Technology in the Classroom and its Impact on Student Understanding of Mathematical Concepts

Marilyn Weaver
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Abstract
Two lesson studies were performed to investigate the impact of technology in the mathematics classroom on student understanding of mathematical concepts. The studies were planned and carried out collaboratively over four days (two days per study) by two eighth grade mathematics teachers in the same building. In the first study, graphing calculator technology was utilized and examined for its ability to engage students and assist in their recognition, visualization, and understanding of linear and nonlinear relationships (including exponential, inverse, and quadratic). The second study employed a computer based mathematical Jeopardy game which students accessed on individual laptop computers from the school’s portable wireless computer lab. The computer game was used as a two day competitive review of algebra concepts such as evaluating expressions, distributing, multiplying binomials, and factoring for a test immediately following. Qualitative as well as some quantitative results suggest that the use of technology engaged the students in the learning activities, provided focused practice, improved their ability to visualize relationships, and enhanced their understanding of the concepts.

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Two lesson studies were performed to investigate the impact of technology in the mathematics classroom on student understanding of mathematical concepts. The studies were planned and carried out collaboratively over four days (two days per study) by two eighth grade mathematics teachers in the same building. In the first study, graphing calculator technology was utilized and examined for its ability to engage students and assist in their recognition, visualization, and understanding of linear and nonlinear relationships (including exponential, inverse, and quadratic). The second study employed a computer based mathematical Jeopardy game which students accessed on individual laptop computers from the school's portable wireless computer lab. The computer game was used as a two day competitive review of algebra concepts such as evaluating expressions, distributing, multiplying binomials, and factoring for a test immediately following. Qualitative as well as some quantitative results suggest that the use of technology engaged the students in the learning activities, provided focused practice, improved their ability to visualize relationships, and enhanced their understanding of the concepts.
Dedication

I would like to dedicate this thesis to my family: my husband, James; our three children, Katherine, Thomas, and Rebecca; and my parents, Warren and Jean Stewart. Without James’ support, encouragement, and willingness to take over many of the family responsibilities, my master’s thesis and degree as well as my new teaching career would not have been possible. In addition, Katherine, Thomas, and Rebecca’s sacrifices, enthusiasm in supporting my efforts, and well-timed expressions of pride in my accomplishments were essential to my success in reaching my goals. The powerful, loving example set by my parents made me who I am and instilled in me the desire to achieve and excel academically. My father, Warren Stewart, whose life and amazing career served as an inspiration to me throughout my life, put the finishing touches on the final chapter of his book and closed the final chapter of his life just as I was completing the final chapter of this thesis.
Acknowledgement

I would like to thank my lesson study partner and colleague, Joe, for his numerous ideas, enthusiasm, and willingness to put in long hours and tireless effort for the sake of a great lesson. I look forward to many more such lessons.
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Technology in the Classroom and its Impact on Student Understanding of Mathematical Concepts

A great deal of attention has been focused on the potential benefits of the incorporation of technology into the classroom. Technological advances offer opportunities for improvements in many aspects of our lives; education is no exception. This paper considers the potential impact of technology use in the mathematics classroom.

My goal in researching this topic was to learn what educational benefits can be gained from technology use in mathematics education and what factors determine whether it is successful. I hoped, through my review of the existing literature, to become aware of the various options for the incorporation of technology, that is, the devices and programs that are available and how they can be used effectively. My intention for my individual research in the classroom was to determine the effects of utilizing technology that is readily available to mathematics teachers in our school (and many schools), without large expenditures.

As the literature indicates, technology in the mathematics classroom includes computers, computer software (especially interactive programs), graphing calculators and associated systems such as computer based labs (CBLs), and programmable hand-held computing devices (otherwise known as personal digital assistants or PDAs). All of these technological tools hold promise for engaging students, helping them visualize mathematical concepts or processes, promoting enhanced student understanding of concepts, and improving knowledge retention. However, the success of any of these
technological enhancements is highly dependent upon the way in which it is implemented by the individual teacher, as the literature clearly shows. Both teacher training and careful planning are crucial.

I have long been interested in incorporating technology into my classroom in meaningful ways to enhance student learning of mathematics. The knowledge that I have gained from my review of the literature and through my classroom research will help guide me in that effort.
Literature Review

Interest in the effect of the incorporation of technology into classroom instruction is significant and the literature deals with many aspects of this educational trend. The following review includes a brief historical perspective of the incorporation of technology into mathematics education, the goals of its use, and possible applications to various grade levels. Next, applications of instructional technology to the needs of special student populations are considered, in particular those with ADD (attention deficit disorder) or ADHD (attention deficit hyperactivity disorder) and those with learning disabilities (LD). The review then addresses specific types of technology: computers and computer programs, graphing calculators, and hand-held computing devices (PDAs). Consideration is given to the importance of appropriate teacher training as well as an effective and realistic process for incorporating technology into the teaching repertoire. Finally, the results of technology use in the classroom are examined from the standpoint of standardized test scores and the experiences of model districts, and a problem with the incorporation of technology into mathematics classrooms is considered. A summary, including expectations of technology, concludes the literature review.

Historical Perspective

Computers have been utilized in schools since the early 1980’s. At that time, there was generally one machine per mathematics classroom, which was used in conjunction with spreadsheet software for mathematical modeling (Johnston-Wilder & Johnston-Wilder, 2004). By the mid 1980’s, David Tall was developing a viewing window that
could change scale, paving the way for personal computer graphing software and
graphing calculators (Tall & West, 1986). At about the same time, SMILE (Science and
Mathematics Initiative for Learning Enhancement) computer software was being
developed with guidance from teachers. Shortly afterwards, in the early 1990s, graphing
calculators started to appear in classrooms, though on a limited basis (Simmt, 1997).

Since then, classroom computers, home computers, graphing calculators, CBLs
(calculator-based labs), and PDAs (personal digital assistants) have become prevalent.
Thus, there is now increased access to technology not only in the classroom, but at home.

With the current availability of technology, Johnston-Wilder and Johnston-Wilder
argued that:

There is the potential to achieve much more mathematics education with ICT than
we are currently achieving, and we need to achieve more and differently. Industry
is asking for ‘integrated mathematics and ICT skills, multi-step problem solving,
complex modelling including constraints, recognising erroneous answers,
communicating mathematics and an ability to cope with the unexpected’ (Hoyles
2002). Pupils are asking for ‘important, difficult content presented in an interesting
attractive way’ and are becoming increasingly disillusioned with what is currently

Jacobson and Kozma (2000) also urged use of technology, but cautioned that it be
appropriate and well-designed. They pointed out that schools and communities are
investing a great deal in computers and the latest technology, but that:
Technology in Mathematics Education

The mere availability of powerful, globally connected computers is not sufficient to insure that students will learn, particularly in areas that pose considerable conceptual difficulties such as in science and mathematics. The true challenge is not just to put advanced technologies in our schools, but to identify advanced ways to design and use these new technologies to enhance learning. (p. xiii)

Goals of Technology in Education

As argued above, the approach to technology use in education must be more than simply throwing money and high tech gadgets at teachers and students. Educational communities must have an idea about what they hope to accomplish with the multitude of resources available. One such goal is the promotion of student engagement in the learning process. Erbas, Ledford, Polly, and Orrill pointed out that “technology can become a vital and exiting tool in allowing students to explore...mathematical situations and relationships (NCTM 2000).” (2004, p. 300-301) Hines related the experiences of a district in Modesto, California in which junior high school students enthusiastically participated in an interactive technology-based learning environment for mathematics and science. Students found the system fun and even eagerly anticipated quizzes (2005). Furthermore, Johnston-Wilder and Johnston-Wilder found that incorporation of technology into the learning process increased motivation, and therefore, engagement. Students felt that they were able to work more effectively and quickly, with improved concentration.

Another goal of technology use, which is intrinsically linked to the first, is improved student understanding. One important way in which technology helps students
comprehend concepts and solve problems is through multiple representations, especially in mathematics (Erbas, Ledford, Orrill, & Polly, 2005; Erbas, Ledford, Polly, & Orrill, 2004). Technology is particularly effective in addressing the visual-spatial sense, which can be instrumental in comprehension, as noted by Sundberg and Goodman (2005), as well as Lopez (2001). All students, especially struggling students and those with disabilities, are empowered by technology that frees them from complex computations and allows them to approach mathematical concepts from new, visual, and perhaps more intuitive and easily understood perspectives. Erbas, Ledford, Polly, & Orrill (2004) noted that technology is no longer "just a simple tool to perform some calculations or engage in drill-and-practice exercises" (p. 305). Its true power is utilized when students investigate concepts using multiple approaches, thus improving their comprehension as well as their problem-solving skills. In fact, providing students with the opportunity to explore a single mathematics problem with multiple technologies allows students to experience authentic problem solving, be creative in their problem-solving approach, examine their data from various perspectives, check the validity of their answers, and find effective representations of their findings. With appropriate lesson design, technological tools foster deep understanding of difficult scientific and mathematical concepts (Lopez, 2001; Jacobson & Kozma, 2000).

