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The Effect of Standards-Based Professional Development in Content Knowledge and Questioning Techniques on Classroom Practices of Fourth Grade Teachers

Elizabeth T. Walker
St. John Fisher College

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Our schools are in crisis. Mathematics learning in the classroom has been examined and deemed to be insufficient. Scores on the Test of New York State Standards (TONYSS) for math and the 4th and 8th grade New York State math test have become stress factors for teachers and administrators. Reform is being dictated from the higher echelons. Status quo is not an option.

Teachers are the key figures in implementing changes suggested by national and state standards. The National Council for Teachers of Mathematics, NCTM, and the New York State Math, Science and Technology Learning Standards promote a constructivist, hands-on, inquiry method of teaching and learning (NCTM, 2000; New York State Education Department, 1996). The NCTM standards and current research also indicate that teacher knowledge of mathematical processes is not enough to encourage student understanding and connections in mathematics. Teachers must have a strong understanding of mathematical content and connections themselves to be effective agents of change in the classroom (NCTM).

Three years ago, I took the position of enrichment coordinator in an elementary school. As a non-traditional teacher, with 15 years of experience in teaching math at the college level, I had more math content knowledge than any of my elementary teaching peers. My concurrent involvement with the GMST program at St. John Fisher College provided a structure and a supportive environment for my ideas of inquiry learning, cross-discipline connections, constructivism and the use of manipulatives. Through observations of classroom teachers, reactions to lessons I taught in conjunction with my colleagues, and through conversations with administrators, teachers and students, I began to realize that
many of the elementary teachers in my building had little or no knowledge of standards-based educational practices, including the use of manipulatives, and many had very low math content knowledge beyond the process skills required for their grade level. Few incorporated inquiry learning or cross-discipline connections in their classrooms. I struggled with how to manage an environment of change in classroom practices that would benefit math teaching and learning at my school.

My opportunity to become a teacher-leader came when our district was awarded a Goals 2000: Educate America Act grant for $25,000 during the summer of 2002. I was asked to develop a two-week math academy that would increase teacher content knowledge through the use of manipulatives, and provide time for reflection and professional development during half of each day. The other half of the day would link teachers with students in direct instruction. Teachers kept reflective logs and shared an exhibition of learning at the end of the academy. The outcomes for both teachers and students were decidedly favorable. Teachers gained content knowledge and classroom strategies that affected their classroom practices. Students came grudgingly the first day and left excited and eager to return (Walker, 2002). Although the atmosphere that the academy created and the results it produced were most encouraging, I began to wonder if they were somewhat artificial; that is, could these results be repeated within the realities and constraints of day to day teaching, through after-school professional development.

Studies have shown that after-school or workshop style professional development can be highly successful in promoting positive change in the classroom if certain characteristics are present. My intent was to use the successful aspects of our teacher
manipulatives while teaching their students. I have collaborated with one of the teachers on regular “problem solving days”.

The professional development was appealing to these teachers as a way to improve classroom practices as they prepared their students for the high stakes New York State fourth grade math exam.

I wrote and received a $1000 Teacher Resource Center grant that provided funding for five hours of paid professional development time for each of the four teachers. This funding also purchased the manipulatives for use during the study. As further incentive, the teachers kept the manipulatives for classroom use at the end of the study. The specific content focus of part and whole was chosen for its presence in national, state and local curriculum requirements. This content falls in the February to March time frame on the fourth grade curriculum map. Teachers were able to immediately implement lesson plans developed during this study in their classrooms. There was time incorporated into the research for follow up and collaboration, a key issue for success.

This research project enabled an entire team of teachers to begin to embrace mathematical reform and to become agents of positive change in the classroom.
The Effect of Standards-Based Professional Development

Review of the Literature

Introduction

I reviewed the Math Academy model and looked at studies of other factors that supported reform. These were: coherence with the NCTM and New York State learning standards, a constructivist approach of inquiry-based learning with manipulatives, timely teacher implementation of learning and reflective practice, collaborative teaching and learning experiences, sustained teacher learning rather than one shot programs, and a math content focus. The literature shows these factors as catalysts for positive changes in teacher classroom practices. If these factors are individually successful when applied to teaching and learning, then a professional development model that incorporates many of them should also be successful.

The following review addresses: the mission of the math reform movement as it pertains to changes in classroom practices and teachers as agents of change, why teachers need opportunities through professional development to embrace change, why professional development has been largely ineffective as a catalyst for change in a standards-based system, and how professional development can be altered to be successful.

In addition, the literature identifies four key areas that promote successful professional development. They are:

1. content knowledge,
2. sustained professional development,
3. opportunities for active learning including collaboration and reflective practices,

and
4. coherence with other learning opportunities involving various topics contained in the NCTM and New York State Learning Standards including the use of manipulatives, effective questioning techniques, and other constructivist teaching techniques.

My summary will describe a professional development model that should be effective based on the results of former studies and current literature.

The Standards: A Call for Reform

In 1989, a community of math educators and other professionals developed the NCTM [National Council of Teachers of Mathematics] standards in an effort to address the low math performance of students in the United States (Maccini & Gagnon, 2002). For over a decade, educators have been encouraged to shift their classroom practices away from an exclusive focus on computation and accuracy and toward a focus on deeper understandings of mathematical ideas and connections (Kazemi & Stipek, 2001; Hiebert & Carpenter, 1992; Lampert 1991: NCTM, 1989,2000). Teachers are seen as the key figures of implementation of national standards by both mathematics and science groups (Lowery, 2002). The burden lies with the teachers, but many are not prepared to meet the challenge.

My goal was to help the teachers of my school begin the journey toward standards-based classrooms by focusing on increased content knowledge and higher level questioning techniques through the use of a constructivist model of professional development.

Teachers as Agents of Change

Because classroom teachers have the greatest capacity to influence classroom practices, systemic reform efforts must focus on them as the primary agents of change (Wise, Spiegel & Bruning, 1999). Many teachers, however, learned mathematics through a
model that focused heavily on memorization of facts without an emphasis on a deeper understanding of subject knowledge (Garet, Porter, Desimone, Birman, & Kwang, 2001). The NCTM Standards (2000) ask teachers to engage their students in ways that teachers have never been taught nor have been taught how to put into practice (Kazemi & Stipek, 2001; Cohen & Ball, 1990; Fullan, 1991).

Before teachers can become advocates for change in the mathematics classroom, they must first evolve into confident and skilled mathematicians themselves (Ward, 2001). They must learn as their students must learn. They must play and explore mathematically as their students will explore and play. “They must become critical thinkers and content specialists as they will guide their students to be. Finally, they must understand the philosophical foundation of the methods they are experiencing so that their advocacy will be complete and sustained” (Ward, 2001).

I chose to work with teachers instead of students because teachers can be such powerful agents for change. As a teacher-leader, I was responsible for leading the way.

**Professional Development as a Method for Change**

Professional development for teachers is one method for achieving a change in classroom practices and has become a major focus of systemic reform initiatives (Garet, Porter, Desimone, Birman, & Kwang, 2001). Many dollars and hours have been spent on workshops and seminars outside of the classroom, facilitated by outside teacher-leaders with special expertise, but lacking in required implementation or follow up. This type of professional development has produced little meaningful changes in classroom practices (Garet et al). Research has begun to look at what can make professional development successful.
Elements of Professional Development that Promote Success

1027 math and science teachers involved in Eisenhower Professional Development Program funded activities were questioned on what makes professional development successful and useful. Based on teachers' responses, Garet, Porter, Desimone, Birman and Kwang, (2001) identified three core features in professional development activities that have significant positive effects on teachers self-reported increases in knowledge and skills, and in changes in classroom practices. These core areas are: a focus on content knowledge, opportunities for active learning and coherence with other learning activities. In the article “Experts: Teacher training key to standards’ success”, Richard Elmore, a professor in the Harvard University Graduate School of Education contends, “effective professional development has a clear mission and instructional purpose; is focused on a specific problem; is focused on academic content; and is sustained over time” (Anonymous, 2002).

Professional development continues to be a strong candidate for empowering teachers, but it must be carefully and systematically planned to incorporate the areas of academic content, active learning, coherence, and sustained professional development over time, to have positive impact and drive change in classroom practices.

Focus on Content Knowledge

The NCTM Standards (NCTM, 2000) are very clear in their expectations for teacher content knowledge. They state,

To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teacher tasks (NCTM).
The NCTM Standards (NCTM, 2000) go on to state that the teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm. This is stated for all classrooms, not just high school math classes. This kind of knowledge is beyond what most elementary teachers experience in standard pre-service mathematics courses in the United States (NCTM). The undergraduate catalog for SUNY Brockport, a highly respected state college with a large math department, lists three math requirements for successful completion of a K-6 teaching certificate. The course descriptions appear below:

**MTH 121 College Algebra (A).** Prerequisite: Two years of high school mathematics, or QNT 110. (Closed to students who have completed more than three years of high school mathematics or MTH 122 or a calculus course). Covers algebra at the intermediate level, including operations on polynomials and algebraic fractions, solution of first- and second-degree equations, graphs of functions and equations, logarithms and exponential functions. 3 Cr. Every Semester.

