The Benefits and Demands of Journal Writing in a Mathematics Classroom

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Abstract

While mathematics teachers believe that student journal writing is beneficial, however, not many have tried to implement writing in their classrooms. I began a project that places journal writing at the center of my Course II class. The main purpose of my project was to see how daily journal writings affected the teacher's practice and to assess its benefits and demands. Even before beginning the project, I had anticipated that time was going to be a factor, which it was. What I had not anticipated was the extent to which good mathematical discourse must be modeled in order to establish a viable discourse community which, in turn, enabled students to write quality journals, to deepen their thinking, to heighten their interest in mathematics, and to establish an atmosphere of trust and respect—in brief, to transform the classroom and to good measure my own teaching. Upon the completion of this project, I came to the following conclusions: writing in mathematics helped me to learn that students are capable of more than we give them credit for and writing in mathematics is a powerful tool that helps to heighten a student's understanding.

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Abstract

While mathematics teachers believe that student journal writing is beneficial, however, not many have tried to implement writing in their classrooms. I began a project that places journal writing at the center of my Course II class. The main purpose of my project was to see how daily journal writings affected the teacher’s practice and to assess its benefits and demands. Even before beginning the project, I had anticipated that time was going to be a factor, which it was. What I had not anticipated was the extent to which good mathematical discourse must be modeled in order to establish a viable discourse community which, in turn, enabled students to write quality journals, to deepen their thinking, to heighten their interest in mathematics, and to establish an atmosphere of trust and respect—in brief, to transform the classroom and to good measure my own teaching. Upon the completion of this project, I came to the following conclusions: writing in mathematics helped me to learn that students are capable of more than we give them credit for and writing in mathematics is a powerful tool that helps to heighten a student’s understanding.
The Benefits and Demands of Journal Writing in a Mathematics Classroom

Statement of Background

Of all the many assignments and examinations I undertook during my college years, never once was I expected to write in a mathematics course. It was assumed that you knew how to communicate mathematically if you could solve numerous complicated mathematics problems. It was not until I became a high-school mathematics teacher that I began to realize the importance of having students communicate their mathematical knowledge and understanding to me and to each other in ways other than solving a problem in strictly “mathematical language.” I began to ask students to explain how they came up with a solution; to justify why this was the correct method to use and why their solution made sense based on the problem context.

Based on their responses, I realized some students were not able to communicate mathematical concepts through writing even though they could work through the symbolic solution with ease. On the other hand, other students struggled to obtain the correct symbolic solutions, but could provide written justification for each step in their problem solving process. Because of this experience, I began to incorporate a variety of writing activities into my daily/weekly classroom routine.

I use journal writing most often in my classes. In journals, there are many ways to give students the opportunity to write mathematics. (See Appendix A for other ways to incorporate writing into your mathematics classroom.) Sometimes I give my students a topic to explain, while other times I have them write about how a certain quote relates what we are learning in class to other events in their lives. Writing helps me to get to know students better as individuals and also gives me the opportunity to correspond with students on a one-to-one basis, which cannot always happen during the 40-minute periods that I see students. Since journals are
always confidential, students have the opportunity to freely express their thoughts and feelings without approbation from their classmates.

The use of writing in my classroom has helped me plan units and pace the class. When students explain how a problem can be solved, I can readily assess where students are in their understanding of the topic and determine if there are any misconceptions that need to be cleared up. This helps address mistakes before an assessment and raises student confidence levels as well as their grades.

**Literature Review**

The idea that writing is a powerful aid to learning mathematics is well supported. The use of writing to help promote conceptual understanding in mathematics is a teaching strategy that has been heavily researched by numerous individuals ever since the National Council of Teachers of Mathematics (NCTM) published its *Curriculum and Evaluation Standards for School Mathematics* in 1989.

The NCTM strongly encourages teachers to incorporate writing into the teaching of mathematics. For many teachers, however, writing in mathematics is a foreign concept because they see no connection between the two. Nevertheless, research in the use of writing in the teaching and learning of mathematics has received considerable attention. This research suggests that teachers should not think of writing as an additional burden but rather as a helpful tool that can be incorporated into their curriculum as an aid to learning (McIntosh, 1991). Moreover, research suggests numerous ways to encourage students to write in mathematics classes. (Again, see Appendix A) and for those teachers who have implemented writing into their mathematics classroom the benefits are becoming evident.
Writing is more than just a means of expressing what we think; it is a means of knowing what we think. Writing helps students shape, clarify and discover ideas (Bagley and Gallenberger, 1992). With the incorporation of writing in the mathematics curriculum, students are given the opportunity to investigate their own ideas and make connections without being told how and why particular mathematical concepts are significant. In addition, writing helps students become accountable for their learning. Myers (1984) states that during the writing process, students are obliged to relate information from the classroom to what they already know and to organize and synthesize it so that the concept becomes their own.

Foremost among uses of writing in the classroom is to promote a deeper understanding of mathematical concepts. The more students express their thinking in writing, the more they are able to internalize mathematical information and make connections between concepts. Students who have learned to verbally express the various connections between mathematical concepts and to accurately describe their interrelationships demonstrate higher-level mathematical thinking skills (LeGere, 1991). Moreover, students who write to learn retain concepts better than students who do not write as part of their course work (LeGere 1991). For these students, mathematics has become a meaningful and purposeful activity.

Because writing in the mathematics classroom is unfamiliar territory for many mathematics teachers, Havens (1989) has proposed guidelines for such writing: (1) credit should be awarded for all writing assignments, (2) writing should replace some of the homework, and (3) writing assignments should include explanation of procedures, defining vocabulary definitions and the use of applications. (In addition to implementing Havens' 5 guidelines, I inform my students that spelling and grammar will not be held against them and that effort and conceptual insight is my goal for writing in mathematics.)
The benefits of writing in mathematics are not restricted to students alone. Journals in mathematics, for example, provide teachers with a unique diagnostic tool. As students write, they not only reveal areas of confusion and misconceptions, they also reveal areas of expertise (Nahrgang and Petersen, 1986). Teachers can then use this writing to assess whether their students are learning what is taught during class. Furthermore, writing provides excellent feedback for pacing and adjustment. Teachers are better able to determine whether students have grasped previous material before introducing new material.

In spite of the abundance of research supporting the use of writing in the mathematics classroom, there is a paucity of specific information concerning the amount of time such writing demands, not only in the classroom but also what is demanded of the teacher outside the classroom. As LeGere (1991) claims, “Collaborative and writing experiences do take more time than just lecturing; however, lecturing plus these [writing] experiences takes little more time than repeated lecturing and enables the [instructor] to learn how the student has interpreted the lecture” (170). This “time” consideration raises the larger issue of how writing in the mathematics classroom impacts the day-to-day activities of the teacher who attempts to integrate it in her teaching.

**Statement of Research Question**

**Question**

In what ways does journal writing affect my daily teaching practices in a New York State Course II Regents class and what are its benefits and demands?

**Secondary Questions**

1. How much time does journal writing demand, particularly of the teacher?
2. How do I get students to write quality journal entries?
3. In what ways do students' reflections change as a result of my modeling mathematical writing?
4. Do students' journal reflections change as their content knowledge develops?
5. How does journal writing influence students' classroom behavior? Does the classroom atmosphere change as a result of writing?
6. Does my relationship with the students' change as a result of the writing process?
7. Is journal writing worthwhile when "technique" units are being taught?
8. In what ways does assessment change as a result of journal writing?

Methodology

Participants

My research took place during the first semester of the 1999-2000 school year. The students involved in my research project were Course II Regents students at Pittsford Sutherland High School, in a suburb of Rochester, New York. There were 14 students in the class, 3 males and 11 females. All students were sophomores, except for one male who was a freshman. The class met 40 minutes a day, every school day of the week.

In this small class, the students know and get along with each other. There are no students with special needs and no special accommodations are needed. In addition, I previously taught 7 of the 14 students in Course I during the 1998-1999 school year. In that class, I exposed students to writing though not nearly to the extent undertaken in Course II, the current classroom of study. As a result, these seven students were already at least familiar with the concept of writing in a mathematics class and were able to help establish the classroom environment in which writing in mathematics was valued.
Approximately 78% of the students in the class project held averages higher than 90% (11 of 14 students). I divided these 14 students into five mixed-ability, collaborative learning, groups which they kept throughout the study: Group One consisted of three males, Groups Two – Four consisted of three females each and Group Five consisted of two females.

Classroom Materials and Conduct of Research Project

For the purposes of this project, I chose to teach the unit on Quadratic Equations, Chapter 10 in our text, Integrated Mathematics. The majority of content in this unit was new to the students, though a few topics at the beginning of the unit were review from Course I. Within the Quadratic Equations unit, students studied the following topics:

1. factoring review
2. solving quadratics by factoring
3. radicals
4. deriving the quadratic formula
5. quadratic formula
6. discriminant
7. geometric applications of quadratics
8. graphing quadratics
9. graphic solutions of quadratics
10. graphic solutions of quadratic-linear systems
11. algebraic solutions of quadratic-linear systems

An in-depth look at the Quadratic Equations Unit is found in the Appendix B.

Since space is very limited in my high school, I had to share a classroom with another teacher. As a result, valuable class time was lost at the beginning of each day rearranging the desks into groups and arranging for students to pick up their journals and any handouts for that day.