Additionally, technology should be a catalyst for the introduction of authentic practice into the classroom (Lopez, 2001; Roth, 1992). Based on a study that he performed, Roth proposed an interdisciplinary approach to science, mathematics, and technology education in which students have the opportunity to experience real-life problems that are messy and ill-defined, requiring true problem solving utilizing all three
disciplines. Education which incorporates authentic practice enhances learning, improves problem-solving skills, and helps prepare students for the challenges of the real world. Furthermore, it aligns with state and national standards for mathematics and science education and is encouraged by the NCTM (National Council of Teachers of Mathematics) and the AAAS (American Association for the Advancement of Science) (Roth, 1992).

Finally, technology can be a very effective tool in the effort to rectify student misconceptions, since persistent misconceptions are best reversed through student discovery of concepts. An example of such use of technology is the Inventive Model (IM) developed by Rezaei and Katz at the University of Calgary. “The IM uses a computer assisted constructivist approach to deal with students’ deeply seated misconceptions that obstruct learning” (Rezaei & Katz, 2003, p. 57).

Across the Grade Levels

The incorporation of technology into mathematics education is effective at all grade levels, but varies in approach. At the elementary level, students may use spreadsheets to graph, analyze, and compare data, or they may play computer games that help develop mental math skills, strengthen number sense, assist with money or time concepts, or develop deductive or spatial reasoning (Bayliffe, Bric, & Oliver, 1993).

At the middle school level, McGehee and Griffith pointed out that technology can be used to help address most of the NCTM content and process standards. In particular, it can “help students make generalizations from arithmetic and develop algebraic thinking” (2004, p. 344), a key objective of middle school mathematics. Graphing calculators or
graphing software can help students draw connections between the patterns of numbers in tables, graphs, and algebraic equations. CBLs, or calculator-based lab probes, can help generate real data for the analysis. Graphing calculators or spreadsheets can support student understanding of decimal representations of rational and irrational numbers (number and operations standard). Dynamic geometry interactive software, such as Cabri and Geometer’s Sketchpad, allows students to gain a deeper understanding of geometry by interacting with figures, changing their size, shape, or perhaps orientation. It is an inquiry process, in which students make conjectures, gather evidence, revise their assumptions, and draw conclusions about the properties of geometric figures. Furthermore, technology facilitates the study of data analysis and probability, providing easy access to the visual graphical representations necessary for making good predictions and inferences (McGehee & Griffith, 2004).

As evidenced in each of the above areas (number patterns in tables, graphs, and equations; understanding of decimal representations; geometric properties of figures; and data analysis and probability), technology allows middle school math learning to be done efficiently, with enhanced understanding. In fact:

The technology prompts student thinking; it does not limit it. The tools...discussed here allow teachers to choose worthwhile mathematical tasks ‘that take advantage of what technology can do efficiently and well--graphing, visualizing, and computing’ (NCTM 2000, p. 26). Because much technology is available and applicable across the Content Standards, it should be considered essential to mathematics teaching and learning. (McGehee & Griffith, 2004, p. 349)
Technology has the potential to promote problem solving and improved comprehension in high school mathematics courses, as well. As at the middle school level, its use can help students gain a deeper understanding of algebra, geometry, and calculus concepts. In fact in a problem solving scenario presented by Erbas, Ledford, Orrill, and Polly, students made connections between algebra and geometry, thus supporting in-depth comprehension in both content areas. The sample investigation which follows (Erbas, Ledford, Orrill, & Polly, 2005, p. 600) indicates the potential of this approach:

In a rectangular field ABCD, Tom and Paul are both at point A and want to arrive at point C. To do so, Tom walks straight from A to C. To get a drink along the way, however, Paul walks first from A to B and then from B to C. What is the distance in yards that Tom travels if Paul travels 40 yards farther than Tom? What are the dimensions of the field? (Adapted from Georgeson 1997)

Students were provided with multiple technologies with which to explore the problem, including interactive geometry software (Geometer’s Sketchpad), computer spreadsheets, a computer algebra system (CAS), and graphing calculators. The dynamic geometry software helped them to visualize the problem geometrically and draw some preliminary conclusions; the spreadsheets offered the students an opportunity to test their conjectures (but rounding error complicated the situation); the computer algebra system supported them in their attempts to solve the equation algebraically, and the graphing calculator made it possible to explore the equation graphically to visualize the critical values of x and the continuities and discontinuities (asymptotes) that occurred in the function. Students did not work through the problem in a simple linear fashion, but revisited the various technologies
as needed, creating multiple connections in their understanding of both algebra and
geometry from varied perspectives (visual, numeric, symbolic, graphical).

As indicated above, current technological tools are well suited to improve
comprehension of high school calculus. In fact, Roschelle, Kaput, and Stroup (Jacobson &
Kozma, 2000) asserted that “the mathematics of change and variation [calculus] can be
made accessible to a much wider range of students through the design of visualizations and
simulations for collaborative inquiry” (p. 2). They argued that MCV (the mathematics of
change and variation) should be included earlier in the high school math curriculum and
made available to students of all abilities. Roschelle, Kaput, and Stroup hope to employ
their SimCalc project to “democratize access to the mathematics of change” (p. 3), in view
of the need to understand such concepts in the 21st century, a time of economic, social, and
technological change. Future citizens will need to comprehend the concepts of “rate of
change, accumulation, approximation, continuity, and limit... not only to participate in the
physical, social, and life sciences of the 21st century, but also to make informed decisions
in their personal and political lives” (p. 47). SimCalc’s first software product is
MathWorlds, which helps students to use motion to explore MCV concepts, including
position, velocity, acceleration, variable rates, accumulation, mean values, and
approximations. It utilizes animation and interactive graphical representations, introducing
students to piecewise linear functions as a conceptual foundation for understanding
calculus concepts.
Special Student Populations

Students with ADD (Attention Deficit Disorder) or ADHD (Attention Deficit Hyperactivity Disorder) have special needs with respect to mathematics instruction. As Rief indicated, such students are encouraged to use calculators to compensate for memory difficulties in recalling math facts (2005). Additionally, graphing on a regular basis is encouraged as “a way to present and organize data so that relationships in the data are seen easily” (p. 296). Furthermore, Rief reported that visual and hands-on approaches to learning are usually quite effective for ADD/ADHD students. All of the above factors suggest that technology in the form of graphing calculators and computer software with an effective visual component are valuable learning tools for these students.

Students with learning disabilities (LD) are also well served by technology-enhanced mathematics education, for many of the same reasons. As with ADD/ADHD students, learning disabled students have difficulty with computation and often respond well to visual and hands-on instruction. Additionally, the latest thinking is that they need to concentrate on learning concepts and problem solving, rather than mastering computation (Woodward & Howard, 1994; Woodward & Montague, 2002). They often are plagued by misconceptions, which are best dealt with by improving conceptual understanding (Rezaei & Katz, 2003; Woodward & Howard, 1994). Instructional technology such as graphing calculators holds great potential for meeting all of these needs (Laughbaum, 2003; Vasquez, 2003; Woodward & Montague, 2002).
As expressed by McGehee and Griffith (2004), "the word technology encompasses many platforms including calculators, programmable hand-held devices, tutorial software, interactive software, and Internet resources" (p. 344). The various tools are considered below, grouped as computers and software, graphing calculators, and hand-help computing devices (PDAs).

Computers and Software

Computers, software, and resources that can be accessed through them provide powerful learning tools for use in mathematics (and other content area) classrooms. Interactive software provides environments that allow students from the elementary grades through high school to explore and discover mathematical relationships on the computer. Dynamic geometry programs such as Cabri and Geometer's Sketchpad are particularly powerful tools at the middle school and high school level (Erbas, Ledford, Orrill, & Polly, 2005; McGehee & Griffith, 2004), where they allow for discovery learning and making connections through multiple representations. Spreadsheets make it possible for students to examine data in a discrete way when problem solving, and to look for patterns and trends that could help them express the data symbolically and graphically. They help students develop number sense, reasoning abilities, and problem-solving skills (Erbas, Ledford, Orrill, & Polly, 2005; Sgroi, 1992). Additionally, a computer algebra system (CAS) helps students solve equations "who have limited symbolic or numeric manipulation skills...[It] allows them to focus on the concepts and meanings rather than become distracted with the
messiness of the symbolic or numeric calculations" (Erbas, Ledford, Orrill, & Polly, 2005, p. 601-602).