**MTH 313 Mathematics for Elementary Teachers I (A).** Prerequisite: MTH 121 or QNT 111 or three years of college-preparatory mathematics. Open only to students seeking elementary teaching certification. Includes: sets, relations, number systems, elementary number theory, mathematical systems, and probability. Uses a problem-solving approach where appropriate. 3 Cr. Every Semester.
In her study of content knowledge, Ma ponders that,

A teacher's subject matter knowledge may not automatically produce promising teaching methods or new teaching conceptions. But without solid support from subject matter knowledge, promising methods or new teaching conceptions cannot be successfully realized (Shreffler, 2002).

Ma states that in China, deep arithmetic knowledge is not owned by the mathematicians. The teachers at the elementary school level own content knowledge. The knowledge is in their minds and they pass it on to the next generation. She did not see this type of content knowledge in American teachers that she observed (Herrera, 2002).

Clearly, pre-service elementary teachers need more math content exploration incorporated into their undergraduate education. In-service teachers need required professional development and higher administrative expectations to promote acquisition of mathematical content knowledge and discovery of mathematical connections.

Lack of content knowledge is the largest single factor in failure to teach mathematics successfully. I have chosen to focus on increasing a specific content level in fourth grade teachers and view its immediate effect on classroom practices.

**Sustained vs. One Time Professional Development**

One of the major complaints about traditional professional development is that it is short in duration of sessions and in the number of sessions involved. Professional development that is sustained over time is important for two reasons. First, teachers are more likely to have an opportunity for in depth discussion of content, misconceptions and pedagogical strategies. Secondly, more teachers are likely to try out new practices in the classroom if there are opportunities for follow up and feedback. The study of the
community of teachers (NCTM, 2000). Reflective thinking before, during and after teaching is considered imperative for a thorough teaching experience and is valued in professional growth and successful teaching (Dewey, 1933; Kraus & Butler, 2000; Schon, 1987; Vallie, 1992). Teachers should be continually encouraged to develop reflective thought as a tool for developing confidence and competence in teaching mathematics and science. Knowledge of self as a learner and as a teacher surface through the reflective process. Reflection can be accomplished through daily journal entries, informal daily evaluations, and other course artifacts (Lowery, 2002).

Teamed with collaboration, reflection on the teaching and learning process is a powerful part of successful professional development. The participating teachers learned, planned and assessed their learning collaboratively. I built reflection time into the hourly sessions of professional development with the teachers. I also set aside time for myself to reflect on their reflections. Immediate reflection time was also built into the implementation segment of our plan. I agree with the literature that reflection defines and refines our thinking and should be a part of good professional development.

**Coherence**

Teachers receive guidance about what to teach and how to teach it from multiple sources, such as material covered in formal professional development, pre-service education, textbooks, national standards, state and local policies and assessments, and the professional literature (Cohen & Spillane, 1992). If these sources provide a coherent set of goals, they can facilitate teachers’ efforts to improve teaching practice, but if they conflict they may
create tensions that impede teacher efforts to develop their teaching in a consistent direction (Garet, Porter, Desimone, Birman, & Kwang, 2001; Grant, Peterson, & Shojgreen-Downer, 1996).

Teachers who experience professional development that is coherent are more likely to change their classroom practices (Garet, Porter, Desimone, Birman, & Kwang). Professional development topics aligned with the NCTM and the New York State standards have included the use of manipulatives to access content and make connections, constructivism, and questioning techniques that promote critical thinking. I examined each of these topics that affect the coherence of the offered professional development.

**Use of Manipulatives**

Lack of materials is considered the greatest barrier to implementation of the NCTM standards in a study by Reston, Maccini and Gagnon (2002). But simply using manipulatives or having them in the classroom does not guarantee a good mathematics lesson (Stein & Bovalino, 2001; Fennema 1972; NCTM 2000; Sowell, 1989). “Manipulatives do not magically carry mathematical understanding. They provide a concrete way for students to link new, often abstract information to already solidified and personally meaningful networks of knowledge, thereby allowing students to take in the new information and give it meaning” (Stein & Bovalino). Teachers are more successful in using manipulatives in the classroom if they have had training and practice in using the manipulatives and if they design their own lessons. By working with the manipulatives, teachers are more prepared for student’s questions and for addressing misconceptions that may arise (Stein & Bovalino).

In “Manipulatives: One piece of the puzzle”, Reston, Stein and Bovalino (2001) talk about “getting students to think about mathematics in ways that go beyond using procedures
to solve routine problems” as an important goal of mathematics reform. They list three factors that influence the success, or failure, of using manipulatives in the classroom:

1. too much teacher direction. Showing students “the way to do it”,
2. too little teacher direction. Leaving students too much to their own devices resulting in unsystematic and nonproductive exploration, and
3. the teacher not fully understanding the mathematical connections because of lack of content knowledge, or lack of personal use with manipulatives.

I have added a fourth factor:

4. The lack of transfer of knowledge constructed with manipulatives to paper and pencil or abstract concepts.

During the math academy that I helped to design and direct, students utilized one inch cubes and tiles to explore volume and surface area. Learning was gradually transferred to paper and pencil work with equations and dimensions and then applied to a performance task. When asked how they determined volume and surface area of their project during the students’ exhibition of learning, none of the 48 students referred to the manipulatives they had used to initially explore the concepts. They constructed the knowledge of volume and area using physical manipulatives, constructed the knowledge as an abstract concept and then discarded or buried the original discovery phase that used the manipulatives. It was no longer needed. If prompted, they could explain how to use the blocks to find area and volume, but they found it too simplistic once the abstract concept was firmly in their minds.

The study of content in my research revolved around the use of manipulatives to build understanding and connections. Teachers were allowed to keep the manipulatives that we used to encourage them to continue using them for self-discovery and with their students.
together and looked at cause and effect in the classroom when a successful critical thinking question was launched. I looked at questioning techniques in the classrooms of the teachers that I worked with as a sign of their increased content understanding.

**Constructivism**

The construction of knowledge, not the memorization of knowledge, is a reoccurring theme in the NCTM Standards. NCTM calls it “The Learning Principle” and states, “students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000). Constructivism as a philosophy of learning is not new, but its application to modern education is still in its infancy. Constructivism is “a belief that all knowledge is necessarily a product of our own cognitive acts” (Ward, 2001; Confrey 1990, p.107).

By building on previously constructed knowledge, student can better grasp the concepts and can move from simply knowing material to understanding it. Constructed knowledge promotes critical thinking, which allows students to integrate concepts within and between disciplines to represent concepts in multiple forms and to justify, defend, and reflect on the concepts. The teacher plays an important role in helping students construct accurate knowledge. Teachers must also be able to understand students’ constructions that differ from their own. (Ward)

Teachers must also construct knowledge as they learn new math content, not simply experience an information transmission from an in-service specialist (Panakuk & Sullivan, 1999). The knowledge they construct must surpass the knowledge of the majority of their students. Many students don’t show their thinking because they lack confidence in it (Ward,
or follow up have been largely ineffectual in sustaining a shift to standards-based education. Teachers remain the greatest potential for being agents of change but current practices to encourage shifts to standards-based education have not always been successful.

Studies show that individual learning opportunities that are successful in empowering teachers to become agents of change towards a more standards-based classroom environment include: content-specific training, collaborative and active learning experiences, reflective practices, sustained professional development, coherent teacher learning, or timely implementation. A professional development model that incorporates all of these facets concurrently should prove effective.

My study highlights teachers as agents of change by increasing specific content knowledge in part and whole, a fourth grade topic. By incorporating all of the positive and powerful aspects of teacher growth shown to be effective in the literature my professional development model included constructivist, inquiry learning using performance tasks and collaborative learning sustained over 5, one hour sessions. Manipulatives were used to make connections in content knowledge. The teachers were able to keep the manipulatives in their rooms during and after the professional development sessions for exploration and implementation of new ideas in the classroom. Critical thinking questioning techniques were defined, modeled and practiced. Teacher reflection, both prompted and unprompted was part of each session. Immediate implementation of new skills with an observer present in the classroom was scheduled followed by a debriefing and reflection time with all participants. Student work was included in the debriefing. Coherence to local, state, and national standards was incorporated into my study to encourage participation and retention.
The Effect of Standards-Based Professional Development

Methodology

The purpose of this study was to assess if increasing fourth grade teacher mathematical content knowledge through exploration with manipulatives using a constructivist model would affect classroom practices. Data focused on an increase in teacher content knowledge indicated by the use of open-ended questions by teachers in the classroom as well as questions that prompted critical thinking and engaged students in dialogue and activities promoting mathematical connections.