Students worked in collaborative groups on a daily basis and all class notes were taken in their journals. I presented major concepts to the entire class, allowing for continual student interactions, such as questions and explanations as to how new material connects to other topics.
we had already covered in class, after which students worked in their groups. The group work encouraged students to rely on their peers to help explain difficult concepts as I circulated around the room. Another advantage in using group work was that I was able to present more challenging problems earlier in the unit because students could brainstorm with each other and sound out their ideas.

Design and Procedure

I used the following data collection instruments; my on-the-spot, written daily observations of classroom activities, student journals, my own reflective journal, student writing assessments and tape recorded interviews with all students following the completion of the project.

As we progressed through the unit on quadratic equations, students were required to record class notes, observations, feelings and thoughts in their journals on a daily basis. The format for each student’s journal is shown in the diagram below.

As the teacher-researcher, my own journal was organized in the same format as that of my students.

Since writing in mathematics class was new for many students, I attempted to decrease their anxiety levels by informing them that they would only be assessed on the effort and thought they put into their responses, not on mathematical or writing correctness. Specifically, I informed them at the very beginning that I was looking for honest thoughts and reactions and
that their grades would not suffer as a result of what they wrote. Rather, I told them that I anticipated that their grades would improve as a result of their writing.

Anticipating that many students would not know where to start when responding to what we had learned or done in class, I designed a journal format that each student could respond to during class. This format would also help guide them through the writing process and provide them with an outline for what I would be looking for in their entries. I hoped this structure would also provide the kind of information I was looking for during the research process. The daily journal format was as follows: **In your journal, please respond to the following questions. Be sure to fully explain your thoughts and feelings on each question. Be honest and open.**

1. What did you learn/do today in class?
2. **“Question of the Day”**
3. Be specific – How are groups going?
4. Is there anything that I should know about?

Each day students responded to the same four questions. The “Question of the Day”, however, changed based on what we were learning/doing in class that day. Each “Question of the Day” can be found in Appendix C, along with the topic covered that day.

As previously mentioned, many of my students were unfamiliar with writing in mathematics class. Anticipating that getting students to elaborate on their thoughts and probe deeply into their individual understanding of mathematics would be difficult to do. I determined that I would have to model in some way how I expected the students to respond to the posed questions, though I was not aware at the onset of the study all the ways and the extent to which I would have to do this. Consequently some of the modeling strategies I used were developed
during the course of the study, thought I described them here and under Data Sources and Procedures in this Methodology section for expository reasons.

Since teacher modeling was so critical for this project, I had to devise several ways to demonstrate what I would be looking for in their responses. One of these ways of modeling that I had determined to do before the study began was to write along with students. After presenting the daily journal entry, I wrote my own response as students were writing theirs. After a few times of collectively writing as a class, I hoped that students would begin to sense the nature of the explanations that I was looking for. Another modeling technique I used was to dialogue with myself in front of students. This enable me to explain how connections are made between what we learned today and how it connected to a prior lesson. In addition, I emphasized the importance of explaining "why" a concept is true and "how" concepts connect together. I hoped that this would help students to deepen their mathematical understanding and foster their desire to know more. This technique also enabled me to give permission to the students to voice their confusions and misunderstandings.

Students took four pop quizzes approximately every four days, completed daily homework assignments and wrote two tests during the unit. After each formal assessment, students wrote in their journals about their thoughts while taking the test, why they might have answered a question incorrectly and then they corrected any mistake that they had made.

At the conclusion of the entire quadratics unit, I conducted tape-recorded interviews with individual students during which I elicited their responses to the following:

1. Their working in groups for an entire unit;
2. Their writing in a daily journal;
3. Any personal benefits they realized as a result of writing in mathematics;
4. If they would continue to write about their concerns in their own notebooks;

5. If they would like other units to be structured in the fashion as the quadratics unit;

6. If they felt their understanding of concepts and mathematical connections had increased as a result of their writing;

7. If there were anything that I could have done differently to enhance this method of learning. These interviews were conducted at the conclusion of the study rather than throughout because I did not want to influence student attitudes and beliefs with regard to the value of writing. That is, I hoped in their way to preclude my injecting a “teacherly” bias in the data during the collection phase of my project.

Data Sources and Procedures

As mentioned above, my personal teaching journal, student individual journals, my field notes, and the student interviews instituted the main data sources for my research. This data was collected during an on-going process of interactive journal writing and response between each student and myself, throughout the unit. A cyclic process of “students write--teacher responds--students write and respond.”

Throughout this cyclic process, I was able to link my own reflections to student thoughts and feelings throughout the unit. This on-going, interactive analysis had a significant impact on how I approached the unit on a daily basis. At the completion of the quadratic equations unit, I began another cyclic process of reading--re-reading--formulating as I looked back on the students’ journal entries. This reading--re-reading--formulating cycle enabled me to reflect on how successful I was at teaching to the needs of the students.

Throughout the unit, as I attended to the students’ emerging understandings, I remained focused on myself as well--my reactions to what was happening in class, how my daily activities
were affected by what students wrote in their journals and to my sense of classroom atmosphere. What changes do I make to ensure the students are learning and understanding quadratic equations? How are students changing throughout the unit as I adjust and make changes based on their writing? How much extra time do I spend outside of class preparing for the next day? What types of problems should I create that require students to not only employ the problem solving techniques under study but also to make connections between mathematical concepts?

Analysis

Introducing the Journals

Although I have not been teaching very long, I have found that students are not very skilled at explaining why something is true or how they arrived at their answer in mathematics. One reason why this is true is that teachers are not teaching students how to put their mathematical thoughts into words, and another reason why this is true is that the teachers themselves were never asked to justify or explain their solutions to mathematical problems. It is very difficult to teach your students to do something that you, as an individual, do not do. When I first began introducing journals to my Course II class, I could see the apprehensiveness in my students eyes. They were not sure what to write, when to write or how much to write.

My obvious goal was to get enough meaningful data to complete the research portion of my thesis. Exposing students to standards their younger siblings are going to encounter and emphasizing the importance of writing in any discipline, especially mathematics, was another goal of mine. Those students who had me as a teacher last year already knew how important justifying solutions are in class because they were previously responsible for performing this task. For a handful of my students, writing in mathematics seemed foreign. By the end of the
project, the majority of students verbalized to me that writing and thinking through their solutions helped them to internalize the information and really piece together concepts from previously learned units.

With prior knowledge of my students' exposure to journal writing in mathematics, I felt confident that they would take the project seriously and use the journals as a learning experience. There were a few moans when I first mentioned journals, however, as we got into the process of writing on a daily basis, students realized there was not any extra work involved, after all, the journals never left the classroom. One student wrote, "I like journal writing, and I think it will help learning come easier for me." Another student said, "Journal writing in math is such a cool idea." For others, writing in mathematics class took a little getting used to.

For some students, I believe that journals were a means of studying. Since they knew I would read what they wrote and respond each day, some students were very conscious of taking detailed notes and trying to think through any questions they had about the material we were covering. For those students who did not normally take notes on a daily basis, a sense of obligation to show they were paying attention in class led to every student being on-task and attentive during the lesson. The note-taking process became very beneficial for those students who, in the past, had chose not to record personal notes on a daily basis in class.

Throughout the modeling process and the beginning stages of helping students become accustomed to daily journal writing in mathematics, I had to continually modify my lessons. I needed to be sure I was meeting the needs of my students and providing them with all the opportunities to help them fully understand each mathematical concept in the unit. A day did not go by where my lesson plans remained unaltered. Since students were using class time to write in their journals, I had to alter the number of days I would spend on the Quadratic Equations unit.
Originally, I planned the unit around a 15-day period, however, with all the additional class time spent on journal writing and problem-solving activities, I added approximately 6 more days to the unit.

Not only was the number of days spent on the unit a change, but the material I had planned for each day also changed, especially once I started receiving feedback from all students. Basing my lessons and the pace of the class directly on the needs of my students proved to be time-consuming, but extremely worthwhile. Students no longer had an excuse for not understanding the material. They did not have to voice their confusion in front of the class. “If someone is afraid to come talk to the teacher, they can write their question in their journal and get an answer that way,” said one student. “Private interaction w/teachers”, said another student. All students had to do was write his/her thoughts and feelings in their journals and request we spend a little more time on that specific topic. It was my sense that higher levels of comprehension and a greater retention rate on the students’ part was occurring. It was evident in the quality of their writing and questions and discussions that took place in class. At the beginning of the unit, when asked what was learned today, one student responded, “how to describe the roots”. This same student, after several days of modeling responses for students, responded to the same question, “the roots of the quadratic equation help you to know more about what the graph looks like. If the discriminant is imaginary (negative number) then it won’t touch the x-axes. But if it is not negative, then the parabola will cross the x-axes one or more times.” It is clear that the student has a greater understanding of the material as is evident by his pointing out varying connections between the value of the discriminant and its graphical interpretations.
For those students who did not always understand a concept after the first time through, the journals provided them the opportunity to tell me this, without exposing their misunderstanding in front of the class. This was particularly beneficial for those shy, reserved students who suddenly found they had a voice in class without having to speak out publicly. Not only did all the students benefit from others’ questions and comments, they were also able to request additional examples that would further develop or explore a particular concept. One student wrote, “[graphing calculators] are a little confusing. I personally like doing algebra better, than working out w/a calculator. Can you show me how to do it w/o a calc?” The following are additional examples of student requests for more time to be spent on a concept in class:

“I don’t understand how a point is the solution to a system. Can we do more of them tomorrow?”

“The quadratic formula is somewhat confusing to me-help.”