Graphing Calculators

The literature is replete with articles addressing the use of graphing calculators and programmable graphing calculators in mathematics classrooms. Consideration will be given here to graphing calculators, that is calculators with the capability to perform numerical calculations, graph functions, manipulate lists of data, and calculate and display statistical graphs. It should be noted that most of the research studies of graphing calculator use have focused on its effectiveness in enhancing algebra instruction, as opposed to statistics.

Overall, the available research suggests that using graphing calculators in mathematics education can enable students to approach situations graphically, numerically and symbolically, and can support students' visualization, allowing them to explore situations which they may not otherwise be able to tackle (and thus perhaps enable them to take their mathematics to a more advanced level). In this way, using graphing calculators can lead to higher achievement among students, perhaps through increased student use of graphical solution strategies, improved understanding of functions, and increased teacher time spent on presentation and explanation of graphs, tables and problem solving activities (compared with students not using such calculators). (Jones, 2005, p. 31)
There was general agreement in the literature on the above points (Erbas, Ledford, Orrill, & Polly, 2005; McGehee & Griffith, 2004; Laughbaum, 2003; Lewis & Farley, 2000; Lopez, 2001; Vasquez, 2003), as well as the critical importance of teacher philosophy and preparation in determining how effectively the technology is used.

It should be noted that the above collection of literature reported the stated common conclusions concerning graphing calculator use based on observations at the middle school level (McGehee & Griffith), in high school classrooms (Erbas, Ledford, Orrill, & Polly), in calculus classes at the college level (Lewis & Farley), and in developmental algebra classrooms with remedial programs (Laughbaum; Vasquez). The common theme among all of these sources was the capability of the graphing calculator to significantly enhance student visual intuition. Lopez wrote that “it is a reasonable assertion that all students learn certain concepts better by thinking visually about them, that is by constructing mental images of the concepts” (2001, p. 117). The graphing calculator provides a concrete image from which students can work to construct their understanding.

Additional technology in the form of CBLs (calculator-based labs) can be used in conjunction with graphing calculators to allow student collection of authentic data that can be used for analysis. McGehee and Griffith (2004) reported on classroom use of the Calculator-Based Ranger (CBR), a motion detecting probe, in conjunction a graphing calculator. The student collected data provided opportunities for analysis using graphical, tabular, and symbolic representations of authentic data that were meaningful to the students.
Hand-held computing devices (PDAs)

Personal digital assistants (PDAs), otherwise known as palm-held or hand-held computing devices, have become very versatile, especially with software that provides them with all the capabilities of a graphing calculator as well as mathematical worksheet and equation solving capabilities. Ostler has reported on two such software packages, Imag1Math Suite (2001) and PowerOne Graph (2002) that he has found to be very powerful, easy to use alternatives to the graphing calculator.

Teacher Preparation and Philosophy

The literature is very clear that without adequate teacher training and planning, technology in the classroom is ineffective, or worse (Higgins, Moseley, & Tse, 2001; Papanastasiou, Zembylas, & Vrasidas, 2003; Simmt, 1997). Individual teachers need to be trained thoroughly in the use of the instructional technology and they need to have a well devised plan or vision for how to incorporate it into their instruction in order to reap the desired benefits of improved conceptual understanding. It is important that teachers use the technology in ways that enhance student understanding, such as inquiry and open-ended problem solving.

In fact, Rubenstein and Bright (2004) prescribed an ordered process for the appropriate incorporation of technology into teaching practice. Teachers must play with the technology to see what its capabilities are, use it for their own purposes, recommend it to a few students for limited use, incorporate it into the classroom setting, and assess the results based on student learning. They emphasized that incorporation cannot be rushed
and must follow these steps in order to be successful. Furthermore, teachers should start small and successively add more technology, not try to do everything at once.

Research Studies and Results

Many national, state, and local studies of varying sizes have been performed to determine the effect of technology use in the mathematics classroom. The standardized test scores have been analyzed and often reanalyzed as part of larger data bases (Johnson, 2000; Martindale, Pearson, Curda, & Pilcher, 2005; Middleton & Murray, 1999; Schacter, 1999; Weaver, 2000). Some general conclusions can be drawn that are substantiated by these studies as a whole. The effect of instructional technology in the classroom is generally beneficial, usually showing positive gains in student achievement, as measured by standardized testing. However, improvement in student performance, as measured by the test scores, is highly dependent not just on the presence of technology in the classroom, but on how and how often it is used. In cases where technology did not result in student test score improvement, the explanations were invariably related to lack of use, use for drill and practice as opposed to concept development, or lack of support by the classroom teacher.

School districts in Iowa (Rigeman & McIntire, 2005), Nebraska (Isernhagen, 1999), and Modesto, California (Hines, 2005) experienced great gains in student engagement and teacher satisfaction when they implemented instructional technology programs. Both districts that had results available to report showed gains in student test performance. Common factors in all three districts were effective teacher training in the use of the technology and a high level of teacher involvement in the program.
Problem with Technology Implementation

Even with all of the benefits of instructional technology, many mathematics teachers have difficulty justifying the instructional time needed (Erbas, Ledford, Polly, & Orrill, 2004; James, Lamb, Bailey, & Householder, 2000). Effective implementation of instructional technology involves providing the opportunity for student discovery and open-ended problem solving, leading to conceptual understanding. Many teachers avoid that commitment because of time and curriculum constraints due to the pressure of standardized testing.

Summary

Expectations of the technology available for mathematics classrooms need to be clear to all concerned.

Technology is not just a simple tool to perform some calculations or engage in drill-and-practice exercises. Technology allows students to interact with and explore abstract and concrete concepts through multiple representations, which will enable them to be better problem solvers. Through the use of multiple computer based applications, students can develop richer understandings of the mathematics they encounter. (Erbas, Ledford, Polly, & Orrill, 2004, p. 305)

Wentworth and Monroe (1996) encouraged educators to inform parents about the innovative ways in which technology is being used in mathematics classrooms, “where students construct mathematical meaning using technology as a tool” (p. 132), in order to gain parental acceptance of this instructional approach. More importantly, individual
teachers must determine their own vision for incorporation of technology into their classrooms, for they determine how, when, and how often it is employed (Simmt, 1997). Finally, students need to adapt and adjust their learning styles. The success of these educational endeavors is of great importance, and is summed up by Sgroi (1992), "the marriage of problem solving and instructional technology in a cooperative-learning setting can be critical to the development of a skilled twenty-first-century work force" (p. 8).
I collaborated with another teacher in my junior high school building who teaches the same grade level and subject as I: eighth grade mathematics. We work in a school district that is a high achieving suburban district with an average population base; consequently, the community attracts residents who value education. My colleague agreed to collaborate with me in performing two lesson studies. Lesson study is a Japanese approach to peer coaching, in which teachers design a lesson, take turns teaching and observing the lesson, and refine the lesson (Fernandez & Yoshida, 2004). Near the beginning of the school year, we checked our teaching schedules and realized that it would be possible for my colleague to observe me teach during first period and for me to observe him teach sixth period. In addition, we had the opportunity to develop or revise lessons fourth period. Our plan was that I would teach the lesson first, during first period, while he observed, we would revise the lesson fourth period, and he would re-teach the lesson while I observed during sixth period. We planned to meet after school the same day to make final revisions to the lesson. This compact sequence was actually used only for the second lesson study, later in the school year, but not for the first study. About two weeks before the first lesson study, our administrators requested that I temporarily slip my lesson schedule so that my classes would be one day behind those of my colleague. The intent was to allow me, in my first year teaching at the school, the opportunity to observe his lessons prior to teaching them myself. Therefore, my colleague and I restructured our format for the first lesson study so that he would teach the lesson first, during sixth period while I observed, we would meet to revise the lesson after school the same day, and I would re-teach the lesson first period the
next day while he observed. Final revisions of the lesson would be made during fourth period the second day. We decided that we would each teach our own students in our own classroom.

Since my colleague had taught the curriculum previously, he was able to suggest a particular point in the curriculum that he felt would be appropriate for our first lesson study of the impact of technology in the mathematics classroom. About a week beforehand, we began meeting fourth period as well as after school to plan the details of the two day lesson (based on material from our Connected Mathematics program textbooks; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998, 2002) that would take place on Wednesday and Thursday October 19 and 20. The expectation of the curriculum at this point was that students learn to recognize different equation types and their associated characteristics. We chose graphing calculator technology as a means for students to investigate various forms of equations and their graphical representations in order to categorize equation types as linear or nonlinear as well as increasing or decreasing.

Since each mathematics teacher in our building has a class set of graphing calculators, this choice fulfilled our goal of using technology already available in our classrooms.