Community

The surrounding townships that comprise the participating school district, are a blend of agricultural, light industry and commuter populations. There is one major employer that processes local seasonal produce. Per capita income in 1995 was $21,218 (Wayne County Demographics http://www.wedcny.org/data.htm).

School

The school district has a teacher to pupil ratio of 1:14. The percentage of students continuing their education post graduation is 64.2%. There are 1169 student enrolled in the school district, with 607 in grades K-6 and 562 in grades 7-12. Average class size is 22 pupils. In the overall school population, .5% of students are Asian, 1.8% are Black (Non-Hispanic), 1.2% are Hispanic and 96.5% are white (Non-Hispanic). 17.5% of the student body receives free or reduced lunch due to economic need, 11-20% of student families receive public assistance (The New York State School Report Card, 2001).

The Test of New York State Standards (TONYSS) math scores for incoming fourth graders show 36% of the population taking the 2001, 3rd grade TONYSS scored at a level 4,
exceeding the New York State standards, 49.3% of students scored at a level 3, meeting the state standards, 12% scored at a level 2, needing extra help to meet the state standards, and 2.7% scored at a level 1, having serious academic deficiencies. The areas that the New York State Mathematical Key Ideas identified as those in which students were most deficient are Modeling and Multiple Representations, and Numbers and Numeration.

Classrooms and Participants

Fourth grade classrooms in this school are grouped heterogeneously with mixed ability levels. There are 21-23 students per classroom. The 4, fourth grade classrooms are housed in the same school building and are the only classes of this grade level in the district. Students with special needs are mainstreamed into the classrooms. Two students are hearing impaired and use audio modifiers. Four students are classified and are accompanied by full time, instructional aides. One student is an ESL Spanish speaking Hispanic with limited English vocabulary.

The teachers participating in this study are all white, Anglo-Saxon females ranging in age from 40 and 55. The number of years the participating teachers have been educators in the public school system range from 5 - 29. All have taught fourth grade for at least three years with the most senior teacher having taught fourth grade for 18 years. Educational concentrations for advanced degrees are in reading or special education. The teachers work together under a team system where they plan lessons and special programming together. The teachers involved were not taught in a standards-based system. They were also not taught to teach using the ideas of constructivism, inquiry or manipulatives in their undergraduate or graduate programs. One of the teachers attended a five-day, Marilyn Burns seminar on the use of manipulatives in July 2000.
The content area of part/whole was chosen because of its placement on the fourth-grade curriculum map. These topics are generally taught during the months of February and March.

**Materials**

Mathematical manipulatives used were 1 inch, color tiles, 1 inch color cubes, and fraction tiles. Teachers recorded their reflections in standard composition notebooks. Professional development conversations and student interactions were recorded on standard cassette desktop recorders with exterior microphones.

**Procedure**

I developed a rapport with the participating teachers over two years through monthly problem solving days, consulting on classroom math topics and daily interaction as math project leader and enrichment coordinator. The students in the fourth grade classrooms had all worked with me at least once a month and 75% of the students had interacted with me through enrichment programming.

The professional development hours occurred in the Learning Center classroom within the elementary building, Monday afternoons from 4:00-5:00 pm for 5 weeks during February, March and April. This is a large room with tables for working and access to many learning tools. It is well lit and pleasant.

The 4, fourth grade teachers participated in three hours of professional development, one hour of lesson plan development and one hour of reflection and discussion. I observed all four teachers present their lesson plan in their classrooms and provided them with release time from their classes so that they were able to immediately reflect on the lesson that had been presented.
The desired targets for the professional development were posted in clear view of participants at all times. I encouraged e-mail and person-to-person communication amongst the participants between sessions.

Before professional development began, teachers provided me with a lesson plan involving part and whole that they had used in the past. Teachers provided a list of five sample questions they had asked students in prior lessons that they thought would prompt deeper understanding about part and whole. These questions were examined for characteristics of critical thinking questions. See the section on data analysis.

The first hour of professional development included assessing prior knowledge and practices and exploring the relationships of whole to part through the use of manipulatives. The second and third hour included intensive exploration of the content area. The second hour included defining critical thinking questions, differentiating critical thinking questions and evaluating student responses to critical thinking questions. A performance-based task entitled "The String Thing" was utilized working from whole to part using manipulatives. See Appendix B.

The third hour utilized a performance task entitled "Color Tile Bonanza" examining part to whole, also using manipulatives. For a description of the task see Appendix B.

The teachers kept a record of any misconceptions that they discovered in their own knowledge during the professional development. Teachers responded spontaneously in reflection journals and to prompts provided by me at the beginning and the end of each hour session. Prompts were chosen based on the interaction and learning that occurred during each hour. Teacher reflection journals were made available to me to photo-copy after each session. Each hour-long session was tape recorded for my use.
I kept a reflection journal at the end of each hour session based on observations, recorded teacher conversations, misconceptions, vocabulary new to the teachers, and teacher reflection journals.

During the fourth hour of professional development, each teacher created a lesson plan on part and whole for use in their classroom. They wrote five, critical thinking questions to incorporate into the implementation of their lesson. The five questions were analyzed using the same criteria as the initial questions. See the section on data analysis. Copies of these lesson plans were kept for comparison to the initial lesson plans given to me at the first session. Lesson plans were implemented within two weeks of their completion. I observed all of the lesson plans being implemented in the classrooms. Lessons were recorded with one microphone positioned near the teacher and one positioned in a cluster of students. Field notes were taken on the questions and responses that occurred during the lessons. A transcript of the questions and responses occurring in each classroom was typed from the notes and audio-recordings for the teachers to use during their debriefing. A section of the transcript that showcased a particularly powerful exchange of critical thinking questions and answers was highlighted and the audio-tapes were set up at that point in the lesson for the teachers to listen to their questioning techniques used in the classroom.

One, half hour of coverage in each classroom was provided immediately after each lesson was taught to allow time for the classroom teacher to reflect on the lesson regarding their questioning techniques, the content of the lesson and the responses of the students to the lesson.

Teachers reconvened for one hour, less than one week after classroom lessons were taught, to listen to taped student conversation, read transcripts and to look at student work.
• Request for information
• Computation
• Connection to other sub-disciplines
• Probing for deeper understanding
• Open-ended essential questions

Requests for definitions, requests for examples, requests for information and computation questions that were followed by additional questions that probed for deeper understanding were placed in the critical thinking question category. For example: “How much is shaded in the pictures?” [showing $\frac{1}{2}$ and $\frac{2}{4}$] was followed by the question, “Why are they the same?” The first question asked for information but was necessary for the critical thinking question that followed. The first and second question together were noted as an information question followed by a probe for deeper understanding and counted as a critical thinking question.

Questions such as, “How would our lives be different without fractions?” are open ended, required deeper thought and had no definite yes or no answer. They were listed as essential questions which are a form of critical thinking questions.

Questions falling only into the first four categories were totaled and converted into a percentage for comparison to the percentage of similar questions in the final lesson plans implemented at the end of the professional development. Those classified in any way as connecting to other sub-disciplines, probing for deeper understanding or open ended essential questions were also totaled and converted to percentages for comparison to the final lesson plan questions. I considered these questions forms of critical thinking questions.
A correlation between the effect of constructivist professional development in content knowledge and questioning techniques on classroom practices would be indicated by a shift toward 100% imbedded, critical thinking questions in written lesson plans: that is from questions asking for information, examples or computation only to questions that probed for deeper understanding. Evidence of a shift toward understanding the nature of critical thinking questions, comfort in using critical thinking questions and probes for deeper understanding by the teachers during student/teacher dialogue were looked for during the implementation of lesson plans. Student/teacher dialogue was examined for evidence of shifts in questioning techniques through the transcription of the audio-tapes and field notes scribed during those lessons, through participant conversations and through the post-assessment questions during the final debriefing hour of professional development.

**Classroom Practices**

Shifts in classroom practices due to exposure to constructivist professional development, including the use of manipulatives and increased content knowledge were assessed by: comparing how the content was presented in former lesson plans and new lesson plans: that is toward a more inquiry method of exploring content instead of a process-oriented lesson, the use of manipulatives to build conceptual understanding, the use of lesson-imbedded critical thinking questions or those similar to the written questions, by the teachers during the implemented lesson, observations and field notes of the implemented lessons with a focus on content understanding, questioning techniques and student/teacher dialogue during the lesson in the classroom, responses to prompts about classroom practices in the teachers' reflection journals, and teacher conversation during professional development. A final debriefing conversation occurred during the last hour that included
Results and Data Analysis

Participants' content knowledge before and after professional development

The teachers had basic content knowledge of fractions as part and whole, as evidenced by their responses to the journal question, "What is 1/2". All four teachers referred to dividing a "whole" into 2 equal parts and taking one of them. They defined the whole as either one entity or a group of objects. They primarily utilized the visuals in their math textbooks for teaching fractions. The pictures are brightly colored but mostly involve one shape, either circular or rectangular, divided into parts with some parts missing or shaded. When asked, the teachers were not familiar with any historical or cross-cultural connections concerning fractions. The teachers were unaware of fraction sequences or patterns in fraction sequences, and exhibited no verbal or written clues that they held an understanding of number theory or characteristics of rational numbers.