I found that many students, even those who were not afraid to ask questions in class, used the journals as an educational resource. As one student wrote, “If I forget why something is true, I can always look back and read the explanation I wrote.” Further evidence that students began to regard their journals as a significant resource surfaced when I asked the students if they would like photocopies of their individual journals—all 14 students wanted me to do so.

Modes of Modeling

Although the students did not complain or express any apprehensions as we began the daily journal writing process, I was not satisfied with how they were elaborating on what they were learning in class or showing an increased understanding of the concepts we were studying. To help prevent the brief and expected one-line responses, I began to model in different ways
what I was looking for in their responses, which were well thought-out responses with some evidence of higher-level thinking. I told the students that "What I [was] looking for is any insight into the development of their understanding of concepts." I explained that I was hoping for mathematically rich entries in which they would write more than "we reviewed radicals". As a lesson came to a close, the mode for modeling responses occurred in many ways. The progression from mode one to mode five (see below) usually occurred naturally during class; however, we did revert back to earlier modes when it seemed necessary because of the amount of material covered and the extent of the questions asked by students.

**Mode 1:** In the beginning of this process I would write the entire response to the Question of the Day (see Methodology) on the overhead as the students and I together verbally developed appropriate answers. We also incorporated any special comments students felt were important. Before writing with the students in class, I thought about what I would want students to emphasize from the lesson in their journals to be sure all key pieces of information were included. I tried to get comments from as many students as possible during the modeling process.

The following teacher-student exchange is an extended example of this mode of modeling from November 29, 1999. During this mode of modeling, I wrote my responses as well as the student responses listed below on the overhead. (Key: T = teacher, S1, S2, etc. are identifiers for students, which change from excerpt to excerpt.)

**Question of the Day:** How would you describe the process of graphing a parabola?

**T [writing on the overhead]:** When you graph a parabola, there are certain characteristics that help you to know what the graph will look like based on its equation.

**S1:** The parabola will open up if the value of a is positive.
S2: It will open down in the value of $a$ is negative.

T: To graph the parabola you need to make a table of values, based on the interval given in the problem.

S3 [looking puzzled]: What happens if they don’t give you an interval for the parabola?

T: Excellent question. We will explore this tomorrow. Why don’t you think about that tonight. Where do the points you plot come from?

S4 [confidently]: You get this by substituting the $x$ value into the equation to get the $y$ value.

T: These values give you the coordinates to plot.

S3 [hesitating]: When the chart is completed... there should be repetition between the $y$ values from top to bottom, with one number that isn’t the same.

S5: The different number is in the middle of the chart.

T: After you plot the points and connect them, remember not to have the bottom or top of the parabola come to a point.

S6: It should be curved.

S7: All parabolas are symmetric.

T: The line that the parabola is symmetric about is called the...

S1 [interrupting]: axis of symmetry

T: Its equation is $x = \frac{-b}{2a}$.

S8: his equation comes from the beginning of the quadratic formula.

S2 [waving hand eagerly]: The point where the parabola intersects its axis of symmetry is called the turning point.

T: If the parabola opens up, then it has a minimum turning point.

S9 [raised voice]: If the parabola opens down, it has a max turning point.
T: You can find the turning point without graphing the parabola. First you find the axis of symmetry of the parabola, which will give you the abscissa of the turning point.

S10: Once you have x, you plug it back into the equation to find y.

The summary of the lesson took about 10 minutes. I found that as time went on and students became more familiar with writing in their journals, the closure/modeling process took less than five minutes each day.

The following is the model writing sample we developed and I wrote on the overhead:

Mode 2: After a few days during which I wrote everything out on the overhead, I merely began to jot notes down as students offered verbal responses. It was the student's job to translate the notes into their journal entries. The following example occurred on November 16, 1999 as we discussed the meaning of the discriminant in class. During this example, I wrote my responses as well as the student responses listed below on the overhead.

T [speaking and writing]: The nature of the roots of a quadratic equation with rational coefficients can be determined by knowing the value of the discriminant.

S1 [loudly]: Formula is \( b^2 - 4ac \)

S2: Discriminant comes from under \( \sqrt{ \) in the quadratic formula

S3 [voice rises]: Roots can be real, imaginary, rational, irrational, equal and/or unequal

T: Roots = x-intercepts, where it crosses x-axis (roots = answers, solution)

S4: Doesn’t cross x-axis, imaginary roots

S5: Imaginary = negative value for the discriminant

S6 [with a firm voice]: describe the roots as:

- Perfect square – real, rational, unequal
- Positive, not perfect square – real, irrational, unequal
- Zero – real, rational, equal

T: Equal happens when a double root exists, only one touch on x-axis

S7: Rational roots touch at a nice number

S8: Knowing its value helps visualize the graph

The following are post-mode 2 samples of students’ entries illustrating how the students’ were beginning to move away from their earlier one-line responses to more elaborate responses. One student wrote, “Before finding the roots and describing them, you need the equation in standard form. Keeps consistent to what we were taught about the quadratic formula.” Another student wrote, “I never understood the difference between equal and unequal roots until I graphed the equations, \(x^2 + 2x + 1\) (root at \(-1\)) and \(x^2 - 5x + 4\) (roots are 4, 1). Now that I see the roots and know the discriminant helps to see the graph, it makes more sense.”

Mode 3: As a class, we would brainstorm a list of important points from the lesson that should be incorporated in their journals (one student would record this list on the board while the rest of the class, including myself, verbalized ideas to include in the list). This list included how new material connected to previously learned material, concerns and frustrations with the problem-solving process, questions and anything else students felt was important. Once the list was generated, students would take certain phrases and expound on them in their journals. Each student would not write about every idea on the list, and not all students wrote about the same ideas. This is where reading all the students journals benefited me as a teacher because, knowing what constituted the list of ideas, I could see how students put them into their own words. This helped me know if
any concepts needed clarification. Here is a brainstorming list from November 17, 1999 as we were reviewing for our first test of the unit.

**Brainstorming list – review for test**

**Formulas to know**
- standard form of a quadratic, discriminant, quadratic formula

**Concepts to know**
- simplifying radicals and rounding to nearest hundredth
- four ways to factor
- given {-2,9}, how to write a quadratic equation (working backward)
- what is a root
- how to find a root
- use the quadratic formula only when you cannot factor
- always reduce to lowest terms
- draw and label your picture in geometry problems involving quadratics
- in geometry, reject negative answer

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Description of Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect square</td>
<td>real, rational, unequal</td>
</tr>
<tr>
<td>Positive, not perfect square</td>
<td>real, irrational, unequal</td>
</tr>
<tr>
<td>Zero</td>
<td>real, rational, equal</td>
</tr>
<tr>
<td>Negative</td>
<td>imaginary</td>
</tr>
</tbody>
</table>
After this list was completed, one student wrote, "I really understand this unit. Everything connects to standard form of a quadratic. If you know a, b, c you can usually find the answer." Another wrote, "I'm ready for the test. I memorized all the formulas and when to use them. All I need to do is practice the applications and learn the trigger words, describe - discriminant, solve - QF, factor, always use standard form," wrote another student. Clearly writing has not only enabled students to make connections between concepts, it has also helped them to organize their knowledge.

**Mode 4:** A student volunteer wrote one idea, concept, piece of information or even a question to explore on the overhead. This student then chose the next student to write down his/her idea on the overhead. This process continued until all students had the opportunity to write an idea on the overhead. Once the list was complete, students were to use these ideas to help them formulate their journal entries. After a day or two of using this mode, students were begging to go first. It was, after all, the easiest position to be in. (No one could take your idea.) The following is an example from December 6, 1999. Our lesson focused on finding solutions to quadratic-linear systems of equations. In this sequence, Ann begins and, apart from one teacher-injected verbal clarification, all but one student had contributed new information or made a different connection in writing.

**T:** Ann, why don’t you start us off today.

**Ann [smiling after a sigh of relief]:** Systems can be solved graphically and algebraically

**S2:** When solving graphically, the line has to be in $y = mx + b$ form to graph

**T:** $y = mx + b$ is the slope-intercept formula for graphing a line

**S3:** After making the chart for the parabola and graphing the line you want to find all the points they have in common
S4: It is possible to have 0-2 solutions depending on the equations you are given

S5: our solution needs to be written in squiggles and as a coordinate \{((x,y))\}

S6: The points where the line and parabola intersect is the solution

S7: Always check

S8: Use substitution to solve a system with algebra

S9: If the graphs don’t intersect, then there is no solution

S10: Solving a quadratic gives you two x values

S11: Only reject an answer if they ask you to

S12 [looking disappointed in self]: I can’t think of anything else

S13 [chuckling]: Once you solve for x, you have to solve for y or you will lose credit

S14: It’s easier to graph and get your solution from a picture

“Even though \(x^2\) tells you to get 2 answers, that doesn’t always happen in systems,” wrote one student after reflecting on the review list. Another student wrote, “This is just like Course I. It’s easy cause you just apply what you learned last year. Solutions occur where 2 lines intersect – now it’s where a line & parabola touch.” A less elaborate, but more typical response was “It shows how many times and where lines intersect.”

**Mode 5:** Our final mode of modeling was a whole class discussion. Various ideas were discussed. Of these, students chose what they wanted to write about. For the most part, students chose similar ideas to write about. Sometimes, however, all the students wrote about different ideas. As is evident from the text below, this mode gave students the opportunity to expand their mathematical thinking to real-life applications of what they has been learning. The following example is a re-creation, based on notes from my
journal, of a verbal discussion we had as a class on December 7, 1999 after we had worked through some real-life examples involving quadratic equations.