We felt that it was important to carry out our second lesson study using a different type of technology, but one that is also readily available within our school. Another colleague suggested that we try a computer version of the Jeopardy television game show. A computer template of the game was available, and since we were working on an algebra unit in preparation for the New York State Eighth Grade Mathematics Test mid March, we decided to build our two day review of evaluating expressions, distributing, multiplying binomials, factoring, and properties around the Jeopardy game, to be played on laptop
computers from our school's portable lab. My colleague and I prepared two *Jeopardy* game boards, each consisting of five categories and a total of twenty-five questions. The lesson was held on Monday and Tuesday, January 30 and 31, immediately preceding our in-class test on the same material.

**Participants**

As indicated above, the subjects of our research were the eighth grade mathematics students in the classes taught by my colleague and myself, with about twenty students in each class. Although the students in my first period class and his sixth period class were the focus of our lesson study due to our opportunity to observe each other at those times and reflect on our practices, all four of my classes of regular eighth grade mathematics and all four of his classes received the lesson. Consequently, we utilized data from all of them when analyzing results and drawing conclusions. In the first lesson study, all eight classes received the technology enhanced version of the lesson; however, for the second lesson study, we decided to create control groups consisting of two out of the four of my colleague's classes and two out of my four classes. Consequently, in the second lesson study, my first and third period classes received the technology enriched version of the lesson both days, while my fifth and ninth period classes (control classes) had the same review, but without the laptop computers. Similarly, my colleague's fifth and sixth period classes had the technology enhanced lesson, whereas his eighth and ninth period classes (control group) received the same review material without the computers. These class choices were made based on convenience, allowing us to move the wireless laptop cart only once each day from my room in the morning to my colleague's room mid-day.
Class assignments made by the counseling department at our school at the beginning of the year were intended to result in classes that were heterogeneous in nature, with all classes having equal ability and mathematical preparation, on average. In reality, my first and fifth period classes were able to understand mathematical concepts more easily and quickly than my third and ninth period classes, making my choice of my first period and third period classes as the experimental classes and my fifth and ninth period classes as the control group very fortuitous. As can be seen in Table 1 on page 34, the year-to-date cumulative average in mathematics (as of March 1) for students in my first period class was nearly identical to the average for my fifth period class, and considerably higher than the nearly identical averages for my third and ninth period classes. With one stronger and one weaker class in the experimental group and in the control group, data comparisons became meaningful due to a balance of student abilities. Unfortunately, my colleague's groups were not as well balanced; his fifth and sixth period classes (experimental group) were made up of weaker mathematics students, on average, than his eighth and ninth period classes (control group). In fact, his fifth period class included a mix of eight special education students and twelve regular ability students, as well as five adults (my colleague as the subject teacher, a special education teacher, her aide, and two one-on-one aides for individual students). Thus, his experimental group had weaker mathematical skills to start with than the control group, making it difficult to measure quantitative gains attributable to technology use. However, qualitative results based on observation were possible.

The classroom environments in my colleague's and my rooms were quite comparable and maintained our usual arrangements. Desks were positioned in rows, with two or three desks adjacent to each other, allowing for partner and small group work. The desks faced
the front of the room, where a long white board, large wall-mounted television, and
overhead projector and screen were positioned. When the wireless computer lab was used,
the cart was placed in the rear of each classroom.

Instruments and Materials

During the first lesson study, each student was provided with a TI-83 graphing
calculator from the classroom set as well as classwork sheets (included in Appendix A for
the first day, or Part I, of Lesson Study I; and in Appendix B for the second day, or Part II.
of Lesson Study I). The first day, students were given a warm-up sheet that we generated,
consisting of sixteen equations which they were asked to place into one of four categories
that matched the four categories they would be investigating in the main body of the lesson.
The second sheet, a slight modification of Connected Mathematics program material in the
Thinking with Mathematical Models textbook (Problem 4.3; Lappan, Fey, Fitzgerald, Friel,
& Phillips, 1998), included four grids for the students to draw the graphs of the four
equation types: linear, inverse, exponential, and quadratic. The second day, the students
received a two-sided classwork sheet at the beginning of class, and subsequently, up to
three extension questions on partial sheets of paper were provided for those who finished
early. The extension questions were prepared by another eighth grade mathematics teacher
colleague in our building. My lesson study partner and I prepared the classwork sheet,
around which the lesson was centered, based on an extension problem in the book,
Thinking with Mathematical Models (1998) by Lappan, Fey, Fitzgerald, Friel, & Phillips,
which we modified to include a summary categorization and explanation.
For the second lesson study, we provided each student in the experimental group (my first and third period classes and my colleague’s fifth and sixth period classes) with a laptop computer from our school’s portable wireless computer lab. The cart housing this lab was wheeled to the back of each of our rooms both days and plugged into a wall outlet to keep the laptops fully charged. Prior to first period each day, the math *Jeopardy* game for that day was copied from a network file onto the desktop of each computer for easy student access. Each *Jeopardy* game was based on a game board with five categories ("Plug and Chug," "Mr. Distribute," "Name that Property," "Wrap Your Answers in FOIL," and "Factoring Frenzy"), including five individual problem slides for each category, and answer slides that the students accessed by a click of a mouse. A modified version of the game board was provided for the special education students in my colleague’s fifth period class. In between classes, the laptop computers were returned to the cart to recharge before the next class. Each of the two days, experimental group students were given a sheet of scrap paper and a score sheet on which to keep track of the points they earned and their total points, as well as a review sheet to be completed for homework in preparation for the test on the day immediately following the study. Control group students (my fifth and ninth period classes and my colleague’s eighth and ninth period classes) were provided with a packet of the jeopardy game questions for that day, to be completed in class, and the review sheet for homework that night. See Appendix C for score sheet (given to experimental group only), game questions packet (handed out to control group only), and homework review sheet for the first day, or Part I, of Lesson Study II; and Appendix D for the same materials for the second day, or Part II, of Lesson Study II.
The above-mentioned lesson materials for the second lesson study were largely generated by my colleague and myself. We created the score sheets for the experimental group, as well as the questions for each day’s game, which were then placed on the Jeopardy game computer template and given to students in the control group classes as a handout. It should be noted that the game for Day 2 was comparable to the first day’s game, with some slightly more difficult questions. Also, a modified version of the questions for Day 2 was prepared for the special education students in my colleague’s Period 5 class by their special education teacher. The five categories used each day for the game questions, evaluating expressions, distributing, multiplying binomials using the “FOIL” method, factoring binomials, and properties, were chosen based on the upcoming test, which, in turn, was developed using the revised New York State Learning Standards for Mathematics (NYS Board of Regents, 2005). The two review sheets assigned as homework were extensions of the algebraic concepts targeted during the in-class review to an authentic problem that involved profit, income, and expenses, based on a similar problem from the test. That test problem was taken from Connected Mathematics program assessment material related to the Say It with Symbols textbook (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002).

Data Collection

Assessment vehicles in the two lesson studies included both qualitative and quantitative, formal and informal, as well as formative and summative evaluations. The assessment of the first lesson study was primarily qualitative and involved no control
groups, whereas assessment measures for the second lesson study were a mixture of qualitative and quantitative (made possible by the inclusion of control classes).

Lesson Study I utilized qualitative, informal, formative assessment as the two teachers, in addition to the school principal (Day 2, my first period class) and the math curriculum supervisor (Day 2, my fifth period class), observed the student investigations. The focus and quality of the student individual and partner work during class as well as the participation during class discussions and the summary at the end of each class period provided a great deal of evidence. Beyond that, the unit test scheduled one week after the lesson study and the “reflection” assignment assigned for homework the night of the test (for my classes only) provided formal, summative, and more quantitative information; however, definitive quantitative conclusions were not possible due to the lack of control groups. The test was adapted with minor changes from the Connected Mathematics Program assessment materials and the reflection questions that students were expected to answer were on Page 59 in the Thinking with Mathematical Models textbook (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998). See Appendix E for assessment material for Lesson Study I (the unit test).

The second lesson study, conducted later in the school year, made use of qualitative, informal, formative assessment in the form of in-class observations made by the two lesson study teachers as well as the school principal (Day 1, my third period class and Day 2, my colleague’s fifth period special education class). Furthermore, an informal class discussion in the experimental group classes near the end of the period on the second day provided student feedback concerning the learning value of the technology-rich lesson and suggestions for further improvements. This qualitative, informal, summative data from the
students proved to be quite thoughtful and illuminating. The quantitative, formal, summative assessments for this study included the test given to students the day following the two day lesson study and the “reflection” questions assigned for homework on the test day. Quantitative comparisons were possible in this study due to the structuring to include control groups. See Appendix F for assessment materials for Lesson Study II.

