to*

keep her informed, you need to write a three- to four-page letter describing what you learned this year in calculus. Keep in mind that your Grandmother never made it out of high school. She knows arithmetic, but nothing more.

I told them flat out that it would be impossible to get all of calculus into four pages. I was interested in the basic ideas of calculus and the connections, not the details. I wanted to see what each of them, individually, considered the most important aspects of the subject we had just spent one year learning. I chose to express the assignment verbally, rather than written in a handout, so that the students would feel like they had more of an opportunity for input in the assignment. As an example of this input, the students quickly came to a consensus that Granny should know how to read a graph, or their job would be too difficult. As a class, we decided to allow Granny this extra knowledge, gained from her years of reading the newspaper. Giving them this small amount of input seemed, from my perspective, to improve the morale of the class considerably. In general, allowing students to have input in their assignments has been shown to improve the quality of work that students produce and to increase student engagement (for example, see [5]).

At this point you can spend some class time brainstorming the letter contents. By this I mean that the students can discuss the type of material that is appropriate for the letter, choosing for themselves the appropriate information. Rather than focusing on the specific content ("should we talk about tangent lines?") students should look for broad categories of content, such as calculations, examples, applications, concepts, definitions, theorems, illustrations, and so forth. This discussion will enrich the class and result in even more interaction and collaboration in the assignment.

I also informed the class that after they handed in their letters (typed, one-and-one-half spacing, etc.) I would read these letters, and "Granny" would then write an individual letter back to each student asking for clarification on points that needed it. The students would then respond with a short, one- to two-page letter addressing Granny's concerns.

5.3 Results of the Assignment

Writing the responses to my students took *time*. The benefit to all this was that I had a much better feel for where my class was in terms of understanding the subject matter. As expected, the explanations of earlier topics

(functions and the like) were more coherent and the students were generally able to explain them at a more conceptual level. They were also much more creative with their connections to Granny's life with regard to earlier topics in the course. Later topics in the course were usually described at a lower level, focused on the procedures and definitions rather than the applications and concepts. This is to be expected.

As long as a student satisfied the requirements of the assignment, I gave full credit for the initial letter. This amounted to about 14% of the overall project grade, which amounts to about 2.8% of the final course grade. I felt that individual style and personal expression were an important piece of the writing, but in cases of particularly poor mechanics, Granny was quick point out that the student needed to consider proofreading more closely. In these cases, I deducted a few points for mechanics. In terms of the content, I was looking for a coherent discussion of the topics that we had covered, with an emphasis on the applications and the ideas of calculus rather than the details. Since each student would get one more chance to clarify and redirect his or her efforts in the final part of the assignment, I was not overly concerned with minor content errors during this first stage.

Some students got lost in the details – formulas, graphs, and definitions – but did little to explain the applications of calculus. Since "Granny" was not likely to compute a second order Taylor series in the near future, she tended to request more "down to earth" descriptions in her letters. If students were inconsistent (an issue when they are writing four pages on such a large topic) or flat out illogical, Granny asked for clarification. If a student provided a particularly cogent description of a topic, Granny tried to think of relevant examples to ask about. This allowed students with strong conceptual knowledge to push to a higher level of Bloom's taxonomy [2], moving toward true synthesis and application.

The "Letter to Granny" assignment identified an audience – Granny – with little or no experience in mathematics. This choice of audience focused the student writing on the basic ideas and the over-arching principles of calculus, rather than the details. It is true that some students got lost in details and tried to report every formula in the book, but I contend that those students have a weak conception of calculus and need help to see the big picture. Some students would mention ideas – like local linearity as seen on the surface of the earth – and Granny's response could then ask about real-world examples of local linearity: "Is that why my table won't sit flat, because of the local linearity of the earth?" The response to the question told me a lot about how far that student had gotten in applying the concept outside of calculus. (In fact, this was one place where creativity got in the way – the student explained

that the table wouldn't sit flat because someone had sawed off one of the legs. However, in a vector calculus setting, this answer could have led to the idea of using three points to define a plane.)