T: Does it help to know there are times in your life where quadratic equations exist?

S1: When you are playing sports, you don’t think about parabolas.

T: True, but now that you realize the path the ball takes makes a parabola...

S2 [interrupting me]: It helps to see why we need to learn about quadratics

S3: This doesn’t affect how you play the game

T: That’s not true. Think about the problem we did involving Natalya Lisovskaya (see Appendix D). Don’t you think her coach helps her throw the ball as far as possible? This requires the coach to calculate the maximum height she needs to get the ball too in order for her to maintain her world record.

S4: Same with Michael Jordan. You can’t get a basket if you throw a brick.

T: Excellent point. There needs to be a certain arc on the ball for the shot to be good.

S3 [speaking softly]: I guess I never thought about that before.

T: What about a contractor? Or a statistics report? Or planting a garden? If your mom wants to plant a garden on the side of the house, she is really using math. Why? How?

S5 [smiling]: She needs to calculate how much of an area she can use so my dad won’t get mad about her digging up the yard.

T [laughing]: Speaking from personal experience?

S5 [laughing]: Yup

T [pointing to number 5 on the sheet] (see Appendix D): Here is an example of this type of situation involving quadratics.
T [smiling]: You guys are always asking me where we use this topic in real-life and now you have an application that applies to quadratics. There are many more out there. All you need to do is think about it a little and you will be amazed with the number of places math appears in your life.

Some of the comments from these journal entries are very interesting. Here are a few examples: “Quadratics gives sports a whole new meaning – one that no one thinks about too much.” “I wish there was an example of where all math concepts are used in real life. It would make math more worthwhile to learn,” and “I guess if you think hard enough, math really is everywhere.”

Journal Transactions

As the teacher, I knew I needed to relate discussion sessions to how students can better respond and make better journal entries. As a result of these five modes of modeling, students were provided a context within which they could develop their writing. My students became responsive to what was expected of them and began to write more elaborate entries in their journals, all of which provide evidence that higher-level thinking was taking place and that had not previously occurred. I truly believe the modeling of my responses, in conjunction with students thoughts and feelings, helped students to understand that nothing they wrote was incorrect and that they could be honest with me and really share their thoughts, understandings, confusions and enlightenments (a-ha’s) through written communication.

Since this is only my second year teaching Course II (my third year teaching), I rely on the information passed on to me by veteran teachers in my department. When we began the unit on quadratic equations last year, I was advised to skip the sections in the chapter on deriving the quadratic formula and completing the square. Because completing the square is used more in
higher-level mathematics courses, we did not need to emphasize it in this unit. Moreover, I was informed that the derivation of the quadratic formula would only confuse students and that the time could be better spent on applying the formula. Following this advice, I planned on teaching the unit in the same sequence as I did the year before, but my students’ curiosity led me in an unexpected direction. As a result of their interest in knowing the details of the quadratic formula, more specifically, what the values of a, b, and c mean in the formula, I modified my existing schedule to make time to discuss these parameters. My following is excerpted from overhead collaborative notes (Mode 3) taken on November 30, 1999 after a lesson on discovering the meaning of a, b, and c in a quadratic function. Following these notes are some samples of journal entries classified by their level of response: one-line, middling or more elaborative. The responses, whatever their level of elaboration, indicate that students made connections and appreciated being able to make sense of an otherwise formulaic mathematical structure.

Example:

Activity (see Appendix D)
- graph equations on the TI-83 graphing calculator, discover what the values of a, b, and c mean in relation to a quadratic equation

Brainstorming (Mode 3, collaborative written notes developed in class)
value of a: positive means parabola opens up, negative means parabola opens down, the bigger the value of a, the closer the parabola becomes to the y-axis and vice-versa

value of b: x-intercepts (roots) of the equation, where the parabola crosses the x-axis

value of c: y-intercept of the equation, where the parabola crosses the y-axis
- easier to visualize what the graph of the equation looks like knowing the meaning of $a$, $b$, and $c$
- connection between algebraic and graphic solutions
- anytime the parabola crosses the x-axis, the discriminant is greater than zero
- parabola only touches the x-axis once when there is a double root

**Responses from journals**

**One-line responses**
- "helps to see how all the parts we have learned about fit together"
- "graphing quadratic functions ties everything together and makes everything else make more sense"

**Middling responses**
- "calculators really help to see many examples quickly, but we should learn how to graph parabolas by hand too"
- "I never knew the connection between finding the roots algebraically and a graphic solution. Now I know that using the two methods for solving a equation can help me to double check my answer...helps me to know my answer is right"
- "seeing a use for the discriminant and how it helps me understand the solution to the equation"

**More Elaborated responses**
- "Since I have always been good at math, I never really cared where equations came from. All I want to know is what the formula is and how to apply it. Discovering the meaning behind the values of $a$, $b$, $c$ in an equation has really helped me to see how all parts of a quadratic equation fit together and why we need to learn details. Besides, every student likes
to know that sure-fire way of knowing his or her answer is right. Using the information from today’s lesson will help."

"The coefficient of c will help me check right away to see if I graphed my parabola correctly along with the direction the parabola opens. Some questions have asked to solve the equation algebraically, after graphing it (too much work) and I can see if my answers match the location where the parabola crosses the x-axis."

Another type of journal transaction involved a written dialogue between the students and me. In addition to the four questions the students answered on a daily basis (see Methodology), I continually wrote probing questions in individual student journals to help guide them to their own solutions. By asking questions such as, "What do [real, imaginary] mean in relation to the discriminant and the quadratic formula? Think deeper" and "What do the words [real, rational, irrational, imaginary] mean with respect to a quadratic?" I hoped to provide students with the opportunity to think deeper into the underlying meaning of the concept. As a result, some students were able to reason out conclusions based on these journal transactions. The following is an example of a written conversation between Margaret and myself that takes place over an extended time frame between November 15 and December 5, 1999.

Margaret [writing]: "I think the significance [of a, b and c] is that it's an easy way to organize the information needed for solving the problem. It also makes it easier to remember."

Teacher [responding]: "You may discover new meaning for a, b and c as the unit progresses. Keep thinking about how these values connect to the concepts we are learning."

Margaret [discussing the discriminant]: "The solution tells you what kind of number the quadratic is. It tells if it’s real or imaginary, rational or irrational and equal or unequal."
**Teacher**: “What do these terms mean with relation to a quadratic? How does its value help you see the connection between $a$, $b$, and $c$ and the discriminant? Keep thinking.”

**Margaret [after an exercise on the values of $a$, $b$, and $c$]**: “We learned what the variables $A$, $B$, $C$ represent in a quadratic parabola.”

**Teacher [responding]**: “What do they represent? Does this help answer your questions from earlier in the unit? How? Why?”

**Margaret [in response to me]**: “It helps because it shows you where your equation comes from and how all the parts fit together. And it helps you figure out where & what the roots are. The values of $a$, $b$, $c$ gives you all of this information.

- **‘a’** - tells whether the parabola opens up or down
- the bigger the coefficient the thinner the parabola
- **‘b’** - tells you what the x-intercept is
- as the value $b$ increases the turning point gets higher
- tells you the roots of the quadratic
- any time the parabola crosses the x-axis the discriminate is greater than zero
- if it has a double root the parabola touches the x-axis once – and that spot is the turning point
- to find the roots – factor & quadratic formula
- to describe roots, find discriminate $b^2 - 4ac$
- **‘c’** - y-intercept of parabola

**Teacher [in response]**: “Excellent. Does this information make more sense now that you discovered the meaning as opposed to me telling you?”

**Margaret [in response]**: “Yes. I think I will remember it more because I found out the information and just wasn’t told it.”
Even as I discovered that I must model the writing that I expected my students to do, so I discovered how powerful writing can be in transforming not only the thinking of individual students but the climate of the classroom as well. As the previous section documents the effect of writing on individual students, so this section will document the effect of writing on the culture of the classroom as a whole.

In order to deepen students' thinking through writing, I often did not immediately offer solutions to their "why" and "what if" questions, rather invited the students to explore their own thinking in their journals. The following is adapted from my own journal and field notes, taken on November 19, 1999, and documents a period of transition both for me as well as for my students.

While students were working in their groups trying to solve some difficult quadratic word problems set in a geometric context, I could sense the level of frustration rising. Several students did not know where to begin and immediately asked for help. Much to their dismay, I did not offer any helpful suggestions. Instead, I told students to think about the problem and brainstorm with their classmates. Since so many students are trained to immediately ask for help instead of working and sometimes struggling through a problem, this was not a response they wanted to hear. Hoping that some interesting responses would develop while groups were thus engaged in the problem-solving process, I made the decision to resist their desire to have me explain the problems. Here are a few student responses from the end of the activity:

"Help", "I can't do this."

"I am so lost. Why won't you help me?"

"I don't know what to do"

"It's not fair you won't help"
“Why are you being so mean, Miss Vaccaro?”

Although not much was accomplished this day so far as progressing through the material, I mark it as the beginning of a significant change. Students finally got over the “I can’t do it stage” and started connecting concepts until they began to be successful. Students learned they could struggle through a difficult problem, both in class and in their journals, and come up with some kind of reasonable response on their own. Writing, as well as brainstorming (which was, as indicated above, a form of prewriting activity), offered students a viable channel through which to express their growing ability to persist in their solution-seeking endeavors. One student’s response to my question, “Name an advantage to journal writing” was “if I have trouble, it’s not a bad thing, b/c you let me go, and be frustrated! Sooner or later, I know I will get it.”