When presented with the sequence $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$, the teachers were able to identify the characteristics of the sequence (the denominator increases by one) and could verbalize that the fractions in the sequence were representing smaller and smaller portions, but could not visualize extending the sequence into infinity without prompting. The concept of a limit was a new concept to them. With help from the teacher leader, the teachers could identify the pattern involved in creating the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$ but also had difficulty projecting this sequence into infinity. The teachers needed prompting to connect the characteristics of this sequence to the prior knowledge of the former sequence.

TL: "What do you observe about this sequence? How is it different from the previous sequence?"
BB: "You are increasing the size of the whole."

TL: "What is happening to the size of the fractions as they progress?"

SA: "They are getting bigger."

TL: "Why?"

BB: "You are combining parts."

SA: "You are adding parts together."

BB: "Your pieces are getting smaller, but the number of pieces is getting bigger.

TL: "Why is \( \frac{4}{5} \), which has a greater number of smaller pieces larger than \( \frac{3}{4} \) which has a smaller number of pieces but each piece is larger? How do you know that three of those bigger fourths will be smaller compared to \( \frac{4}{5} \)?"

CA: "I am trying to think of what I know of \( \frac{4}{5} \)." (Audio-tape. February 24, 2003. Session 1)

The teachers' knowledge on the limit of \( \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ... \) was tentative. They were unsure what number this sequence would approach when extended into infinity. Some thought it would approach the number one. Some thought it would be greater than one.

The first performance task involved breaking "trains" of 1" tiles into fractional parts and keeping a data table on the number of parts and the length in tiles of each part. The teachers were familiar with reading information from data tables but were unfamiliar with the creation of a data table, or the significance of listing data in a logical sequence to look for patterns. When the data tables were complete, the teachers observed the factor pair
relationship between the number of fractional parts and the length of each part but needed to be prompted to see any other patterns. Their verbal responses, when prompted for further observations of the data tables, were hesitant and incomplete. They were unable to use the patterns between the data table for a 12" train and a 24" train to extrapolate the size of fractional parts for a 6" or a 48" train.

TL: “What observations can you make about your data tables [of factor pairs of 12 and 24]?”

BB: “When the numbers are small they are close together and then they get farther apart.”

TL: “Why do you think that is?”

BB: “They’re factors?”

TL: “What about the factors?”

BB: “Like the factors of 12 are 1,2,3,4,6 and 12.”

TL: “Consider the 24 tile train. When it is broken into fourths, how long is each piece?”

SA: “6”

TL: “How long is one fourth of a 12 tile train?”

CS: [Consults data table] “3”

TL: “What is the relationship of the size of the parts and the number of tiles you began with?”

BB: “They are both fourths.”

CS: “You have to divide by four”

TL: “Did you figure that out by looking at your data table?”
CS: "No, I just divided by 4."

TL: "What if our original train was only 6 tiles long. How big would a fourth be?"

BB: "Um. I don’t think you could do it."

CS: "You can’t divide 6 by 4."

TL: "What if we followed the pattern shown in the data table for one fourth of 24 and then one fourth of 12. What would one fourth of 6 be then?"

BB: "That is a very hard question. I don’t think you can do it." (Audio-tape. February 24, 2003. Session 1)

The teachers were then asked to make a graphical representation of their factor pairs for both trains. The teachers had never graphed on the Cartesian plane and were unable to make a prediction as to the shape of the graph. They were intrigued with the results and were anxious to try other factor pairs. A portion of their conversation while graphing follows:

SA: "Oh, wow!"

BB: "Look at that!"

SA: "Does yours look the same as mine?"

CS: "I never thought it would look like that!"

BB: "Can we do both (data tables)?"

BB: "Now I have a question and you’re going to tell me I’m silly. Will our graphs look alike because I started with 12 groups of 1 and she started with 1 group of 24? Is that going to make a difference?"

TL: "Not as long as you keep your axes the same."
“whole” being divided into all halves, all thirds, or all of any other one unit fractional size. They had not explored a mixture of different fractional sizes with different denominators to make one whole; indeed, two of the teachers did not think such a task was initially possible.

**Individual teacher growth in content knowledge**

**BB:** BB’s self-assessed her initial knowledge of part and whole as being weak. When asked on the pre-assessment what most influenced her comfort with asking students questions that might spark discussion, BB responded, “My personal content knowledge in some areas of part to whole is weak, so that also is an influence” [in choosing 3 on a scale of 1-5 measuring comfort in asking probing questions]. (BB, pre-assessment. February 24, 2003)

She worked very diligently through the performance tasks offered during the professional development hours, increasing her content understanding of part and whole and took her learning directly back into the classroom. When asked to make an observation about the day’s learning after completing “The String Thing” she wrote, “I had not realized I could combine fifths and tenths with twelfths, sixths and fourths to get a whole.” (BB, journal entry. March 3, 2003)

Teacher BB’s initial and final lesson plans were evidence of her great shift in understanding of the content of part and whole. Her initial lesson plan consisted of the students making fractions strip kits out of pre-made strips of paper and templates. The students then would be directed to show fractional parts such as \( \frac{1}{2} \) or \( \frac{4}{16} \) using the fraction kit. BB would assess learning using paper and pencil by having the students identify the part of a shape that was shaded, or shade in a shape given a fractional amount.
while exploring math content. In professional development sessions two and three, the teachers briefly explored Egyptian fractions, and fractions as they related to music. Being strong in music and interested in other cultures, SA was very intrigued with both of these connections. The March 17, 2003 entry in her reflection journal states, “I never thought of using music while teaching fractions. I never had heard of Egyptian fractions. I am excited to teach my class all about these new concepts and strategies.”

SA’s initial lesson plan focused on dividing whole objects: a cake, a large tootsie roll, and a pizza into equal unit fractions. The students would be encouraged to show that

\[
\frac{1}{2} + \frac{1}{2} = 1 \text{ whole}, \quad \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \text{ whole and so on for other magnitudes of unit fractions.}
\]

SA’s lesson plan, written after professional development, expanded the concept of a whole to include groups of objects as well as one singular entity. She utilized the pre-made colored fraction tiles and encouraged her students to find different ways to make 1/2, both with congruent unit fractions and with a mixture of different unit fractions. She spent time working with the fraction tiles herself and making discoveries about possible solutions before the students completed the exercise.

The use of standards-based modeling during professional development prompted SA to use manipulatives to continue to build her own understanding of part and whole and to prompt the understandings of her students. The enthusiastic response of SA’s lower achieving students to her inquiry task reinforced a desire for SA to continue using manipulatives and inquiry in her classroom. She wrote of two of her struggling students in her post-lesson reflection:
A. [student] surprised me. He really was working hard to get the fractions to equal $\frac{1}{2}$ in a variety of ways. T. [student] at times also was using a variety of fraction pieces to cover $\frac{1}{2}$. (SA. Journal reflection. March 17, 2003)

SA’s post-assessment responses confirmed her acquisition of knowledge through the use of manipulatives. In response to the question, “Would you and could you have written both of these questions [in the lesson plan] without the tasks that we did that explored your own content knowledge of part and whole?” She writes,

Probably not!! Some of the knowledge that I gained was through the manipulatives that we used and worked through along with the questions that you asked of us. (SA. post-assessment. April 18, 2003)

This was SA’s 29th year of teaching. She was excited about learning new content and exploring it with her students.

CS: Teacher CS also chose personal math content knowledge as the greatest impediment to challenging her students in math. The constructivist approach of the professional development sessions helped CS in constructing her own knowledge. In responding to a reflection prompt about the value of teacher’s using manipulatives for learning while exploring their own content knowledge, she writes,

“Yes, it allowed me to, through experimentation with manipulatives, see possibilities, concepts, etc. with part and whole I otherwise would not have thought of (well tried to think of)!” (CS. journal reflection. March 17, 2003)
The initial lesson plan of CS focused on examining the fractional parts of paper, folded into four equal areas, and on reviewing fraction vocabulary. For independent work, the students were instructed to draw rectangles and to shade in various fractional parts. Fractions with unlike denominators were not shaded in the same rectangle.

CS's final lesson plan exhibited a greater understanding of the need for students to construct knowledge through the use of manipulatives and exploration. She valued the learning she acquired through the performance tasks that were presented during the professional development sessions and used them as a basis for her own lesson. She writes,

> It was helpful to be presented with performance-based projects because they forced me to see part and whole in new and different ways. Also being able to have a "hands-on" experience was very helpful. (I'm one of those folks who needs to "see it"!) I really thought of the concept of whole in a "whole" new light as well as looking at the parts of that whole! Learning how to use manipulatives and learning new math terms and concepts was also helpful.