I would not recommend giving the students a list of the "big ideas" that you are interested in (like differentiability, continuity, convergence, approximations). The students need to identify these ideas for themselves, and they should spend a great deal of time struggling to put all the ideas into a single framework. This struggle is often at the core of learning and is certainly part of the foundation for classrooms modeled on constructivist theories [4].

6. Conclusions

All three of these assignments proved valuable in the course. They are each very different in nature, providing variety in both purpose and audience. Each of these assignments helped both my assessment of the students and the students' self-assessments. I find that although the writing takes time to interpret and assess, I agree with Dorothee Blum: It is "an indispensable part of learning calculus" [3, p. 128]. By reading what my students have produced, I have learned more about the way they think and learn than I did from grading a lot of identical problems from the textbook or from grading a few application-type problems. This is not to say that practice work should be ignored, but it is only one part of the learning process, and it is applicable primarily to procedural knowledge [9, pp. 310-314]. Examples of procedural knowledge in calculus include taking the derivatives of functions, computing definite integrals and computing the location of critical points. However, there is a great deal of information in calculus that is declarative, rather than procedural. Examples of this include local linearization, approximations and continuity. The reform calculus puts great emphasis on this type of knowledge, and students are often uncomfortable due to the lack of "practice problems". Assignments must be designed with the distinction between procedural and declarative knowledge in mind. Good writing assignments are an essential part of assessing declarative knowledge.

At the end of assignments like these I am left with two tasks. I must assess the students' work to determine the level of understanding that each student demonstrates. Then, I must assign a grade. These two tasks are often at odds; many students do not have the entire picture. Even if a student has a grasp of the material, that student may not express that understanding. In order to survive the conflict, I try to apply the following principles.

If I am interested in assessing students' understanding of one particular concept, then the assignment must focus on that concept alone. This results in a small assignment with a narrowly defined purpose. Nothing prevents a

student from going beyond that purpose and exhibiting connections to other topics, however. This extension often provides insight into that student. At the same time, the narrow focus of the original assignment ensures that the students have an opportunity to express their understanding of that concept. Assigning a grade to the piece of writing can then be based on how well the students express their understanding and at what level. Other criteria, such as mechanics or reference material, can easily be included in the grading scheme. Always remember that you are assessing the way that the students have expressed their understanding. By allowing re-writes, or follow-up assignments, students can respond to your questions and comments. This allows the students to learn about their writing in a more cooperative way than a fire-and-forget assignment would. This shared approach also compensates for poor writing skills by allowing students to respond to specific points. At the same time, the questions that are asked of them indicate where the writing was weak to start.

If I am interested in the ways that many topics tie together, then I must be open to many different interpretations. I must allow the students some leeway in expressing their understanding. The "Letter to Granny" assignment is an example of such a case. No two students covered exactly the same set or subset of topics. Each student looked at the material in slightly different ways. If I felt that an initial letter lacked evidence of certain topics, I had an opportunity to elicit responses concerning that topic. The interactive, almost collaborative, nature of the assignment allowed the broader assignment to narrow its focus differently for each student. For assignments of this nature, grading becomes more fluid initially, since you must allow each student to express their understanding in very different ways. However, the individual nature of the focusing that takes place after this provides a more "objective" criterion for evaluation. Did the student respond to the questions you raised? How did the response delve into the subject more deeply? How did the student modify his or her understanding to incorporate the new points that you brought attention to? Answering these questions involves very little subjectivity.

As I read the papers that students produce, I keep a running record of my comments in a word-processing file. I divide the papers into "excellent", "good", "acceptable" and "unacceptable" piles. After an initial pass through all of the papers, I have produced two essential devices for grading: the students have been separated by quality and I have a general notion from my notes as to what the class learned from the assignment. This helps me answer both of my tasks – assessment and grading – at the same time. The comments can be filed away to help re-design the assignment, to inform the development of future assignments, to direct class time in a more productive way, or to

assist with grades at the end of the semester, where the collected comments help me make more informed decisions about grades and often help in the "border" cases.