I remember one instance in class, on November 15, 1999, when Sarah finally realized how the quadratic formula connects to the standard form of a quadratic. The expression on her face was priceless as she exclaimed, “I get it!” From her verbal and facial expressions, you knew that a light bulb had gone on inside her head and all the pieces to the puzzle finally fit together. For the remainder of the class, her glowing smile lit up the room. As a result of her experience being deep, she was able to make sense of the topic.

As we progressed through the journal writing process, I could sense students’ confidence building. Their writing was getting stronger and they were letting their personalities begin to show in their writing. Even though some students were not interested in writing in journals, they took the time to answer all the questions I posed and thought-out their responses. My trust in the students and the atmosphere of our classroom are the main reasons why everyone in the class,
including myself, appeared to grow as a result of this experience. Without the journals, I do not believe this would have been possible.

**Emergence of a Discourse Community**

Despite the evident success resulting from my modeling responses for the class, some students were still struggling with putting their thoughts into words. On November 22, 1999, I decided to pose selected questions from individual’s journals to the entire class (without using names) in the hope that other students would be willing to respond to the questions and offer their insights into what they believed were appropriate conclusions. Students helping students, I reasoned, would help to further our discourse community that I saw emerging from the classroom interplay between discussion and the journal writing. This proved to be more successful than I had anticipated, the original intention of which was for students to help each other develop their mathematical understanding by clearing up misconceptions and deepening their personal understanding of the material.

For example, Tom had asked in his journal on December 3, 1999, “When is there more than one answer to a system?” I brought up Tom’s question the following class period, without identifying him. Julianne answered the question by offering an example that involved the conditions under which there is and is not more than one solution to a system of quadratic-linear equations. On the following day, December 4, 1999, Tom had responded in his journal to his original question with, “I am not sure why I didn’t get it yesterday but knowing the solutions are when the lines intersect really helps.”

As a result of students responding to each other’s questions, they had more to write about after their question was answered in class. On occasion, additional questions arose which led to a deeper understanding of mathematical connections for all students. For example, after Tom’s
question was answered in class, another student (John) wrote, “What would happen if you
graphed 2 parabolas as a system?” I thought this was a very interesting question, the very sort of
conjecturing I hoped would emerge from the journal writing, and certainly would be well worth
investigation the next day. As I posed the new question to the class, the puzzled looks on several
students’ faces told me they had not thought of this question. After students thought for what
seemed to me like a long time, John asked if we could try an example. Below is a re-creation of
that day in class based on my journal entry.

I pulled out the TI-83 graphing calculators and we proceeded to graph the equations
\[ x^2 + 2x + 1 \quad \text{and} \quad -x^2 - 3x + 4. \]
As we looked at this system of equations, some students offered
suggestions as to what was happening on the screen. Since the graphs intersected at two points,
they reasoned there were two solutions to the system and moreover, the maximum number of
solutions a system of quadratic equations could have is two, based on the shape of a quadratic
function. They observed that one difference between quadratic-linear systems and quadratic
systems are that the maximum number of possible roots is four. Again, due to the nature of the
graph of a parabola, John’s question turned into an unintended, yet excellent discussion on
systems of equations that threaded its way from journal to class to journal to class.

As students were finding solutions to factorable quadratic equations, they came across a
problem that could not be factored. As I posed this problem to the class, several students
immediately shot their hands into the air as if it were a race to respond. The first student called
upon came to the board and proceeded to put the quadratic formula up for all to see. Since I had
not taught this topic, I asked her how she knew the formula. Smiling, she answered, “I learned it
last year.” I hid (I hope) my disappointment since I had planned on introducing the quadratic
formula with specific examples that I hoped would help students see the connection between the
equation and the formula. Now with half the class already familiar with the formula, my lesson plans needed to be altered once again – a teachable moment gone, I thought.

It was obvious students were not having any difficulty plugging numbers into the equation but accepting without question the results of this classic, straightforward algorithm – until November 10, 1999 when Katie wrote “I know a, b, c guide you to put each number from the problem into the formula and solve, and it makes it a lot easier, but where did the formula come from?” My teachable moment had returned.

In class the next day, I posed the question about the derivation of the quadratic formula and was pleasantly surprised by the responses. In whole-class guided discovery, we were able to derive the quadratic formula from the standard form of a quadratic. After we completed the process, the students were visibly affected. The expressions on several students’ faces told me that they had made connections and that they understood the quadratic formula at a far deeper level. One student even wrote, “Why don’t other math teachers show us where formulas come from? It would help me to understand how different concepts connect together.” Although I had not planned on taking two days of class time to show students how the quadratic formula was derived, I am glad the question arose and I believe these students not only will have a better understanding of solutions to quadratic equations but will have understood something about the nature of mathematics. When asked by a colleague why I was spending so many days on the quadratic formula, my only response was how natural it felt to teach this formula to students who had a desire to know. For me, this was a learning experience as well – students had taught me that they were capable of learning material many (myself included, before having this experience) feel they are not ready for. Instead of just teaching my students to “plug and chug” with the quadratic formula, I became convinced that they are able to comprehend material at a
much deeper, hitherto unsuspected, level. As a result of my success with this class, I plan on incorporating the derivation of the quadratic formula in all my future journal-writing Course II classes and to rethink other topics as well.

Through the class discussions and journals, a viable discourse community was being created where students were learning from other students through verbal and written exchanges. I believe this is attributable to the journal writing and the modeling done in class. As I communicated to my students via public writing, they responded publicly as well as privately. This balance helped enhance our discourse community. Students were challenging their own understanding of concepts by having to write them down and make connections between topics. From this stemmed many questions that we were able to explore as a class. Without the journals, I do not believe this would have happened. In contrast with my other Course II class, who were not involved in the daily journals, these discussions did not take place. Students were not asking probing questions, nor wanting to know where formulas came from. They were content to know how to use the formula to get the right answer and that was all. For example, on November 12, 1999, the following conversation took place with the non-journal writing class after we had derived the quadratic formula in my other Course II class. It began by my asking if they too were interested in knowing the derivation of the formula. As I began to show them where the quadratic formula came from, they asked me to stop because it was just going to confuse them. One student responded sarcastically, "...show us, it will take [up the time] time." In contrast, students in the journal class were eager to make connections between topics we were studying—evidence that the journals contributed to students' levels of curiosity and desire to understand how topics connected. Their openness and motivation to learn new things stemmed from questions they posed while writing and the questions that subsequently resulted from classroom
discussions. From them, our journals helped my students become active learners and motivated them to learn and thrive in a discourse community.

Other Changes In My Teaching

In addition to the changes previously mentioned, I found myself going into more depth in certain topics. As I read and responded to the questions students posed, I was able to use this writing as a basis for taking the class to the next level of understanding and to even challenge them with concepts that I previously would not have considered suitable for their level of understanding. I found that not only were the students intrigued by the questions that their classmates raised, but I was also surprised by the wealth of information and the number of connections the students were able to establish. For example, Course II teaching materials do not typically have students discover for themselves the roles that a, b and c play in a quadratic function. As a result of my students’ interest and success in understanding how the graphic solution connects to the algebraic solution, I now know how to integrate this component into the unit. As a result, I have developed a guided discovery lesson that uses the TI-83 graphing calculator as the main tool to help students come to their own conclusions with regard to the function of these parameters (see Appendix D). When I shared my discoveries with two other teachers in my department, my colleagues were surprised that my students expressed so much interest in knowing all the specifics. I feel this interest stemmed partly from my students being offered the opportunity to pose written questions about the derivation of the quadratic formula and partly from their heightened desire to make sense of the material that evidenced itself in their journals. Indeed, one teacher thought he might add this activity to his unit. Encouraged by this conversation, I decided to share this activity with all Course II teachers in my department. In
summary, not only has journal writing changed my day-to-day teaching practice but it also has had an impact on my teaching considered within the larger context of my professional life.

The amount of time that went into the preparation of the quadratic equations unit was more than I originally anticipated much more. Between planning for each class, preparing the daily journal entry questions, grading papers, reading and responding to students’ journals and writing in my personal journal, I was spending an unusual amount of time on this particular class. Normally I spent approximately 20-30 minutes of outside preparation for each class. Since the introduction of daily journals, I have been spending between one-to-two hours an evening preparing for the next day’s lesson. As I revised the components of the unit I found myself neglecting my other classes and focusing on this one class, as it held the highest importance to me at the time.

Before I could have all my classes use daily journals, I would need an ample amount of time to sit down and really plan out the unit. In doing this, I would also need to realize that the unit would require continual modification throughout the teaching process, as students are now responding to what is happening on a daily basis. Students are going to be much more honest and open in writing than there were verbally. In addition, I will receive feedback from each student each day, which is not usually the case. Once I feel comfortable with the amount of time I devote to reading and responding to the journals of one class, I may then want to begin the process of journal writing with another class. This will allow me time to alter my schedule so that I will have enough time each day to get all the journals read and still have time to prepare for my other classes. With practice, daily journals in each class will be valuable for both the teacher and the students. The time involved in getting to this point is enormous, but in the end, the pay off is worth every minute of preparation. After reading my students’ entries, I felt as if I had an
insight into my students that I otherwise would not be privy to. Because my students opened up and shared their thoughts, feelings and emotions with me, I was able to better meet the needs of all my students.