(CS. reflection journal. March 17, 2003)

CS’s lesson after the first 4, one hour sessions of professional development utilized finding fractional parts with twelve, 1” tiles per student. CS directed her students to explore the different possible fractions that the twelve tiles could be broken into. She included unit fractions and multiples of unit fractions. She prompted her students to try to explain why certain denominators did not result in equal sharing with the twelve tiles.

CS was disappointed at this particular point in the lesson. When students began to explore denominators that did not result in equal sharing, CS began to doubt her own ability
the opportunity to collaboratively work together to increase their content knowledge and felt that the professional development gave them knowledge and skills that they could take back to their classrooms.

**Teachers' ability to write critical thinking questions before and after professional development**

The preliminary critical thinking questions provided by the teachers for a lesson on part and whole where categorized as shown in Table 1. Over half of the teachers' questions fell into the categories of: connections to other sub-disciplines, probing for deeper understanding, essential questions or were questions that were followed by questions that probed for deeper understanding. These questions could all be considered a form of critical thinking questions.

None of these teachers felt that they could identify, write or spontaneously use critical thinking questions about part and whole in the classroom during the entire research period yet they did so quite naturally in their preliminary lesson plans. The teachers' journal entries consistently showed a lack of confidence in this area.

In response to the journal prompt, "What do you still need to create critical thinking questions for classroom lessons?" during the third hour, BB wrote:

- Lots of modeling - I need examples, examples, examples.
- Begin by talking and coming up with some as a group.
- Practice, practice, practice. Writing them with support. (BB, journal entry, March 10, 2003)
information but were followed by prompts for deeper understanding were noted as a subcategory of questions followed by a probe for deeper understanding. These questions were included in the percentage of critical thinking questions.

Table 1

Analysis of Preliminary Lesson Plan Questions from Fourth Grade Teachers

<table>
<thead>
<tr>
<th>Type of question/participants</th>
<th>SA</th>
<th>BB</th>
<th>JL</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Request for Definition/ followed by a probe for deeper understanding</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Request for Example/ followed by a probe for deeper understanding</td>
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<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Request for Information/ followed by a probe for deeper understanding</td>
<td>1 / 2</td>
<td>1 / 0</td>
<td>2 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Computation/ followed by a probe for deeper understanding</td>
<td>1 / 0</td>
<td>0 / 1</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

Percentages of Request for Definition, Example, Information or Computation not followed by a probe for deeper understanding

<table>
<thead>
<tr>
<th></th>
<th>SA</th>
<th>BB</th>
<th>JL</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connections to other sub-disciplines</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Probe for deeper understandings</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
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<tr>
<td>Essential question</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Percentages of Critical thinking questions: Connections to other sub-disciplines, Probes for deeper understandings, and Essential questions

<table>
<thead>
<tr>
<th></th>
<th>SA</th>
<th>BB</th>
<th>JL</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75%</td>
<td>80%</td>
<td>75%</td>
<td>100%</td>
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</table>
The questions included in the final lesson plans of the teachers were tallied in the same fashion as the initial lesson plan questions.

Table 2

Analysis of Final Lesson Plan Questions from Fourth Grade Teachers

<table>
<thead>
<tr>
<th>Type of question/participants</th>
<th>SA</th>
<th>BB</th>
<th>JL</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Request for Definition/</td>
<td>0/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0/0</td>
</tr>
<tr>
<td>followed by a probe for deeper understanding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Request for Example/</td>
<td>1/2</td>
<td>0/0</td>
<td>0/0</td>
<td>0/0</td>
</tr>
<tr>
<td>followed by a probe for deeper understanding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Request for Information/</td>
<td>0/1</td>
<td>0/3</td>
<td>0/1</td>
<td>0/2</td>
</tr>
<tr>
<td>followed by a probe for deeper understanding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computation/</td>
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<td>0/8</td>
<td>0/0</td>
<td>0/2</td>
</tr>
<tr>
<td>followed by a probe for deeper understanding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentages of Request for Definition, Example, Information or Computation not followed by a probe for deeper understanding</td>
<td>12.5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Connections to other sub-disciplines</td>
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<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>Probe for deeper understandings</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Essential question</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Percentages of Critical thinking questions: Connections to other sub-disciplines, Probes for deeper understandings, and Essential questions</td>
<td>87.5%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
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</tbody>
</table>
Teacher BB found that a focus on critical thinking questions altered her perspective of all her questioning techniques. In her post-assessment, she commented:

"I am much more aware of the types of questions that I am asking my children, not only in math but in other areas" (BB. post-assessment. April 18, 2003).

When asked to choose two questions and comment on her ability before and after professional development to write these questions, BB reflected, “The tasks we did make me think about questioning and the importance of letting them [the students] explore.” (April 18, 2003)

The response of BB’s students to her critical thinking questions that were included in a hands-on, inquiry lesson gave BB insight into how each of her students was processing information about part and whole. It gave her access to different ways to view part and whole through her students’ solutions, which increased her own understanding of part and whole. She found critical thinking questions engaged students that very often were distracted or whose engagement was limited during math class.

BB asked her students to respond to some of her imbedded, critical thinking questions in writing. She followed the writing exercise with a whole class discussion. In reflecting on her lesson, BB discovered that the written responses and the collaborative, oral responses differed significantly, with the oral responses being more coherent and elaborate. This seemed to indicate that critical thinking questions have a greater impact when used in a collaborative environment where student/student and teacher/student dialogue can ensue. The percentage of critical thinking questions in her initial lesson plan compared to the percentage of critical thinking questions in her final lesson plan rose from 80% to 100%.
BB, as well as the other teachers participating in the study, discovered that critical thinking questions consume more class time as students construct understanding. With a great deal of content to cover in fourth grade, these teachers see the value of critical thinking questions but struggle with the dilemma between covering content required by the administration or helping their students to develop deeper understandings of that content.

Teacher SA relied heavily on the examples of critical thinking questions provided in the outlines used for the professional development. She also relied on the hand-out that defined what a critical thinking question was, the need to differentiate critical thinking questions, and the response of students when critical thinking questions were successfully launched, while constructing her own critical thinking questions. Her initial attempts to create critical thinking questions after “The String Thing” were successful. The questions she created in her reflection journal exhibited her ability to probe for deeper understanding. She wrote:

- Will the fractions you came up with work for any length of string?
- We used 4-5 fractions [to make up the whole]. What is the smallest number of different fractions you could use?
- What is the largest number of unit fractions you could use? (SA. reflection journal. March 3, 2003)

Imbedding the critical thinking questions in her lesson plans forced SA to examine the content deeper herself, before she engaged her students. Her percentage of intentional critical thinking questions imbedded in her lesson plans rose from 75% to 87.5%.

Teacher CS had already written 100% of the questions in her initial lesson plan as critical thinking questions. An example of one initial critical thinking question written by
CS did not increase the percentage of critical thinking questions in her lessons but used these questions more efficiently to achieve her goals.

Teacher JL had experienced critical thinking questions when she was a student of pre-service teaching. She was affected by the disequilibria these types of questions caused in her own mind and included them in her own teaching style. She rated herself as "very comfortable" in asking questions that sparked discussion in her classroom. JL loved to rile her students with questions that make them think, and encouraged acceptance of all justifiable answers.

JL also chose to limit the parameters that her students were working with in this introductory lesson. She kept her "part" fixed at $\frac{1}{2}$ and varied the "wholes" that the students examined. Each different whole (fabric, water, suckers, nails, dog biscuits) presented its own set of critical thinking challenges. Her question type shifted more toward essential questions at the end of the lesson. These critical thinking questions have no definite yes or no answers. The students felt that they had really conquered the concept of $\frac{1}{2}$, but these essential questions challenged their thinking and created a desire for more understanding. Some of these questions were,

Can we divide everything in half?

Can we divide a dollar in half, a sock in half, a house in half, maybe a tree or flower? Why are we able to divide some things and not others? (JL final lesson plan. April 10, 2003)

JL's comfort level with critical thinking questions came from looking at the content from many different directions which may or may not indicate a higher level of math content.
knowledge. Her success was to take something very basic and examine it in depth with questions, examples and counterexamples. While she was developing her lesson plan, JL referred to examining what $\frac{1}{2}$ really meant during our professional development sessions.

Even though she was already comfortable with higher level questioning techniques, she was able to utilize new information from the professional development to enhance her ability to probe for deeper understanding.

Teachers BB, SA and CS found that it was much easier to write critical thinking questions than it was to implement them in a lesson. All of the teachers found that imbedding the questions in writing in their lesson plans helped them to think through the content before they taught their lessons and helped them to implement the questions when presenting lessons to the students. Probes for understanding were limited to a given critical thinking question and one or two student responses. The teachers found it difficult to spontaneously continue dialogue to probe for deeper understanding. Practice in simulated teacher/student dialogue to probe for deeper understanding during the professional development sessions may have increased success in the classroom. JL was already very comfortable challenging her students with critical thinking questions and did so readily.