In short, the use of writing assignments in my calculus classes has provided insight as to what concepts my students are taking from the course. I have made an effort to create writing assignments that engage the students more by assigning an audience to their writing. At the same time, I have focused their attention to a specific purpose for their writing, narrowing down the possibilities and providing them with more information about "what I am looking for" in their papers. The bulk of this paper consisted of three examples I have provided from my experience. All three were used with college freshmen, in a reform calculus course. Each of them could be adapted to upper division courses. Each of them incorporates purpose and audience in a different way to create a stronger assignment and to aid the instructor in grading and evaluating the assignment. A further example was provided to illustrate how a typical writing assignment can be modified to include these writing characteristics. As a result of including purpose and audience in my assignments, I have seen an improvement in student engagement. I also have a much better understanding of my students – both as growing individuals and as students of mathematics.

Appendix A. The Text of Writing Assignment II

The Setting: The International Bureau of Mathematics (IBM) has recently convened a task force on organized mathematical families. As a member of the task force, your first assignment is to complete a dossier of information on one of the notorious families of functions that is listed below. This information will be provided to the district attorney in order to speed prosecution.

The Task: Create a dossier for one of the families of functions that we have discussed. Be sure to use proper mathematical terms and notations. The dossier should include the following categories:

- Introduce the family of functions. This should include the name of the family and the mathematical description of this family.
- The history of this family of functions. This could include a description of when you first encountered the family. This could include a brief description of problems from the book where the family proved useful. It could include real world applications of this family. Be creative, but include mathematical content.
- The family tree. This could include a discussion of the number of parameters in the family. Different choices of these parameters lead to different functions from the family. Create an organized description of the family that shows how the parameters pick out different members of the family. Are there any prominent members of the family, which are easy to recognize? Describe them and show where they fit into the family tree.
- Connections. Does this family relate to any of the other families on the list in any way? Are any of the other families children or ancestors of this family? Demonstrate or explain this relationship.
- Evidence. Where has this family been involved? What areas of math, science and the world at large give this family a hiding place? Where will the task force begin searching for them.
- Glossary. Provide definitions of the mathematical terms that are used in your report. For example, if you refer to an asymptote, define this term, in your own words. Concavity, the idea of an increasing versus a decreasing function and many other terms (domain, range, etc.) will be common to all of the dossiers. Be sure to include information about these.

The Families: You are free to choose any family of functions from the list below.

- Exponential Functions
- Power Functions
- Logarithms
- Sine Functions
- Cosine Functions
- Tangent Functions
- Rational Functions
- Polynomials

The Format: Your dossier should include at least the above-mentioned topics. Be thorough, but do not get overly verbose. Some of the topics will only require one or two lines, maybe. Others (like the family tree) might be better as an appendix with a graphical layout of the family tree. Pictures, graphs, tables of data and verbal descriptions should all be included. It should be several pages long. You should use at least two other references besides the textbook. Remember to reference other material as you use it and include a bibliography. (Good sources are physics books, *Scientific American*, newspapers, *The Wall Street Journal*, science texts and the like.) Be creative!

Due Date: Your dossier will be due on _____ at the time you take the first test.

Appendix B. Sample Scoring Sheet for Assignment II

Student:

Family Investigated:

Category	**Description**	**Score**
Completeness	Includes introduction, history, family tree, connections, evidence and glossary	30 25 20 15 10 5 0
Quality	Paper is typed, includes a cover sheet, is well-oganized, neat, and demonstrates that the writer has consulted several different sources.	30 25 20 15 10 5 0
Correctness	The mathematical facts are explained correctly; historical references are accuracte. Examples are thorough and correct.	30 25 20 15 10 5 0
Mechanics	The grammar, spelling and style conventions are followed.	10 5 0
Creativity	The paper is particularly creative in its approach; stays within the context of the assignment rather than becoming a textbook	+ 10
Total		