Findings

Secondary Question 1

At the present time, I cannot imagine using daily journals in all of my classes. I feel it would be so onerous that the efficacy of journal writing to promote student’s ability to develop their own understanding and come to their own conclusions would soon dissipate. Instead of taking the information students record and using it to modify an existing lesson or delve deeper into a current topic, I might read their response and just respond with a comment—and this would be a disservice to my students. My concern is this: how could a teacher, who is also a varsity coach, a class advisor and a returning student find time to read and respond to 100+ journals a night, while keeping up with all of her other teaching duties? I have not figured this out. But, despite this last sentence, I believe in the benefits of using daily journals in all disciplines, especially mathematics. Not only does the teacher get the students to write in a mathematics class, but she also helps the students to formulate their own “what if” questions, thereby fostering inquiry and leading them through the process of arriving at an answer on their own. Instead of telling my students “how” or “why” a concept is true, through writing in journals and the development of a discourse community, students were able to conduct their own investigations and come to their own conclusions. Not only did journals help to make learning more meaningful for the students, but it also forced them to become more responsible learners.

“Writing in journals makes you realize what you don’t know and need to practice more,” one
student commented. This comment reveals in a stroke one of the most important benefits that journal writing fosters: if you do not have to communicate your understanding of a concept, it is easier to make yourself believe you understand what is happening in class when in fact, they do not. Or, on the other hand, students may believe they are not capable of probing deeply into material when, in fact, they are.

In spite of my conviction of the effectiveness of journal writing, I have not come to any conclusions as to how this process can be easier for myself – both in workload and time management. I will continue to search for ways in which daily journal writing can retain its benefits and yet be less of a time management issue. This issue of time will have a major impact on my decision whether or not to use daily journals in more than one of my classes.

Another challenge that I encountered throughout the project, which is worth mentioning, was balancing the journal writing time, the instruction time, the group work time and dealing with homework questions during each class. Since most writing was done at the beginning and the end of the class period, I always felt rushed. Certainly I did not want to spend too much time going over homework and examples in class. Yet, I wanted to make sure I gave students enough time to really expound on what they were thinking and feeling during each lesson. I sometimes felt as if I was rushing the learning process so they would have ample time to write, a critical factor since students were not able to take their journals home with them. There were days when I felt as though I cheated them out of extra examples or cut their questions short just so they would have enough time to tell me this in their journals. Then there were days when I pushed aside the writing time because I did not want to interrupt the discussion we were having in class. And yet when there was not enough time left for journals at the end of class, I felt their recollection of key issues or how they felt at the moment was lost when they wrote the next class
about what had happened the previous day. It was quite clear that next-day entries were shorter and less affective. One day, while discussing the discriminant, we ran out of class time. Students only had a few minutes to write. I could tell many students felt rushed because they all used one-line responses. For example, “we talked about the discriminant” or “it helps to know what the graph will look like”. There was simply not enough time for them to elaborate. As students walked into class the next day, I handed them their journals and asked them to finish their entries from yesterday. Not too many students had anything to add because the lesson was old news and they did not really remember how they felt while learning about the discriminant. One student wrote, “it helps to know what the graph will look like” on the previous day had this to say the next day: “I told you what I knew yesterday.”

Since students’ journals also served as their class notebooks, I had to make sure copies of the notes were available for students to take home with them on days when they learned a new concept. At times this was difficult to do, since my lesson plans changed frequently in response to student needs. Being flexible and planning at least a week ahead is vital to staying afloat throughout the process of daily journals despite the fact that these carefully laid plans were usually altered.

Secondary Questions 2-6

I found many reasons for the establishment of the viable discourse community described above. The students wrote quality journal entries and their reflections deepened as a result of my modeling mathematical writing. Their understanding of the material increased as their content knowledge developed, and the relationships between myself and my students grew in mutual respect and trust, thereby transforming the classroom atmosphere. The discourse community, in turn, would not have been possible without the modeling of good written mathematical
discourse, which, I soon discovered, took form in ways I had not anticipated before the study began. To reiterate, these were:

**Mode 1:** I would write the journal entry on the overhead projector with students at the end of class.

**Mode 2:** I would record notes on the overhead based on students’ verbal responses. From these notes, students would write their journal entry.

**Mode 3:** A student would record the list of brainstormed notes on the board for students to use in their journal entries.

**Mode 4:** Each student in the class would be responsible for putting a concept or idea on the overhead. Once the list was completed, students would use this list to complete their journal entry.

**Mode 5:** In their journals, students would expand on ideas mentioned in our whole-class discussion.

But it was through the interactions of these modes of modeling that the expression of the discourse community took place. It came into being through the flow of discourse, through the written to the oral to the written, and again to the oral as ideas moved from my overhead writing to student journals to students’ overhead writing to students’ journals to whole-class discussions that prefigured both overhead and journal writing.

**Secondary Question 7**

Since the unit on quadratic equations is traditionally taught as a “technique unit”, I was not at all certain that it would be conducive to daily journal writing due to its algorithmic nature. Moreover, I wondered how much of this material could be “discovered” by students through investigation since the unit entails so much fact-driven information. After going through the
review material at the beginning of the unit, I discovered that the writing helped students to make
connections between concepts and to question what they were learning through reflection in their
journals. I learned that any topic is conducive for writing as long as the questions are well
thought out and students have models to help guide them through the writing process.

Secondary Question 8

I did not change my assessment practices to accommodate journal writing because of the
highly experimental nature of my project; that is, I was not sure of what assumptions I could
make as to what my students could or could not do. However, I am convinced that students who
do extensive journal writing would succeed quite well on more conceptually driven assessments.
For example, as we move toward the new New York State Math A test, it would seem that daily
on-going journal writing in Course II classes would be highly beneficial for students success.
That is, in order to prepare our students for exam questions requiring justification of solutions
and explanations as to why an answer is correct, writing in mathematics classes will become
more a matter of course. Based on my research, it would seem that students who do extensive
journal writing should be more successful on the Math A test.

Although I did not change my assessment practices during the project, I nevertheless
gave the same "traditional" exam to both my journal-writing class and my non-writing class.
Even though I had spent less time on symbolic manipulation in my journal-writing class than I
did in my non-writing class, the journal class did slightly better on this conventional, symbol-
manipulation exam than did my non-writing class, even though the latter had received more
direct instruction, a result that is consistent with other research findings involving non-traditional
teaching approaches.
Primary Research Question:

Journal writing had a positive impact on my teaching. Through journals I was able to ascertain a fuller picture of my students' capabilities. I learned that not only are they capable of more than I had previously given them credit for but I also learned how rewarding it is to try something new in the classroom--both for me and for my students. It helps me grow professionally, as well, by teaching my how to show students that there is more to math than they had realized--that understanding and being able to communicate why a concept is true is just as important as knowing how to apply formulas. Written communication helped my students become more active learners; in return they committed themselves to understanding "how" and "why" a concept is true. Providing my students with additional modes of modeling good writing that further students' knowledge and understanding of concepts placed demands on me that I had not previously experienced; for example, modify the course content to include more challenging problems and thus cope with all the increased communication with my students on a daily basis.

Summary

In retrospect, I learned a lot about my students during the journal writing process, both those who are vocal during class discussions and, perhaps more important, those who are not. For those students who always participate in class discussions, I was able to read their thoughts on paper and follow them through the problem-solving process as they struggled to reach a conclusion. For those students who are very reserved in class, I was able to gain access to their thinking, which was hither to inaccessible to me, so that I was able to determine whether they really understood what was going on and then to engage them through written communication. In this way, I found that the journal writing process helped me to meet all the needs of my students on a daily basis and so to become a better teacher. For not only did I learn a lot about
my students through the journal writing process, I also learned something about myself. Not only did my students teach me to strive for continual improvement of my understanding of concepts and connections, they also helped me realize that veering away from the norm for an extended period of time is refreshing and rewarding. I have always felt that students will meet my expectations if I convey them, but never really tested this theory out until this unit. Because of this, I put more faith in my students and raised my expectations of them and they have met and exceeded those high expectations I set.

Many members of my math department are veteran teachers (I am the youngest member of my math department). It was very interesting to see their reactions to my work with daily journals. Some of the teachers were glad that it was I struggling with the time management issue and not they, while others saw beyond the issue of time and were intrigued with what I was doing in class. They were interested in knowing about the reactions I received from my students and the amount of dialogue that I was able to achieve with each student on a daily basis. One of the teachers in my department decided she wanted her students to experience the benefits of writing in mathematics class.

Now that the project is completed, I can tell the members of my department that I am glad it was I who struggled with the time management issue of reading and responding to daily entries. If I had not done this, I would still believe that students would not be able to handle the derivation of the quadratic formula or want to know how a, b and c related to a quadratic equation. I would strongly encourage mathematics teachers to try daily journals with one of their classes, even if it is only for a short period of time. I found it to be an eye-opening experience to get into the heads of my students. If I had not undertaken this project with my Course II students, I would still be teaching the same way and using the typical quadratic equations unit
that I and my colleagues have always used. Establishing of a discourse community through exacting and demanding journal writing made mathematics more exciting for both my students for me.
Appendix A

Ways to use Writing in Your Mathematics Classroom

1. journals
2. learning logs
3. expository writing
4. creative writing
5. diary entries
6. letters to friends
7. advertisements
8. writing word problems and solutions
9. poetry
10. impromptu writing prompts
11. create test questions
12. Posed questions
   a. Write a paragraph about ____
   b. Write a questions containing the following words ______
   c. List some common mistakes you have made in class, on homework, on tests…
   d. List four topics from the chapter and summarize one of them
   e. Explain how to solve a problem
   f. How can algebra be improved
   g. What do you do best in mathematics class?
   h. How can you be sure you mastered the information you have learned today?
13. projects
14. summarize class

15. visual image translation

16. synopsizing tactics for solving a problem

17. describe steps in a mathematical algorithm

18. invent your own word problem
Appendix B

Daily Lesson Plans for Quadratic Equations

Day One: Students will be given a Factoring Review Sheet.