These assessment materials for the second lesson study were prepared by my colleague and myself, with some assistance from another eighth grade mathematics teacher in our building who was not involved in our lesson study. My colleague and I developed the reflection assignment sheet; the test was prepared by the third mathematics teacher in conjunction with us. The authentic profit problem at the end of the test was adapted from the Connected Mathematics program assessment materials for the Say It with Symbols textbook (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002); however, we created the remainder of the test based on the revised New York State Learning Standards for Mathematics (New York State Board of Regents, March 15, 2005) as well as the Pre-March/Post March Mathematics Testing document (NYS Education Department, 2005). In particular, we targeted the eighth grade pre-March outcomes taken from the following strands and performance indicators:

**Number Sense and Operations Strand**

8.N.2 Evaluate expressions with integral exponents

**Algebra Strand**

8.A.7 Add and subtract polynomials (integer coefficients)

8.A.8 Multiply a binomial by a monomial or a binomial (integer coefficients)

8.A.10 Factor algebraic expressions using the GCF
We designed the test to emphasize evaluation of expressions; simplification of expressions using the distributive property and combination of like terms, including distributing a negative sign in order to subtract a polynomial; multiplication of binomials; factoring binomials; and properties that included the distributive property. All of these topics were directly related to the state standards listed above, which guided our assessment and, in turn, our lesson. A modified version of the test was prepared by the special education teacher for the appropriate students in my colleague’s fifth period class; the revised test is included in Appendix F.

From our observations and assessment data, we hoped to determine whether technology can make a difference in student engagement and focus as well as whether it can aid in visualization and enhance the learning process. The Results section will address these questions.

Procedures

Lesson Study I, using graphing calculator technology, was carried out as indicated earlier in this section, with my colleague teaching each part of the lesson first while I observed, and my re-teaching occurring the next day while he observed. Consequently, this two day lesson actually took place over three days. The first day, my colleague taught Part I during sixth period and we revised it after school. The second day included my re-teaching of Part I first period, our final revisions of Part I during fourth period, his teaching of Part II sixth period, and our revision of Part II after school. The third day consisted of my re-teaching of Part II first period and our finalization of Part II during fourth period.
The second lesson study, using the math Jeopardy game loaded onto laptop computers, was performed in much the same way, using the same teaching periods. However, I taught each part of the lesson first, with my colleague teaching it later the same day, so this two day lesson took place over only two days. The first day, I taught Part I first period while my colleague observed; we revised the lesson fourth period; and I observed while he taught it to his sixth period class. After school the same day, we finalized the lesson and completed preparations for Part II. The next day, I taught Part II to my first period class while being observed by my colleague; we revised the lesson during fourth period; and my colleague taught the revised lesson to his sixth period class. Final revisions were made after school.

As noted earlier, my colleague and I each taught both days of each lesson to all four of our regular eighth grade math classes. It was just our mutual observations that took place first and sixth periods. See Appendix G for the observation protocol form that we completed when observing each other; the example included is a completed form from the second day of Lesson Study I when my colleague observed my class. Additionally, in the case of the second lesson study, each of us incorporated two control classes which received the same lesson without the computer technology. Students in those classes were provided with a handout of the jeopardy questions which they either completed with a partner or used as reference while participating in a team competition focused around the classroom television or the overhead projector. The lesson was performed using three large teams in my colleague's eighth period class the first day and in my ninth period class that same day. He allowed his students the choice of working in pairs on their packets or participating in teams eighth period the second day and ninth period both days; consequently he had a
mixture of methods in those classes. Students worked in pairs with the packet in my remaining control class periods.

All experimental class periods for Lesson Study II as well as all class periods for Lesson Study I were organized using partner work. Students were paired into heterogeneous ability pairs for the first lesson study so that they could support each other, while the student pairs were chosen to be homogeneous in terms of ability in the second lesson study so that they would work at the same pace, allowing for discussion of the problems and lack of competition between partners. We intended that competition exist between student pairs only, so as to encourage teamwork between partners in understanding the concepts. Thus, we asked students to keep track of points earned during each day’s game for purposes of identifying a winning team from each class to be awarded a prize.

Each day, the lesson was presented in the “Launch, Explore, Summarize” format that the Connected Mathematics program recommends and which our school’s mathematics department has adopted. The intent is to provide students with a hook and to access their prior knowledge during the launch phase, to allow for an extended investigation or explore stage, and to facilitate a brief, but effective, student-led summary at the end of the class period. Lesson plans for the four days are included in Appendix H. Note that in Lesson Study I, the warm-up or launch each day asked students to predict the outcomes of the investigation portion of the lesson.
Results

The impact of technology on student learning was assessed in the two lesson studies using both qualitative and quantitative measures, formal and informal evaluation, as well as formative and summative assessment. Results from the first lesson, which incorporated graphing calculator technology, were more qualitative in nature; whereas, the second lesson, enhanced through the use of computer software technology, made use of qualitative and controlled quantitative evaluation.

Although unit test results and grades for a writing "reflection" assignment were available, it was difficult to draw quantitative conclusions from Lesson Study I due to the lack of control groups. We considered comparing the class average grades for those two assessments to other test and reflection grades during the same time period, but decided that the comparisons were not fair since the other tests and reflections were of varying difficulty and in some cases were set up as partner instead of individual assessments. However, my colleague and I were convinced from our observations of students in class that learning was improved through the use of the graphing calculators. First of all, the students were much more engaged in the lesson than usual; we, our building principal, and the mathematics curriculum supervisor all concluded that student focus was considerably improved as compared to other lessons in which technology, in particular graphing calculator technology, was not utilized. The students approached the lesson with enthusiasm, probably due, in part, to the fact that it employed a learning tool that had not been used previously. In addition to engaging and motivating students to learn, the graphing calculators allowed students to visualize the relationships represented by the
equations. From comments that students made individually during the investigation phase of the lesson as well as during the class summary at the end of each day, it was clear that students were gaining a better understanding of the classifications of equations through the opportunity to visualize the graphs of those equations. In fact, they were drawing connections between the equations and their corresponding graphs, noting how equation types translated into graph characteristics. Although they had had units on linear, inverse, and exponential relationships previously and we had done some cross comparisons of the equations, tables, and graphs, students had not had the opportunity to compare them in as direct and concentrated a manner as the graphing calculator technology allowed. With the ease and quick processing of the graphing calculator, they were freed from the slow, laborious process of graphing the equations by hand and were able to generate many graphs quickly and concentrate on the features of their equations and graphs.

The second lesson study was designed to provide two quantitative forms of assessment, in addition to informal, qualitative observation of students during the course of the two day lesson and student feedback at the end. Recall that we employed control groups to obtain more meaningful comparisons and allow for more definitive conclusions. Again, my colleague and I noted, as we observed our own and each other’s classes, that the experimental group classes were highly enthusiastic about the lesson and thrilled that they each received a laptop computer to use for the entire class period. They had not had that opportunity in math class previously. This student engagement led to a very focused lesson, with the students working on their individual laptops, but comparing answers and discussing solution methods with their partner. Comments from our
building principal and mathematics curriculum supervisor confirmed the engagement and focus that we were observing in our experimental classes. Given that the unit included several algebra concepts that the students had been struggling with, their concentration and focus was particularly noticeable and rewarding. This high level of engagement and focus was not matched in the control classes.

Since the two day study was structured as a review of algebra concepts prior to an in-class test that was given the following day, the student test scores provided evidence of student understanding resulting from the study. See Table I (page 40) for a summary of grade results from the test. In addition, homework the night of the test was a one page guided reflection on the five concepts emphasized in the test; the resulting grades are also shown in Table I.

An analysis of the class test and reflection assignment data presented in Table I indicates some trends. The data for the algebra test given the day after the two day Jeopardy game review provided by Lesson Study II, shows higher average class performance for my two experimental class periods (83.5 and 77.1) as compared to my two control class periods (76.3 and 68.3). As mentioned in the Methodology section, the year-to-date class averages for those class periods (also included in Table I) indicate that my experimental and control groups were diverse but quite well balanced, each comprised of one stronger class and one weaker class in terms of mathematical ability and preparation. Consequently, the differences in the class test averages should be meaningful. Unfortunately, my colleague’s classes were not as well balanced; in fact, his two weaker classes, including one class made up of a blend of special education students and regular education students, were chosen as his experimental group for reasons of
convenience in moving and setting up the wireless laptop lab. His two stronger classes formed his control group. Without a common starting point in terms of mathematical proficiency, it was not possible to draw conclusions from his classes based on the test data. Any improvements in student understanding were not large enough to overcome the difference in ability between the two groups. The data analysis was further confounded by the fact that the special education students in the blended class (Period 5) were given a modified test and used a modified Jeopardy game. However, my colleague was convinced that the students in his experimental classes understood the concepts better and performed better on the test than they would have without the technology.