Three of the four teachers indicated through conversation and journal reflections, that the professional development sessions that provided modeling of critical thinking questions and content knowledge using hands-on, inquiry tasks were crucial to their ability to use critical thinking questions in their classrooms. The fourth teacher saw the professional development sessions as reinforcement of what she already knew.

*Collaboration in learning*
the same building, they were able to plan to continue their journey towards more standards-based classrooms.

*Sustained professional development*

The teachers were asked to complete a different task during each of the three hours of professional development. The first task was to use manipulatives to observe the relationship between the number of parts and the size of the parts in a given whole. The second task was to divide a string of a given length into 5-8 fractional parts and to verify their answer. They then cut the string into those parts. The third task was to determine the fractional parts represented by quantities of colored tiles given to them in a bag, and to verify that the fractional parts equaled one and the number of tiles equaled the whole. The teachers then translated this data onto a circle graph using a compass and protractor.

The teachers completed the first task but were hesitant to use the manipulatives and asked many questions about the directions. They relied heavily on the teacher-leader for directions and support. The teachers needed much prompting to see patterns and connections.

In the second task, the teachers utilized the fraction tiles and the 1" blocks to aid them in their visualization of dividing the string. They were confused by the many different ways to represent the whole, but persevered, going back to their experience with the task of the week before, until they had completed the project. They began to compare answers and to ask each other questions. The teacher-leader was utilized less during the second session for support than during the first session.

The teachers completed the third task easily. Math vocabulary was evident in their dialogue as they referred back to the previous tasks. The atmosphere of anxiety and
paper cookies, and thought hard about how to help their students construct knowledge. The very nature of critical thinking questions requires a student-centered lesson that utilizes exploration and inquiry. The insight into student thinking and the learning that the teachers experienced from their students' discoveries produced ideas for future lessons that the teachers had already begun to talk about and plan by the final professional development session.

When asked if they taught differently now, all teachers indicated that they could not return to a totally teacher-centered classroom 100% of the time. All of the teachers indicated that this shift was possible because of the professional development, the presence of teachers to collaborate with, and the proximity of a teacher-leader with strengths in math content and inquiry learning, that was available as a resource.

Conclusion

By providing sustained, standards-based professional development focused on increasing content knowledge in participating, fourth grade teachers and by modeling, defining and using critical thinking question techniques during the presentation of the professional development, I expected to see a shift in the classroom practices of the fourth grade teachers toward a greater intentional use of critical thinking questions. I expected that these questions would be supported and defined by the teachers' increased content knowledge.

After the professional development sessions, teachers were able to recognize, write and implement critical thinking questions in their classrooms beyond their ability to do so before the professional development. The teachers verified this shift in knowledge and ability in their journals and in their written and oral post-assessments. Increased content
knowledge supported most of the critical thinking questions imbedded in their final lesson plans.

Implementing critical thinking questions in the classroom was more difficult for the teachers than writing the questions. The teachers lost confidence in their ability to continue prompting students toward deeper understanding when student-teacher dialogue continued beyond the initial written questions. Practice in sustaining threads of knowledge exploration during our professional development sessions may have made the teachers more confident in the classroom and would have helped the teachers to further explore their content knowledge.

However, the greatest change in classroom practices, due to the professional development sessions, was not in the use of critical thinking questions but was in a shift from a process-oriented, teacher-centered practice towards an inquiry and hands-on, student-centered pedagogy.

The very nature of examining content understanding through critical thinking questions requires student and teacher inquiry into the content. Listening to students' responses to questions and probing for understanding implies a more student-centered environment. There would not be time or context for critical thinking questions in a traditional, teacher-centered and process-oriented classroom. By experiencing the use of manipulatives in an inquiry-based learning environment themselves, the teachers constructed new content knowledge but also constructed new understandings about standards-based teaching. While agreeing to examine and implement critical thinking questions in their classrooms, the teachers shifted their classroom practices toward more inquiry-based lessons and therefore toward more standards-based practices.
References


Appendix A

Participant agreement letter

Dear ________________

As you may have read, I was awarded a Teacher Center Grant for $1000 to research the use of manipulatives as a tool for helping teachers make connections in mathematics and to increase math content knowledge for teachers.

My focus for this grant is on the content area of part and whole, and patterns.

My hopes are to begin research in February and finish it by the end of March.

I will be using the results of this research as the focus of my Master's theses in May. (You could be part of this milestone!)

I am currently looking for four teachers who will be willing to work with me on this project. I would prefer teachers who already work on a grade level team but it is not imperative. The agreement between the teachers and myself would include the following:

- 5 paid hours ($20 per hour) of professional development time for each teacher.
- This would include
  - 3 one hour sessions after school at a mutually agreed on time to explore content,
  - 1 hour after school to develop a lesson plan for the content, and
  - 1 hour after school at a mutually agreed upon time to review the implementation of the lesson.
- Teachers will be able to each keep 2 sets of 1" tiles, 2 sets of 1" cubes and share 25 sets of fraction tiles at the conclusion of the research.
- Teachers would agree to let me, or another of the four teachers of their choice involved in the research, observe the lesson plan created during the research as it is implemented in the classroom.
- Teachers would agree to keep reflective journals during the hour-long professional development time, and allow me to keep these journals for data when the 5 hours is completed.
- Teachers would agree to allow audiotapes of the professional development sessions.
- Teachers would agree to share student work from the lesson presented.
- Teachers would agree to a pre and post interview of learning styles and content knowledge.
- Teachers would be asked to complete all terms listed here

The information and data collected during the implementation of this proposal is purely for research purposes. Names of the participants and of the school system will be purged from the documents before publication.
If this appeals to you, maybe even as way to fulfill your own teacher proposal, please fill out the information below and return it to me.

Name ________________________________

Grade Level ________________________________

Total years teaching ________

Years at Marion ________

Years teaching current grade __________

Focus of your Master's Degree (e.g.: reading, special ed, etc.) __________________________

Age:

_____20-30

_____30-40

_____40-50

_____50-60

I agree to the terms of this professional development research activity as listed on the attached page.

Signature ________________________________

Date _________________________________

Thank you,

Beth Walker
C.T. Questions:
   a) Why do the lengths of the pieces get further apart as you progress down the data table?
   b) Based on the data tables, could you predict the length of one piece of a 48-tile train, which is broken into 4 pieces? A 6-piece train?
   c) If the number of pieces was graphed against the length of each piece, what do you think the graph would look like?

7. Graph both sets of the data.

C.T. Questions:
   a) Did the graph look like you expected it to?
   b) Why do you think that the graph looks the way that it does?
   c) Have you ever seen another graph that looks like this? What did it represent?
   d) We graphed integers. What about the graph between the integers?
   e) Do you think that all graphs of tile train pieces and lengths would look like this?
   f) What is the smallest train that you could graph?

8. Final journal prompt:
   How much is 1/2?

Data Table for Tile Trains

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What is a critical thinking question?

A critical thinking question is a question that requires a logical thought process, using prior knowledge, to develop a cohesive answer.

Examples:

Consider the sequence $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
- What is happening to the size of the fractions in the above sequence?
- Why?
  - What would happen if the sequence continued for a very long time?

Consider the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
- What is happening to the size of the fractions in the above sequence?
- Why?
  - What would happen if the sequence continued for a very long time?

A critical thinking question is a question that prompts a connection across Key Ideas, multiple intelligences or sub-disciplines.

Example:

- Why do you think the Egyptians wrote fractions the way that they did?

A critical thinking question is a question that has multiple paths to an answer or may have no right or wrong answer.

Example:

Multiple paths: What are 5 or more fractions (all but two of them different) that you can add up to 1.

No right or wrong answer: Will $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ ever get so close to zero that it will disappear?
If a question prompts an answer that is easily available and readily explained, without engaging in a logical thought process, it is not a critical thinking question.

Critical thinking questions must be differentiated for different levels of thinkers. Students who process and see connections more quickly will need a different level of critical thinking question.

What behavior in your students indicates that you have successfully launched a critical thinking question?
March 10, 2003
Session 3 - lesson plan
Part and Whole

Targets:
1. Unit Fractions from an Egyptian perspective
2. Cross-discipline connection - Fractions and Music
3. Parts as quantity
4. Unequal parts that make up a whole quantity
5. Verification of solutions
6. Transference of fractional parts to another format (circle graph)
7. Use of measurement tools (compass and protractor)

Materials:
Music
Fraction Tiles
1” tiles
compasses
protractors
colored pencils

Journal Question: How young can children begin to process critical thinking questions?

Lesson:
1. One explanation of Egyptian Fractions.