Instructions for Worksheet: Students are to find someone who can successfully solve a problem on the sheet. Once the problem is solved, the student is to sign the sheet. You cannot have the same person sign more than one square on your sheet. After getting all the squares solved, students are to sit down. Once all students are seated, the teacher will go over the problems on the sheet (use an overhead of the worksheet) using class input. This will give the teacher an idea of what students do and do not remember about factoring from last year. Begin the lesson by asking students what FOIL is. (First-Outer-Inner-Last) Review numerous examples using FOIL to help foster recall. Once students have a firm grasp of FOIL, move to steps need to factor an equation:

METHOD 1 – Take out what is in common

Allow students to discover this method given examples placed on the board. Move from easy examples to more complex examples. Once students are able to factor these problems, review the steps necessary for solving a quadratic. Take a minute to discuss some vocabulary terms with students – binomial, trinomial, quadratic and standard form of a quadratic. The last example on the board cannot be solved by using method one, so students must brainstorm as a class to discover another method of factoring that will allow them to solve the problem.

METHOD 2 – Backwards FOIL

As a class, we will practice method two using simple equations. Begin by reviewing homework answers and answering questions based on the homework. Use the beginning of the period to move into more complex quadratic problems that involve backwards FOIL. Throw a problem at
the students that is not a trinomial but can still be factored by using a special method, similar to backwards FOIL.

**METHOD 3 – Perfect Squares**

Take this opportunity to review what perfect squares are. (Mention the benefit of having a calculator on their desk to help them check to see if a number is a perfect square.) Place a variety of problems on the board for students to solve individually (use all 3 methods) and have student volunteers come to the board and solve the problem for the class explaining the steps they used to obtain the answer. Begin by reviewing homework answers and answering questions based on the homework. Place a 2-step factoring problem on the board. Discuss the steps necessary to factor the equation (combination of method one and method two or method three).

**METHOD 4 – Factoring Completely**

Ask students to get with their assigned partner. With their partners, students are to solve the following quadratic problems: pages 428-429, #1,9,15,17,19,21,22, pages 428-429, #25,29,32,33,34,36,38,43

**Homework:** pages 432-433, #1-6,7,11,13,17,23,26,27,28, 31,34. These problems are to be turned in by the end of the period for a grade.

**Day Three:** Students will get back together with their partners. Yesterday’s in-class activity will be handed back and students will have the opportunity to ask questions on any of the problems they missed. Tell students that we are going to put a new twist on quadratics, but first we need to review the use of radicals in mathematics. (*radical* is another name for $\sqrt{}$ - *square root*).

Work through the problems on the overhead with students to make sure they remember the rules associated with solving radical problems. Once students feel comfortable with radicals, place a quadratic equation on the board and ask them to solve it. Immediately students should recognize
that it is not factorable. Referring back to the standard form of a quadratic \((ax^2 + bx + c = 0)\), inform students that a special formula has been developed for solving quadratic equations that are not factorable. Present the quadratic formula. Teach students an easy way to memorize the formula: to the tune of "Row, Row, Row Your Boat", sing the quadratic formula. Practice this song with students. Work through a couple examples as a class, if time permits.

**Homework:** Memorize the quadratic formula \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

**Day Four:** 5-point quiz on quadratic formula and standard form of a quadratic. Guided practice in using the quadratic formula when solving an equation that is factorable (to be sure students are using the formula correctly – check by factoring) and then for equations that are not factorable. Emphasize the importance of keeping answers in lowest terms by reducing the radicals or by rounding answers to the nearest hundredth – advise students to use their calculators for assistance. **Note:** Read problem carefully to find out in what terms the answer should be left in.

**Homework:** Page 443, #10-13 (solve by factoring and using the quadratic formula)

Study for a mini quiz tomorrow.

**Day Five:** Quiz on the quadratic formula, standard form of a quadratic and the application of the formula into a problem that is not factorable. Today is student-as-the-teacher day. Everyone in the class is given a worksheet with 7 problems on it. Each problem has been assigned to a group of students (3 per group). Students are to work individually on solving the problem their group is assigned and then they get together as a group to compare their solutions and the process they went through to solve their problem. After about 15-20 minutes, the class re-groups and one person from the group (alternates throughout student-as-the-teacher days) is responsible for walking the rest of the class through the problem explaining the procedure used to solve the
quadratic equation. After each group presents their problem, the other groups will peer evaluate the presenters the problem.

**Homework:** journal entry – self-reflection of the group activity and personal understanding of the topic discussed in class

pages 443-444, #16-28 even

**Day Six:** Collect journals. Begin by reviewing homework answers and answering questions based on the homework. Begin the lesson by asking students if they know another word for solution (ROOTS). Explain to students that sometimes all you need to know about a quadratic is the roots of the equation. When that happens, you do not want to have to solve the entire quadratic just to get at the roots. A short cut exists – finding the discriminant. The discriminant represents the equation underneath the radical sign in the quadratic formula \((b^2 - 4ac)\). All you need to do is focus on the value of that equation to describe the roots. Explain the 4 ways of describing the roots:

<table>
<thead>
<tr>
<th>value of (b^2 - 4ac)</th>
<th>description of the roots</th>
<th>what this means</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive number</td>
<td>real, irrational, unequal</td>
<td>crosses x-axis in 2 places</td>
</tr>
<tr>
<td>perfect square</td>
<td>real, rational, unequal</td>
<td>crosses x-axis in 2 places</td>
</tr>
<tr>
<td>zero</td>
<td>real, rational, equal</td>
<td>touches x-axis in one place</td>
</tr>
<tr>
<td>negative number</td>
<td>irrational</td>
<td>never crosses the x-axis</td>
</tr>
</tbody>
</table>

With assigned partners, complete the following problems: pages 443-444, #1-9, 34-39

**Homework:** review worksheet, TEST on Day Seven.

**Day Seven:** TEST on quadratic equations.

**Day Eight:** Pass back test on quadratics. Tell students that test corrections and written explanations of questions they got wrong. Ratio review and geometry review, plus geometric applications of quadratics. Ask students what they remember about geometry. Make a list on
the board as students recall ideas while brainstorming. (Be sure list includes lengths must always be positive and lengths can be positive irrational numbers.) Model process for solving geometric word problems on the chalkboard. Use student responses and suggestions to help set up each model problem. Once set up, students are to solve the problem in their notes and as a class we will go over the process involved in obtaining the solutions.

**Homework:** page 448-450, #1,2,4-7,9,10,29

**Day Nine:** Begin by reviewing homework answers and answering questions based on the homework. Once all questions are answers, ask students to get with their partners. The in-class assignment for today is pages 448-450 #3,8,30,31,32,34,35,36. Solutions will be presented at the end of class for students to comment on and ask questions about (use overhead). Remind students that with geometry, there are times when two different approaches will allow you to reach the same, correct conclusion. Note: Checking answers is a good way to be sure you solved the problem correctly.

**Day Ten:** In cooperative groups of 3, ask students to make a concept map based on all prior knowledge of graphing. After about 10 minutes, groups will present their maps to the class and a recorder will place all key words on the board. This list of words will be copied into student notebooks, where students will have the opportunity to jot down reminders after the words, if they feel it is necessary. Once the list is completed, students will be given graph paper and will practice graphing linear equations using the slope-intercept formula or point-slope formula. After recall of graphing lines has occurred, students will be asked to come up with a method for graphing a quadratic equation. This first method that we will use is making a T-chart to represent (x,y) coordinates on the graph of the function, when integral values are given for the x domain. As a class we will work through one example. From this example, another list of
characteristics of the graph will be developed. This list will lead into discussion of axis of
symmetry, minimum and maximum points, turning point and the term parabola. Do other
examples if time allows.

Homework: on graph paper, do page 455, #3-15 odd

Day Eleven: Review homework by placing already made graphs on the overhead so students
may compare their work with the teachers' work. More guided practice with graphing equations
using T-charts and finding the characteristics of the parabola. The introduction of a
technological tool, the TI-83 Plus Graphing Calculator, will help students visualize the changes
that occur when certain numbers are changed in the quadratic equation. Students will see what
an advantage there is to having technology incorporated into a mathematics classroom. Allow
student time to explore functions on the graphing calculators.

Homework: on graph paper, do pages 455-456, #18-20,23,28,30

Day Twelve: Check homework by having students compare their paper and pencil graphs with
graphs produced by the calculator. Re-visit the discriminant. Introduce the term x-intercept to
students and the connection between the discriminant and the number of x-intercepts a parabola
has. Pass out handout that shows examples of different points of intersection. Ask students to
come up with 3 methods for finding the solution to a quadratic equation. Discuss all three
methods:

1. use graphing calculators
2. use the discriminant
3. graph the parabola and use the quadratic formula to find the value of the roots

In-class project – pro/con activity
Go to the computer lab for class where I will explain how to use the computer program Graphmatica. After exploring the program, tell students they are to follow the instructions on the worksheet while working with Graphmatica. Collect completed worksheet at the end of the period.