The class average data for the reflection assignment, shown in Table I, do not reveal any discernable trends. One explanation for the flatness of the data is that quite a few students did not turn in the reflection assignment. Those students were primarily the weaker mathematics students, and since their scores were not included in the class averages (which included only the non-zero grades), the class averages were only representative of the stronger mathematics students. Furthermore, although the Jeopardy computer game addressed all five concepts included in the reflection assignment and provided students with focused practice and an opportunity to understand how to carry out each of the processes, it did not support them in verbally expressing the nuances and connections between the concepts. Consequently, the game did not have a noticeable effect on the quality of the student reflection assignment.
Table I

Results of Lesson Study II

(Class Averages out of 100)

<table>
<thead>
<tr>
<th>Class Period</th>
<th>Algebra Test</th>
<th>Reflection Assignment</th>
<th>Year-to Date Average (March 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MY CLASSES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period 1</td>
<td>83.5</td>
<td>78.6</td>
<td>83.4</td>
</tr>
<tr>
<td>Period 3</td>
<td>77.1</td>
<td>68.2</td>
<td>79.7</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period 5</td>
<td>76.3</td>
<td>74.5</td>
<td>82.4</td>
</tr>
<tr>
<td>Period 9</td>
<td>68.3</td>
<td>73.6</td>
<td>79.9</td>
</tr>
<tr>
<td>COLLEAGUE'S CLASSES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period 5 Total*</td>
<td>77.4</td>
<td></td>
<td>82.6</td>
</tr>
<tr>
<td>Period 5 Regular Ed</td>
<td>81.5</td>
<td>75.5</td>
<td>—</td>
</tr>
<tr>
<td>Period 5 Special Ed</td>
<td>71.6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Period 6</td>
<td>79.2</td>
<td>71.4</td>
<td>82.7</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period 8</td>
<td>80.6</td>
<td>74.1</td>
<td>85.2</td>
</tr>
<tr>
<td>Period 9</td>
<td>85.3</td>
<td>83.6</td>
<td>88.1</td>
</tr>
</tbody>
</table>

*20 total students, with 8 special education students and 12 regular education students. The special education students played a modified Jeopardy game, took a modified test, and did not do the reflection assignment.
Finally, an informal student feedback session was conducted in the experimental classes at the end of the period on the second day of Lesson Study II, providing insight into their feelings and thoughts on the value of using the "Jeopardy game" and laptop technology for review. When asked whether the computer game technology helped them learn the algebra and how they might change or improve the lesson, they shared some insightful thoughts. Their responses centered around several common themes. Nearly all described the lesson as fun due to the use of computers, especially laptops. They indicated that they learn better when they are enjoying the activity, and that this was much better than sitting and listening because it kept their attention. Several expressed that the computer game helped them understand some of the concepts that they had been confused about previously and that it even helped them memorize property names and terms. They also liked the fact that the technology allowed them to work at their own pace instead of being tied to the pace of the rest of the class; this thought was expressed by students that are capable of progressing at a faster pace and well as those who need more time. The theme of independence also carried over to their choice of what to work on; the fact that students could work on the concepts that they needed the most help with appealed to many of them. Some indicated that the lesson helped them review, but that they did not learn anything new, while others said that the game helped their understanding. The few who did not enjoy or learn much from the lesson stated that they would have preferred using whiteboards or paper because they didn’t like computers in general.

The feedback session for Lesson Study II also entertained student ideas for improvements in the lesson. Their suggestions ranged from as simple as being allowed to
pick their own partners or competing as individuals in a whole class oral competition to much more technological improvements. Some wanted “more problems so they could learn more,” more difficult problems, or a change in the rules so that getting an answer wrong would result in a loss of points (like the television game). Along the same lines, students suggested that we incorporate “Daily Doubles” and “Final Jeopardy” into the game format. Some didn’t like the Jeopardy game and would have preferred using the computers to play a different game. Many would have liked more forgiving technology that would have allowed them to click anywhere on a Jeopardy board square to initiate a new problem, instead of just in the center, and more consistent highlighting of the squares as they were completed. While some suggested sound with the game, others would have liked individual headphones to eliminate annoying noises from the computers. One of the most frequently voiced concerns was that of student accountability. This particular Jeopardy game was not interactive. Students were supposed to bring up a problem on the computer screen, work it out on their scrap paper, and then check it with the answer that could be accessed on the next computer page by simply clicking the mouse. If their answer or their partner’s answer did not agree with the one given in the game, they were expected to discuss it and ask for assistance from the teacher if necessary. Some students admitted that they or others were looking at the answers first and then working backwards. Suggestions to remedy this problem included having the students go to the teacher for the answers or setting up the game so that students would have to input an answer before the computer would show them the correct answer. Ideally, they said, the computer, should respond to an incorrect answer by saying “Incorrect” and “Try Again.” Basically, they wanted a more interactive game. Finally, many students suggested that
the computer game show how the answer was obtained as part of the answer screen, since students who get the wrong answer probably do not know what they did incorrectly. Alternatively, the process to the correct answer could just be shown to those who answer the problem incorrectly.

During the course of the two lesson studies, as my colleague and I observed each other and reflected on the lessons, we made some adjustments that enhanced the effectiveness of the lessons. In the case of the first study, we decided to make a small, but significant modification to Part I (Day 1) of the lesson after my colleague taught it so that students would not have the graphing calculators at their desks during the warm-up at the beginning of the class period. We noticed that the students were playing with the calculators and were distracted from the warm-up and the directions for the investigation, so we decided that the calculators should not be handed out until the investigation (explore) portion of the lesson. When I taught the same lesson, I did not hand out the graphing calculators until after the warm-up and the modeling of how to graph the first equation. In our final revisions, we called for calculators to be handed out after the warm-up but before the modeling and directions so that students would have the calculators in hand and be able to practice using them as the directions were given and the process was modeled.

When Part II (Day 2) of Lesson Study I was taught first by my colleague, we had the students not only graph each of the twelve equations, but also determine an appropriate window on the graphing calculator (xmin, xmax, ymin, ymax, and the scale for each) so that the graph would be visible. Clearly, that is a skill that they need to develop; however, as we realized after the first teaching, the students needed to
concentrate on exploring the equations and categorizing the relationships, not the details of the graphing process itself. Consequently, we decided to post appropriate window settings on the overhead projector for student reference throughout the investigation. Additionally, we wanted to lengthen the explore stage after the first teaching, so we decided that I would not go over homework from the previous night. This modification increased the time for student investigation from eleven to twenty-two minutes, allowing nearly all students enough time to complete all twelve equations and categorize them in preparation for the whole class summary at the end of the period. Any students who finished the investigation early were given up to three additional small sheets of extension or “back-pocket” questions.

When I first taught Day 1 of Lesson Study II, my colleague and I noticed that in spite of what we thought were clear directions and my demonstration of one of the problems, the students were confused about how to play the game. We realized that the directions for the game needed to be more specific, and in fact, our principal suggested that the directions be written out. Consequently, when my colleague taught it later that day, the directions were posted on the whiteboard, step by step. Furthermore, he carefully accessed the students’ prior knowledge of the television Jeopardy game and pointed out the similarities and differences of our game. On the second day, we decided that the lesson as I taught it should be revised to include more closure at the end of the period, so my colleague allowed time after the student feedback session for a whole class summary of the five concepts targeted in the Jeopardy game. Our final revisions of the second lesson study as a whole centered around improvements in student accountability and lesson closure. We decided that, in the future, students should be provided scrap
paper divided into sections numbered for each of the twenty-five game questions. Students would be expected to show their work as well as the points earned for each problem in those sections and turn the papers in at the end of each period. This would not only hold students accountable for their individual work, but would also allow the teacher to discover areas of student confusion to be addressed the following day. In fact, the review session the morning of the test would be dictated by the student data from their scrap sheets.
My colleague and I were impressed by the student engagement that technology afforded in both of our lesson studies. The enthusiasm and focus of the students was palpable for anyone present. My building principal noted one hundred percent engagement of the students for a period of twenty minutes in one of my classes that he was observing during the second day of Lesson Study I, with only a slight reduction (to ninety-six percent) for the remainder of the class period. That level of focus is very difficult to achieve with eighth grade students, especially in a mathematics classroom. They were focused because the technology caused them to become engaged in the lesson; it made learning fun. I believe that it is not only the novelty of the technology that attracts students, but its ability to open up learning opportunities for them. As they stated so well during the feedback sessions, technology allows them the freedom to learn at their own pace and in their own way, concentrating on areas where they need help. They are no longer tied to a single pace, waiting for others to catch up or being hurried to move on so as not to hold up the rest of the class. Furthermore, technology in the classroom relinquishes students from the monotonous teacher-led direct teaching that often occurs in mathematics classrooms. Students are better able to explore things that interest them and tailor their learning to meet their own learning style and needs.