2. Music as part and whole.

3. Part and whole in multiple contexts
   string
   tiles
   fraction tiles

4. Part and whole as quantity
   Task: Part 1 Colored tile bonanza

5. Part and whole as quantity
   Task: Part 2 Same fractions, different look

6. Critical Thinking questions

Journal Question: What do you still need to be comfortable with using critical thinking questions?
Colored Tile Bonanza
Session 3
Part and Whole

Key Ideas: KI 1 – Mathematical Reasoning, KI 2 Numbers and Numeration, KI 3 Operations, KI 4 Modeling and Multiple Representation, KI 5 Measurement

Materials:
Fraction Tiles
1" tiles
compasses
protractors
colored pencils

Task 1:
Goal: From part to whole. Determine the parts of a whole using quantity.

1. Examine your bag of colored tiles.
2. Determine the whole.
3. Determine what part of the whole is represented by each color.
4. Verify that your fractions add up to 1.
5. Verify that your quantities add up to the number of tiles that you began with.

Task 2:
Goal: Use what you know about fractional parts to transfer the idea of whole and part from one group of things to another.

You need to know that:
Protractors measure in degrees
A complete circle contains 360 degrees
Compasses can be used to draw circles but should never be used to poke holes in people.

1. You determined the fractional parts of your collection of tiles.
2. Determine a way to accurately represent your collection of tiles in a circle graph.
3. Explain what techniques and strategies you used to complete this project.
March 17, 2003
Session 4 - Hand out: review of content
Part and Whole

Content we have explored:

- Factor pair connections in part and whole
- Patterns in fraction sequences such as 1/2, 1/3, 1/4 ...
- Patterns in fractions such as $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ ...
- Limits (When a number gets closer and closer to a goal)
- Representation of the same fractional parts in multiply ways
- Verifying answers that relate to part and whole
- Egyptian fractions
- Music and fractions
- Graphing on coordinate axis
- Constructing data tables and using them to see patterns
- Measurement of fractional parts, with consequences (not enough string)
- Whole to part (The String Thing)
- Part to Whole (Fraction Tile Bonanza)
March 17, 2003
Session 4 - lesson plan
Part and Whole

Targets:
1. Briefly review content explored.
2. Write a lesson plan, to be implemented in the classroom in the next 2 weeks
3. Imbed critical thinking questions in the lesson.

4:00 Journal Question
Was exploring the content of part and whole necessary for you to begin writing and using critical thinking questions in this area?

Task:
1. Write a lesson plan around the concept of part and whole.
   • It can be an introductory lesson, a middle lesson or a summative lesson.
   • It can be a teacher centered lesson, or a student centered lesson.
   • It can be directed or exploratory (project of performance based).

2. Your lesson plan can span two days if necessary.

3. Please include 5 critical thinking questions with your lesson. They can be imbedded in the lesson plan format or listed at the end.

5:00 Journal Question
What was more helpful to you in building a base for using critical thinking questions in the area of part and whole:
   learning how to use the manipulatives for part and whole
   or
   being presented with performance based projects that used the concepts of part and whole
   or
   learning new math terms, math concepts and math connections such as limits, music and fractions and Egyptian fractions?
3. Please choose two questions from your new lesson plan and tell me why you wrote these questions. What was your thinking about the content? What were your goals for your students thought processes? What understandings did you hope they might come to?

4. Would you and could you have written both of these questions without the tasks that we did that explored your own content knowledge of part and whole? (Be honest, I will not be offended!)
Appendix E

Initial and Final Teacher Lesson Plans
Lesson Plan
Cookie Sharing Problem

Materials: Direction/question worksheet, envelopes with cookies (1-2), 12x18 paper on which they will glue the solution for each envelope, scissors, glue

Anticipatory Set: What does it mean to share equally? What do you think you would do if the ‘wholes’ didn’t come out evenly?

Steps of the lesson:
- Divide the children into groups of 3
- Pass out the direction/question sheet and discuss.
- Give each group their envelopes, large paper, glue, and scissors.
- As the groups work, wander and monitor progress. Ask questions as needed
- Be sure to leave time to have the group answer the questions

Closure:
- Pull the group back together and discuss their results.
Today you have a sharing problem

Each envelope has a number on it. Inside the envelope there is that number of ‘cookies’. Your job is to share the cookies in each envelope equally among your group members. (I encourage you to use the envelopes in order.) You may use the cookies whole or cut them as needed. The large paper is numbered to correspond with the envelopes. When you have your solution for each envelope, please paste your solution on the paper and give a written explanation as to what you did and why.

When you have completed the sharing activity, as a group please answer the following questions.

1. What do you think will happen as the number of cookies increases? Why do you think this?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
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2. Did you see any patterns as you were completing this activity? Explain.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

3. Did you see anything that surprised you as you worked? If so what?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________
Procedure: "KWH Chart - Brainstorm"  
1) Chart what students already know about fractions.  
2) Show a 9 x 13 whole cake. Discuss that if half was to share this cake with 62 people, we would  
\[
\begin{array}{c|c|c|c}
\hline
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\hline
\end{array}
\]
\[\frac{1}{2} \div \frac{1}{62} = \frac{62}{2} = 1 \text{ whole}\]  
3) What # do you write as the denominator?  
4) What does the denominator tell you?  
\[\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1 \text{ whole}\]  
\[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}\]  
\[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{8}{8} = 1 \text{ whole}\]  
(etc.)

How will we go about dividing the whole cake into equal pieces for 19 students and 1 teacher?  
- Ask for responses  
- Divide up the cake (discuss numerator)  
- Denominator, multiple denominator  
[seconds later]  
- Hand out toothpicks rolls  
- Discuss how it is included that way but it
A pizza comes in parts (fractions).

Mention $1.00 whole dollar

4 quarters  \( \frac{1}{4} = 25 \, \text{c} \)
\( \frac{2}{4} = 50 \, \text{c} \)
\( \frac{3}{4} = 75 \, \text{c} \)
\( \frac{4}{4} = 1.00 \)

10 dimes = $1.00

\( \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} = 1.00 \)

Assessment: Teacher observation
Questions

1. What do you know about fractions? (KWL) or students write on an index card.

2. How do fractions play a role in our lives?

3. How would our lives be different without fractions?


5. Are there any particular math equation that we use when working with fractions? Are there any patterns?
# Standards used

1. Number & (operation)
2. Problem solving
3. Communication
4. Connections

# Mathematical Reasoning
2. Number & Numeration
3. Operation: $x, \div, +$
4. Modeling & Multiple representation (cake, toothpicks, money)
5. Measurement - [cake] to get = parts
6. Patterns $x_2, x_3, x_4, x_5$ (etc.)
Math

Objectives: Fractions: What is a whole?
  What is a part of a whole?
  As part of a set
  As part of a whole.
  How can ½ be shown using different fractional pieces?
  Compare the sizes of fractional parts.

NCTM Standards: 1,2,5,6,8,9

Materials: Journals Students
  Textbooks
  Fraction tiles
  Pencils
  Chart paper
  Bag of onions

Procedure: Opening Journal Prompt: We hear about wholes of things and parts of wholes- fractions in a variety of circumstances in our daily lives. Please think about the following questions and answer them in your math journals.

1. What are some things that we think of as a whole?
2. What are some things that we think of as parts of a whole or that we divide into parts? Include other items that are not food.

Share and chart some examples on chart paper.

Fractions as parts of a set: Using a bag of onions discuss that this is a whole bag of onions. How many are in the bag? Now I take 1 out to use for cooking. I write 1/6 on the board. What do you think the 6 refers to? What do you think the 1 refers to? Name them Denominator and Numerator.

Continue this discussion using a variety of textbooks. What fraction of the set are Math books? Social Studies books? Scholastic books? Then have a group of students stand in the front of the class. What fraction of the group are boys? Girls? Are wearing blue? Are not wearing blue?

Fractions as parts of a whole. How can ½ be shown using different fractional pieces?

Hand out a bag of fraction tiles to each student or pair of students. Have them take out the 1 whole strip and the ½ tiles. Put the ½ tiles below the whole tile. Discuss that ½ plus ½ equals 1 whole. Also that each of the 2 tiles are the same size. Now we are going to focus on just one side of the fraction bar. We are going to find other fraction tiles that add up to 1/2.

Using your fraction tiles what are 4 or more fractions that can add up to 1/2?
Which ones did not work?
Why did some fractions add up to 1/2 and some did not?
Do you notice any pattern in the order of your fractions?
What happens to the size of the pieces as the denominators get bigger?

The students will respond to the questions in their journals. I will assess their understanding through observation and their journals.
Math
Fractions

Using your fraction tiles please answer the following questions in your journal.

1. What are 4 or more fractions that can add up to 1/2?

2. Why did some fractions add up to 1/2 and some did not?

3. Which fraction pieces did not work?

4. What happens to the size of the fraction pieces as the denominators get bigger?
Lesson: Parts Of A Whole

NCTM Standards: 1, 6, 8, 9
New York State Math Standards: Standard 3 (see attached sheet)

Objective: Students will identify, read, and write fractions which represent parts of a whole.