Appendix C. A More General Writing Rubric

I adapted the rubric below from the "6+1 Traits" program [12]. It is a simple, yet comprehensive method for assessing writing and is easily customized to a specific assignment. Each of the seven traits (they are referred to as "6+1" for historical reasons) has a description of how good writing demonstrates the particular trait. Bold terms in the description emphasize certain points and can be circled to indicate those that need work. The scores to the right of the description are from 1 to 5 (if you absolutely must, think 5=A, 4=B, etc.). Each trait is also weighted, based on its perceived importance. The weighted scores can be added to produce a total score between 10 and 50. Descriptions of each scoring level are provided below the matrix of traits. At the bottom is a place for you to customize the rubric to meet the needs of specific assignments. Simply list the content that you are looking for (definitions of certain terms, examples, etc.) and check them off if they are included. The top portion contains information about the assignment and the student. Note that this is a very general rubric. You will probably want to adjust the weights or eliminate some traits altogether. If your students submit their work in electronic format then this rubric could simply be pasted into the work and the comments typed into the appropriate spaces, thereby saving many a tree. This rubric also emphasizes the assessment portion of the instructor's job, rather than the grading portion. After the rubric is filled out for each paper, the results can be tabulated in order to convert the combined scores to a grade.

Student		**Assignment**	
Version		**Overall Grade**	

Trait	**Comments** (areas that need improvement are circled)	**Score**	**Weight**	
Ideas	This paper is **clear** and **focused**. It holds the readers **attention**. Relevant **anecdotes and details** enrich the central theme.	1 2 3 4 5	x2	
Organization	The organization **enhances and showcases the central theme or idea**. The **order**, **structure**, and **presentation** of information is compelling and moves the reader through the text. There is a **clear introduction**, **body** and **conclusion**.	1 2 3 4 5	x2	
Voice	The writer speaks to directly to the reader in a way that is **individual**, **compelling** and **engaging**. The writer crafts the writing with an **awareness of** and **respect for** the audience and purpose of the writing.	1 2 3 4 5	x1	
Word Choice	Words convey the intended message in a **precise**, **interesting**, and **natural** way. The words are **powerful** and **engaging**.	1 2 3 4 5	x1	
Fluency	The writing has a **easy flow**, **rhythm**, and **cadence**. Sentences are **well built** with **strong and varied structure** that invites expressive oral reading.	1 2 3 4 5	x1	
Conventions	The writer demonstrates a good grasp of standard writing conventions (e.g., **spelling**, **punctuation**, **capitalization**, **grammar**, **usage**, **paragraphing**) and uses conventions effectively to enhance readability. **Errors tend to be so few** that just minor touch-ups would get this piece ready to publish.	1 2 3 4 5	x2	
Presentation	The **form** and **presentation** of the text enhances the ability for the reader to understand and connect with the message. It is **pleasing to the eye**.	1 2 3 4 5	x1	

Explanations of Scores

5	Strong	Shows control and skill in this trait; many strengths present.
4	Competent	On balance, the strengths outweigh the weaknesses; a small amount of revision is needed.
3	Developing	Strengths and need for revision are about equal; about half-way home.
2	Emerging	Need for revision outweighs strengths; isolated moments hint at what the writer has in mind.
1	Not Yet	A bare beginning; writer not yet showing any control.

Specific Content Issues Checklist (checked items are present in the author's work)

	Item number 1 (could be "Bibliography with at least three different types of sources")
	Item number 2 (could be "Defined the following terms:…")
	…
	Item number *N*

Comments from the reader:

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Biographical Sketch

Kris Green is an assistant professor in the Mathematics/Science/Technology and Computer Science department at St. John Fisher College. During his first year of full-time college teaching he was selected to be a national Project NExT fellow (Brown Dot). His Ph.D. is in Applied Mathematics, although most of his friends believe that it is actually a Ph.D. in Star Wars Studies. Lately, his interests have been focused on mathematics and science education at all levels. When not engaged in scholarly activities, he enjoys taking walks with his wife and trying to find his cat.