Our research results also supported the contention that a strength of technology lies in its ability to allow students to approach mathematical concepts from a visual, often graphical, perspective as well as the traditional auditory and numerical or algebraic approaches. Students who are visual learners benefit greatly, and all students are
provided with an additional representation of the concepts, thus deepening their understanding. We noticed this phenomenon, particularly as we observed students making connections between the equations and graphs of various linear and nonlinear relationships in Lesson Study I.

We also witnessed improved student understanding of linear, exponential, inverse, and quadratic relationships (Lesson Study I) and of algebraic concepts and processes such as evaluating expressions, distributing, multiplying binomials, factoring, and properties (Lesson Study II). Test results from the classes where effective control groups made comparisons possible (my classes in Lesson Study II) supported our perceptions of improved student learning with technology that we gained as we worked and interacted with students throughout the studies. Presumably improved student understanding results from increased engagement and focus as well as new approaches that enhance understanding.

The opportunity for differentiation was clearly another benefit that my colleague and I observed during our research. Especially with the homogeneous grouping used in Lesson Study II, students were able to proceed with the computer Jeopardy game questions at their own pace, in step with their partner, devoting their time to question topics where they needed the most help. This benefit of technology was referred to by students, during the feedback session, as learning “freedom,” and is an asset of technology that we as teachers appreciate as we strive to differentiate the learning in our classrooms.

Furthermore, the opportunity for interactive learning exists with technology. The technology that we utilized in our research, graphing calculator and computer game
technology, was minimally interactive. However, when giving feedback for Lesson Study II, many students suggested that we make the Jeopardy game much more interactive. Interactivity allows student learning to become even more highly differentiated and tailored to individual needs, a benefit that both students and teachers recognize.

My colleague and I gained a renewed awareness of the importance of teacher planning in implementing technology-rich lessons. This principle became obvious when we revised our lessons several times to minimize distractions and maximize student learning simply by altering when we distributed graphing calculators. Additionally, the importance of very clear directions that can be revisited by students was apparent during the Jeopardy game lessons.

Finally, we found that competition and suspense were characteristics that can easily accompany lessons based on technology. Our Jeopardy game made use of competition to keep students' interest by promising prizes to the high scorers, and suspense was a natural element of the graphing calculator investigation. The suspense aspect was heightened in that study by asking students to predict outcomes as a warm-up each day prior to the investigation. Thus, they were anxious to determine if their predictions were correct.

The literature clearly addresses the capability of technology to attract student attention and engage them in the learning process as well as the benefits resulting from concept visualization utilizing technology. Thus, our results and conclusions are consistent with the observations of others in the field. In addition, we concluded, as the literature purports, that student understanding benefits from the use of technology.
literature also stresses the importance of teacher training and planning in the implementation of technology, indicating that it is an often forgotten weak link; our experiences were consistent with that conclusion. On the other hand, although the promise of interactive software is mentioned in the literature when discussing dynamic geometry programs such as Cabri and Geometer's Sketchpad, the power and potential benefits of this aspect of technology that I envision beyond isolated programs were not discussed. Furthermore, although the literature indicates that graphing calculator and computer programming technology can be very effective when teaching students with learning disabilities or attention deficit disorder, very little is published concerning differentiation of general classroom learning with the aid of technology. Again, I anticipate that technology will play a major role in the trend toward greater classroom differentiation. And finally, although the benefits of suspense and competition as aids in engaging students are well known, their connections to technology were not mentioned in the literature.

I am truly excited about the endless possibilities for incorporating technology in the classroom. I think that it holds great promise for engaging, teaching, and empowering students within the mathematics curriculum. However, as the literature emphasizes, the effectiveness of the teacher in implementing the technology is an important factor in determining the impact of technology on student learning. I am convinced, from my experience and my study of the literature, that in order for the true potential of instructional technology to be realized, the teacher must not only be well trained in the use of the technology and prepare lessons carefully, but they need to have a vision of what effective use of technology would ideally look like in their classroom. Thus, I
would encourage others to contemplate how technology could enhance their particular teaching in some novel way, and beyond that, to share their ideas and discuss them with colleagues.

One of the most rewarding and meaningful aspects of my research using lesson study, was the opportunity to collaborate with another colleague. We were able to not only support each other through the process and share the lesson preparations, but, more importantly, gain new insights and perspectives from each other as we thought out loud and refined our practices. I see incorporation of instructional technology in the classroom as a powerful force toward enhancing the learning process, but one that is at the same time, fragile and susceptible to minor details of implementation. With on-going interaction and sharing of successes and failures, the community of teachers will be able to develop and refine an array of strategies that will unleash the potential benefits of technology and avoid many pitfalls along the way.

As we move forward from this point, my colleague and I are anxious to continue to use and improve our Jeopardy game lesson. I recently received a more advanced Jeopardy game template from another teacher who is a member of the laptop technology group in our school district to which I belong. This new version is much more interactive, providing students with feedback on their answers and what they might have done wrong, much like what the students requested. We also are planning a statistics lesson using the program Excel on the computers in the school computer lab. More globally, I hope to involve more of my colleagues in the mathematics department, both seventh and eighth grade teachers, in using technology to support instruction in their classrooms. I have recently made a first step in that direction by recruiting my lesson
study partner and possibly another mathematics teacher for next year’s laptop technology group.

Longer term, I plan to investigate the features and potential of Texas Instruments’ “Navigator” system for implementing graphing calculator technology in whole class interactive lessons, monitoring individual student interactions with the technology, and managing the data. Besides the obvious benefits of student engagement, visualization of concepts, and improved understanding, such a system could serve as an important assessment tool, allowing a teacher to determine individual student strengths and weaknesses early in the instructional process, thus informing teaching and allowing for differentiation.

My ultimate goal is to utilize ever-changing state-of-the-art technology to thoroughly engage students in intricate, suspenseful problems, much like detectives, as they compete to problem solve real-life or fictitious situations. The problem-based learning scenarios will be carefully constructed such that students will encounter the need for required course content during their investigation, thus providing strong motivation and enthusiasm for learning!
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Appendix A

Classwork sheets for Part I of Lesson Study I
Math 8
Warm-up for Problem 4.3

Place each of the following equations into one of the 4 categories at the bottom of the sheet, matching the equations by type.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 8.7x )</td>
<td>1</td>
</tr>
<tr>
<td>( y = -4.5x )</td>
<td>2</td>
</tr>
<tr>
<td>( y = -40x + 480 )</td>
<td>3</td>
</tr>
<tr>
<td>( y = \frac{x}{2} )</td>
<td>4</td>
</tr>
<tr>
<td>( y = \frac{160}{x} )</td>
<td></td>
</tr>
<tr>
<td>( y = 3x + 20 )</td>
<td></td>
</tr>
<tr>
<td>( x = \frac{100}{y} )</td>
<td></td>
</tr>
<tr>
<td>( y = (x + 2)(3 - x) )</td>
<td></td>
</tr>
<tr>
<td>( xy = 300 )</td>
<td></td>
</tr>
<tr>
<td>( y = 120x )</td>
<td></td>
</tr>
<tr>
<td>( y = 10 - 2x )</td>
<td></td>
</tr>
<tr>
<td>( y = -\frac{x}{3} )</td>
<td></td>
</tr>
<tr>
<td>( y = 5^x )</td>
<td></td>
</tr>
<tr>
<td>( y = (x - 8)(2 - x) )</td>
<td></td>
</tr>
<tr>
<td>( y = \frac{2}{x} )</td>
<td></td>
</tr>
<tr>
<td>( y = 3x )</td>
<td></td>
</tr>
<tr>
<td>( y = 2.7x )</td>
<td>1</td>
</tr>
<tr>
<td>( y = \frac{1}{x} )</td>
<td>2</td>
</tr>
<tr>
<td>( y = 2^x )</td>
<td>3</td>
</tr>
<tr>
<td>( y = (x - 1)(5 - x) )</td>
<td>4</td>
</tr>
</tbody>
</table>

WARM-UP SHEET FOR DAY 1
Using the graphing calculator, graph the following equations on the grids provided.

1. \( y = 2.7x \)
2. \( y = \frac{1}{x} \)

Is the graph linear or non-linear?
Why? (use the equation)

3. \( y = 2^x \)
4. \( y = (x-1)(5-x) \)

Is the graph linear or non-linear?
Why? (use the equation)

CLASSWORK INVESTIGATION SHEET FOR DAY 1
Appendix B

Classwork sheets for Part II of Lesson Study I
Using the graphing calculator, graph the following equations on the grid provided.