**Day Thirteen:** Pop review quiz on graphing systems of equations. Verbal discussion of student responses to the pop quiz. Grade papers in class and discuss results of the quiz. Lead into a discussion about systems of equations (Course I review). Ask students to jot down the steps they believe will be used in graphing systems of equations involving a quadratic equation and a linear equation. On the board make a master list for the steps involved in a graphic solution of a quadratic-linear system of equations. Remind students that their answers must be written in the form of a solution set, \( \{x, y\} \). Walk through a couple examples with students (overhead graph paper).

**Homework:** on graph paper, pages 475-476, #3-5, 8,11,13,17,18

**Day Fourteen:** Review homework problems and questions related to the homework. Ask students if they can think of any other way to solve systems of quadratic-linear equations that would be more efficient and less time consuming. After this discussion, demonstrate how to solve a system of equations on the TI-83. As students practice how to graph a quadratic-linear system of equations by hand, demonstrate how to find the solution on the calculator.

**Homework:** begin studying for the end of the unit test

**Day Fifteen:** Now that students are comfortable solving systems of equations graphically, tell students we are going back to algebra. Students now have to solve quadratic-linear systems of equations algebraically. Remind students there are 2 ways to do this:

1. addition method
2. substitution method

Model some examples using both methods and have students work individually in notebooks on examples. Toward the end of class, have selected student's place their solutions on the board and discuss the process used as a class.

**Homework**: page 481, #7,9,13-15,17,22,25

**Day Sixteen**: Go over homework and answer any questions that students may have. Pass out review sheet and work through problems as a class, where each student is responsible for explaining the process involved in at least one problem.

**Homework**: study for test

**Day Seventeen**: Unit TEST on Quadratics

**Homework**: Answer last journal question.

**Day Eighteen**: Written response based on reaction to the test. Student interviews about groups and journals for the unit.
### Appendix C

#### Table A1

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic in the Unit</th>
<th>Question of the Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>Discuss journals, put students into groups, factoring review</td>
<td>What are your feelings about journal writing? Do you think this is a beneficial way to approach learning?</td>
</tr>
<tr>
<td>Day 2</td>
<td>solving quadratics by factoring</td>
<td>Is it necessary to review concepts year after year or do you feel that students should be responsible for remembering previous learned material?</td>
</tr>
<tr>
<td>Day 3</td>
<td>radicals and deriving the quadratic formula</td>
<td>How important is it for you to know and understand where/how a formula was developed?</td>
</tr>
<tr>
<td>Day 4</td>
<td>quadratic formula</td>
<td>What do you think the significance of the $a, b, c$ are in the quadratic formula?</td>
</tr>
<tr>
<td>Day 5</td>
<td>quadratic formula, pop quiz: formula, standard form</td>
<td>What does your solution to a quadratic (roots) tell you about a quadratic?</td>
</tr>
<tr>
<td>Day 6</td>
<td>discriminant</td>
<td>Describe the connection between the value of the discriminant and the roots of a quadratic equation.</td>
</tr>
<tr>
<td>Day 7</td>
<td>TEST</td>
<td>Has your group work helped you to be more or less prepared for the test? Explain.</td>
</tr>
<tr>
<td>Day 8</td>
<td>geometric applications of quadratics</td>
<td>Describe the importance of working through your frustration with the geo-quad word problems. How difficult is it to keep from giving up and asking for help?</td>
</tr>
<tr>
<td>Day 9</td>
<td>geometric applications of quadratics</td>
<td>Describe a real-life example of when quadratics and geometry simultaneously exist. Be sure to describe the problem and include its solution.</td>
</tr>
<tr>
<td>Day 10</td>
<td>graphing quadratics – TI -83</td>
<td>What is your initial reaction to the graphing calculators? Should they be an integral part of your mathematics curriculum? How do they help connect concepts together?</td>
</tr>
<tr>
<td>Day 11</td>
<td>graphing quadratics</td>
<td>Describe the process of graphing a parabola?</td>
</tr>
<tr>
<td>Day 12</td>
<td>graphic solutions of quadratics – Graphmatica</td>
<td>Compare and contrast the TI-83 graphing calculator and the computer program Graphmatica.</td>
</tr>
<tr>
<td>Day 13</td>
<td>real-world problems – students choice of technology, pop quiz: graph quadratic, axis of symmetry, TP, find and describe roots</td>
<td>What are the benefits of incorporating technology into the classroom? How does this technology allow you to go beyond what is possible on paper and pencil?</td>
</tr>
<tr>
<td>Day 14</td>
<td>graphic solutions of quadratic-linear systems</td>
<td>It is necessary to revisit a mathematical concept each year, such as solution to a system of equations?</td>
</tr>
<tr>
<td>Day 15</td>
<td>algebraic solutions of quadratic-linear systems, pop quiz – system of equations</td>
<td>Explain what the solution to a system of equations means.</td>
</tr>
<tr>
<td>Day 16</td>
<td>review and catch-up</td>
<td>What are your feelings about journal writing? Do you think this is a beneficial way to approach learning? How have your thoughts/feelings changes since the beginning of the unit?</td>
</tr>
<tr>
<td>Day 17</td>
<td>CULUMATIVE TEST</td>
<td><strong>Homework question:</strong> Would you like to use this method of learning/instruction for another unit? Explain why or why not.</td>
</tr>
<tr>
<td>Day 18</td>
<td>write about test and corrections</td>
<td>personal interview with teacher</td>
</tr>
</tbody>
</table>
Appendix D

Investigating Graphs of Quadratic Functions

Class Discovery Worksheet

How do the coefficients $a, b, c$ affect the shape of the graph of the quadratic function $y = ax^2 + bx + c$?

1. Using a graphing calculator to graph $y = ax^2$ using -2, -1, -0.5, 0.5, 1, 2 as values of $a$. Adjust the viewing window if necessary. Discuss your results with others in your group.

Write a sentence that describes how the value of $a$ affects the graph of $y = ax^2$.

2. Using a graphing calculator to graph $y = x^2 + bx$ using -4, -2, -1, 0, 1, 2, 4 as values of $b$. Adjust the viewing window if necessary. Discuss your results with others in your group.

Write a sentence that describes how the value of $b$ affects the graph of $y = x^2 + bx$.

3. Using a graphing calculator to graph $y = x^2 + c$ using -5, -3, -1, 1, 3, 5 as values of $c$. Adjust the viewing window if necessary. Discuss your results with others in your group.

Write a sentence that describes how the value of $c$ affects the graph of $y = x^2 + c$. 
1a. Graph the equation \( y = x^2 + 2x - 8 \) for the interval \(-5 \leq x \leq 3\).

   b. Write the equation of the axis of symmetry.

   c. Write the turning point.

   d. From the graph, find the roots of \( 0 = x^2 + 2x - 8 \).

   e. Graph the line \( y = -2x - 11 \) on the same axes with part a.

   f. Find the intersection of the two graphs.

   g. Does part a reach a maximum or a minimum value?

2. A soccer player chips a ball that follows a path given by \( y = 48x - 16x^2 \), where \( y \) is the height (feet) of the ball and \( x \) is the time \((0 \leq x \leq 4, \ 0 \leq y \leq 50)\).

   a. At what time is the ball on the ground?

   b. How high is the ball 1 second after it is kicked?

   c. How long is the ball in the air?

   d. What is the greatest height the ball reaches?

3a. Draw the graph of \( y = -x^2 + 3x + 4 \) for the interval \(-1 \leq x \leq 4\).

   b. Write the equation of the axis of symmetry.

   c. Write the turning point.

   d. From the graph, find the roots of \( 0 = -x^2 + 3x + 4 \).

   e. Algebraically find the roots of \( 0 = -x^2 + 3x + 4 \).

   f. Graph the line \( y = -x + 4 \) on the same axes as part a.

   g. Find the intersection of the two graphs.

   h. Does part a reach a maximum or minimum value?

   i. Is the turning point the same as the roots?
4. Suppose a volleyball is served with a path of \( y = -4.4x^2 + 17.6x + 2.5 \), where \( y \) is the height (feet) of the ball and \( x \) is the time \( (0 \leq x \leq 5, \ 0 \leq y \leq 25) \).  
   a. At time 0, where is the ball? Why?  
   b. How high is the ball after 2.5 seconds?  
   c. How long is the ball in the air?  

5. If there was a ceiling 20 feet high, would the ball clear the ceiling? A square and a rectangle have the same area. The length of the rectangle is 5 inches more than twice the length of a side of the square, and the width of the rectangle is 2 inches less than the length of a side of the square. Find \( x \) to 3 decimal places.  

6. An insurance company reports that \( y = .4x^2 - 36x + 1000 \) relates the age of driver \( x \) (in years) to the drivers accident rate (accidents per 50 million miles driven).  
   a. What are reasonable \( x \) values? \( (0 \leq y \leq 1800) \)  
   b. According to this model, what age group has fewest accidents per mile driven?  
   c. Which age group tends to have more accidents, 17 or 50 years old?
Natalya Lisovskaya holds the world record for the women’s shot put. The path of her record-breaking throw can be modeled by \( y = -0.01347x^2 + 0.9325x + 5.5 \), where \( x \) is the horizontal distance in feet and \( y \) is the height (in feet). 5.5 feet represents the height at which the shot (a 4-kilogram metal ball) was released.

a. What was the maximum height (in feet) of the throw thrown by Lisovskaya?

b. What was the distance of her throw to the nearest hundredth of a foot?
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Watson, M. Writing has a place in a mathematics class. *Mathematics Teacher, 73*, 518-519.