Materials: McGraw-Hill Mathematics Textbook (pp. 470-471), graph paper, drawing paper, crayons, graph paper transparency, Practice Worksheet 11-1

Vocabulary: fraction, numerator, denominator

Procedure:

1. Provide students with drawing paper- have students fold paper until corners meet, then fold it again. Have students open paper and ask how many equal parts the paper is divided into. Have approx. 1/3 of students color one part. 1/3 color 2 parts. and 1/3 color 3 parts. Review vocabulary by asking students what numbers they would write as the numerator and denominator to show what part of the paper they colored.

2. Do Try It (p. 470) and Practice (p. 471) in textbook orally together.

3. Using graph paper, have students draw rectangles with the following fractions shaded: 2/3, ¼, 5/8, ¾, 5/9, ½, 2/4, 7/8 Note: Model example using graph paper transparency.

4. Assign Worksheet 11-1 for independent practice.

Assessment: Teacher observation, successful completion of work during lesson, successful completion of assigned worksheet

Enrichment: Enrich Worksheet 11-1
Critical Thinking Questions

1. Could you write a fraction if the whole was not divided into equal parts? Why or why not?

2. What happens if the numerator and denominator are the same number?

3. Could you still write a fraction if no parts of the whole were shaded? If so, how would you write it?

4. How would you figure out how to write a fraction for the unshaded part of a whole?

5. What do you notice about the amount shaded in to show ½ and 2/4? Why are they the same?
Students will:

* use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances
* use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions
* relate counting to grouping and to place-value
* recognize the order of whole numbers and commonly used fractions and decimals
* demonstrate the concept of percent through problems related to actual situations

Students will:

* understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded and scientific notation)
* understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations
* relate counting to grouping and to place-value
* develop an understanding of number theory (primes, factors, and multiples)
* recognize order relations for decimals, integers, and rational numbers
* understand the concept of infinity
* recognize the hierarchy of the complex number system
* model the structure of the complex number system
* recognize when to use and how to apply the field properties
* understand and use rational and irrational numbers
* recognize the order of real numbers
* apply the properties of the real numbers to various subsets of numbers
Understanding Fractions

A fraction is a part of something. 1/2 is a fraction. Look at the rectangle above. It is divided into two equal parts. It is divided in half.

Look at the rectangle above. It is divided into 3 parts. The blue area is 1/3 of the whole area. Each part of the rectangle is 1/3 of the whole. There are three 1/3's in the whole area.

The green area is 1/4 of the whole rectangle. There are 4 parts to the whole area. Each part is 1/4 of the whole rectangle.
What is the fraction for each area below?

1/3
Correct My Answers

If you have found an error or would like to make comments on this lesson Click Here

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Lesson Plan

Explore Finding Parts of a Group

Grade 4

Date Taught: 4/4/03

NCTM Standards: 1, 3, 6, 8

Objective of Lesson: Students will be able to identify parts of a group using both manipulatives and paper and pencil

Materials needed: Math journals/pencils, one-inch square tiles, teacher made worksheet

Procedure:

- Review what a fraction is with students, including the meaning of numerator and denominator. Emphasize the denominator represents the whole. Discuss 2-3 examples written on board.
- Explain that we can find the fraction (or part) of a group (whole). We need to consider the whole when doing this.
- Distribute a bag of 12 one-inch tiles to each student. (Briefly explain rules for using manipulatives.) Elicit from students that 12 is the whole.
- Say we want to find \(\frac{1}{4}\) of our whole- 12. The denominator of the fraction tells us how many equal groups to make with the 12 tiles. Students will put tiles into 4 equal groups. Have students count one of the groups. How many? (3) Explain therefore, \(\frac{1}{4}\) of 12=3 or 1 group of 4 contains 3 tiles. Do other examples with the unit fractions: \(\frac{1}{2}\) of 12, \(\frac{1}{3}\) of 12, \(\frac{1}{6}\) of 12.
- Have students predict: Could we find \(\frac{1}{5}\) of 12? Why or why not? After discussing this with students, explain that it depends on the whole. We couldn’t with the 12 tiles, but what about 12 cookies. (We can break cookies.
- Ask students: How can we find a fraction of a group when the numerator is not one? For example, how could we find \(\frac{2}{4}\) of 12?
- Have students make predictions, then demonstrate using tiles. Have students do the same, explaining that after dividing the 12 tiles into 4 equal groups, they need to count 2 of the groups. Do the same to find \(\frac{3}{4}\) of 12, \(\frac{2}{6}\) of 12, and \(\frac{2}{3}\) of 12. Ask: What happens if we try to find a fraction of a group when the numerator and denominator are the same, for example \(\frac{3}{3}\) of 12? Ask students to try this using the tiles. Discuss that they would find the whole.
• Ask: **How could we find the fraction of a group (whole) without using the tiles?** Discuss student responses, then explain the process using paper and pencil. Explain that you can use division and multiplication to find the fraction of a group. Demonstrate using the example 2/4 of 20.
  - Use the denominator. Divide the total into that many groups.
  - **Think:** 20 divided by 5 = 4
  - Multiply the numerator
  - **Think:** The numerator is 2. 2 x 4 = 8
  - So 2/4 of 20 is 8.

• Do examples together with students from p. 485 in text, having students write them in their math journals.

• Close/Ask: **How do you think you could find 7/5 of 25?** Discuss responses and solve.

• Distribute teacher made worksheet (attached) to students for independent practice.

**Methods of Assessment:** Teacher observation of student participation during lesson, participation in class discussion, successful completion of worksheet

**Note:** Critical thinking questions are in bold type.
Find the fraction of each number.

1. \( \frac{1}{3} \) of 15 =
2. \( \frac{3}{5} \) of 10 =
3. \( \frac{3}{10} \) of 30 =
4. \( \frac{1}{9} \) of 18 =
5. \( \frac{5}{6} \) of 12 =
Lesson Plan: Fractions

Goal: Students will learn that fractions are parts of a whole.

Anticipatory Set: Name some things that come in pairs, in groups of three, in groups of four, and in groups larger than four.

Step 1. We're going to talk about fractions. What might fractions have to do with pairs, triples, etc.? Tell me everything you know about fractions while I list some of your thoughts.

Step 2. (determined by step 1) In your journal, let's write the definition of a fraction. Let's draw some examples. Draw two boxes and color in one. Draw three cones and color in one. Draw six balls and color in two. What fraction is represented in the first picture? The second picture? The third picture? Now draw one of your own and name the fraction.

Step 3. Why do we learn about fractions? Where do we find fractions in our lives outside of the school? Are there any natural fractions found in our world and what are they? Is there anything in our world that could not be expressed in a fraction or a part of the whole? What might be one of the largest fractions we could think of? Could our grade compared to the whole school be expressed as a fraction?

Step 4. Open your math books to page 469. This new chapter is called fractions and probability. Let's take a minute and look through the chapter to see what we will learn. On your 3x5 card write down something you think will be challenging for you about fractions. (Allow 3-5 minutes) Let's begin at page 470. Work on page 470-473 together.

Step 5. Assign independent work

Assessment: Discussion plays a very important part in evaluating what they already know so I can move on to what they need to learn. Observation, while working with the slates and completing the math pages together, will provide a clue to what they understand and can illustrate through drawing. The 3X5 card will act as a forewarning to what they might need more help in understanding.
Math Lesson: Fractions

Goal: Students will understand that fractions are parts of a whole and are written in a certain manner. The top part is the numerator (the part of something) while the bottom is the denominator (the whole of something).

Standard 3: Students will apply math to real world settings.

Materials: Variety of items
- Counters, cookies, material, egg, orange, bottle of soda, yarn, dollar bill, box of crayons, m&m’s, box of nails, box of cat food

Introduction: Students will be asked to respond in their journals to questions about fractions. What are they, where can one find them, how do they know they are a fraction, and how do you write them. Some journals will be shared and ideas listed.

A second question will pertain to halves. How do we know if we have 1/2, if all 1/2 are the same and where would we find 1/2 in the real world.

Step 1: We will look at several items and decide if we can divide them in half. We will begin easy and move to more challenging. Answers will be charted for further discussion.
- Half of an apple 1/2
- Half of a dozen of cookies 6/12
- Half of a piece of string 9/18
- Half of a box of crayons 12/24
- Half a dozen of eggs 6/12 ½ of an egg?
What is happening with the fraction number? Aren’t they all ½?

Step 2: Groups of four will be given items to divide in half. They will write their fraction down and be ready to share their results.
- Bottle of soda, box of cat food, box of nails, counters, material
(Allow five minutes) Discuss

Step 3: Can we divide everything in half? Can we divide a dollar in half, a sock in half, a house in half, maybe a tree or flower? Why are we able to divide some things and not others?

Step 4: If you could have your choice, would you pick ½, 6/12, 13/26, or 18/36? Why?
Name________________________

Directions: In Column I, name ten or more things in your home that can easily be divided in half.
In Column II, name ten or more things in your home that can be divided in half equally but require some effort.
In Column III, name ten or more things in your home that cannot be divided in half equally.
* (Parents may help.)

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