- Then determine if the graph is linear, non-linear, or neither.

1. \[ y = 8.7x \]
2. \[ y = -40x + 480 \]
3. \[ y = \frac{120}{x} \]
4. \[ y = \frac{8.7}{x} \]
5. \[ y = 100(1.08)^x \]
6. \[ y = \frac{40}{x} \]
7. \[ y = 1.08x \]
8. \[ y = 100(0.5)^x \]
9. \[ y = 100 - 1.08x \]
CLASSWORK INVESTIGATION SHEET FOR DAY 2, PAGE 2

\[ y = 2^x \quad y = 2x \quad y = \frac{2}{x} \]

Summarize your findings:
Place each equation in an appropriate category:

A) Linear - Increasing
B) Linear - Decreasing
C) Non-Linear - Increasing
D) Non-Linear - Decreasing
E) NONE of the above

Determine why each equation fits in the category and write reasons next to the group.
What part of the equation tells that your graph will look like this?

What part of the equation tells that your graph will look like this?

What part of the equation tells that your graph will look like this?

What part of the equation tells that your graph will look like this?

Compare the graphs of the equations $y = \frac{3}{x}$ and $y = \frac{x}{3}$.

How are they similar?

How are they different?

Make your own equation to have someone else categorize.
Appendix C

Classwork Sheets for Part I of Lesson Study II
I. Evaluate

1) \(3x - 2(x+5)\) when \(x = 3\)

2) \((10-4x)(7+6x)\) when \(x = 4\)

3) \(\frac{75-3x}{5+2x}\) when \(x = 5\)

4) \(x^2 - 8x - 22\) when \(x = -4\)
5. \(x^2 - 4x + 6\) when \(x = 10\)

II. Simplify

1) \(4(2x - 3)\)

2) \(-5(9x + 4)\)

3) \(2(3x - 5) + 3(7x + 6)\)

4) \(-7(-6x + 11) - (2x - 9)\)

5) \(19(-5x + 2) - 3(x + 6)\)

III. Name That Property

1) \((12 \cdot 7) \cdot 8 = 12 \cdot (7 \cdot 8)\)

2) \(9 \left( \frac{1}{9} \right) = 1\)

3) \(-2(6x - 5) = -12x + 10\)

4) \(234,567 + 0 = 1,234,567\)

5) \((4x + 13) + 9 = (13 + 4x) + 9\)
Multiply the binomials

1. \((x+7)(x+5)\)
2. \((x+3)(x-9)\)
3. \((x-7)(x-12)\)

4. \((2x-5)(4x+7)\)
5. \((-5x+3)(6x-1)\)

Factor (Expanded → Factored)

1. \(x^2 + 6x\)
2. \(4n + 3\)
3. \(7n - 2\)
4. \(3x^2 + 12x\)

5. \(8x^2 - 40x\)
Hilary, the class treasurer, is responsible for keeping track of the finances for the next after school party at Dake. She is trying to determine the number of people necessary for the Student Government to make some money on the event. To do this, she makes the following estimates of expenses and income. Notice that some of the expenses and income are fixed and some depend on the number of students who attend.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Music (fee for DJ): $250</td>
<td>Ticket for 8th grade student: $4</td>
</tr>
<tr>
<td>Chaperones: $300</td>
<td>Ticket for 7th grade student: $2</td>
</tr>
<tr>
<td>Refreshments: .75 cents per person</td>
<td>Donation from PTSA: $150</td>
</tr>
</tbody>
</table>

Suppose that \( x \) represents 8th graders and \( y \) represents 7th graders. Hilary wrote the following expression for expenses:

\[
250 + 300 + 0.75x + 0.75y
\]

and the following expression for income: \( 4.00x + 2.00y + 150 \)

A. Write an equation for profit, \( P \).

B. Simplify your equation

C. Evaluate one of your equations for 180 8th graders and 225 7th graders, to see what the profit would be. Show all work!
Evaluate:

\[-x^2 + 5x - 12 \text{ when } x = 4\]

Simplify:

\[6(x + 3) - (x - 5)\]

Multiply:

\[(2a - 7)(9a - 1)\]

Factor: (Go from Expanded Form to Factored Form)

\[6c^2 + 28c\]

Name that Property:

\[(3j + 22m) + 80p = 3j + (22m + 80p)\]

\[17(1/17) = 1\]

\[25(8) - 25(x) = 25(8 + x)\]
Appendix D

Classwork Sheets for Part II of Lesson Study II
Score Sheet (Day 2)

Name _______ Period ______

Total Pts. from Day 1 ______
Total Pts. from Day 2 ______
Total Score ______
I. Evaluate

1) \( \frac{4x - 60}{2x + 5} \), \( x = 10 \)

2) \(-x^2 + 5x\), \( x = 3\)

3) \( \frac{2x + 9x}{17 - 4x} \), \( x = 3\)

4) \( x^2 - 7x + 13\), \( x = -10\)

5) \(-x^2 + 3x - 14\), \( x = -3\)
II. Simplify
1) 7(4x-1)  
2) -8(5x+3)  
3) 4(qx-2)-5(3x+7)

4) -3(-7x+6)-(2x-5)  
5) 22(-3x+1)-5(2x+15)

III. Name That Property
1) 5(4+q) = 5(4)+5(q)  
2) 7(1) = 7  
3) (-6+41x)+15 = -6+(41x+15)  
4) (4)(3x)(8) = (4)(8)(3x)  
5) 10(4/10) = 1
IV. Multiply the Binomials

1) \((x + 10)(x + b)\) \hspace{1cm} 2) \((5 + n)(6 - n)\) \hspace{1cm} 3) \((2x - 11)(7x - 3)\)

4) \((4x - 5)(3x + 8)\) \hspace{1cm} 5) \((-9 + 2k)(3 - 5k)\)

V. Factor

1) \(x^2 + 27x\) \hspace{1cm} 2) \(10n + 85\) \hspace{1cm} 3) \(9n - 21\)

4) \(8x^2 + 20x\) \hspace{1cm} 5) \(6x^2 - 27x\)
Steve, the school treasurer, is responsible for keeping track of the finances for the next homecoming dance at the high school. He is trying to determine the number of people necessary for the Student Council to make some money on this event. To do this, he makes the following estimates of expenses and income. Notice that some of the expenses and income are fixed and some depend on the number of students who attend.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Music (fee for band): $165</td>
<td>Ticket for 9th/10th grade student: $2.50</td>
</tr>
<tr>
<td>Decorations and Posters: $120</td>
<td>Ticket for 11th/12th grade student: $1.25</td>
</tr>
<tr>
<td>Refreshments: $1.00 per person</td>
<td>Donation from PTSA: $225</td>
</tr>
</tbody>
</table>

Suppose that \(x\) represents 9th/10th graders and \(y\) represents 11th/12th graders.

Steve wrote the following expression for expenses:

\[
165 + 120 + 1.00x + 1.00y
\]

and the following expression for income:

\[2.50x + 1.25y + 225\]

A. Write an equation for profit, \(P\).

B. Simplify your equation.

C. Evaluate one of your equations for 350 9th/10th graders and 500 11th/12th graders to see what the profit would be. Show all work.
Evaluate
\[ 3x^2 - 8x + 26 \quad \text{when} \quad x = -3 \]

Simplify
\[ 4 \cdot 5 + 3x - 1 \]

Multiply
\[ 15 \quad 4 \cdot 3 = 2 \]

Factor (Go from Expanded Form to Factored Form)
\[ 10 \cdot 5 \]

Name that Property
\[ 3 \cdot (x + 4) = 3x + 12 \]
\[ 6 \cdot 8 = (8 \cdot 6) = 10 \]
\[ 45 + 0 = 45 \]

HOMEWORK REVIEW SHEET FOR DAY 2, PAGE 2
Appendix E

Assessment for Lesson Study I
Thinking with Mathematical Models Unit Test

Directions:

- Actively Read each question carefully.
- For each written response, be sure to use turn-around style and make sure explanations are clear, correct and concise.
- You may wish to include examples to help support your responses.
- Show all work necessary to complete each question.

Good Luck!!!
1) When Garret was born, Aunt Darla put a quarter (0.25) in a piggy bank for him. She said that she would double the amount she gave him on each birthday until he became a teenager (at age 13). Uncle Owen gave Garret $50 the day he was born. He said he would give him that same amount on every birthday until Garret became a teenager.

a. Make a table for each gift plan showing how much Garret will receive from Aunt Darla and Uncle Owen on each birthday.

<table>
<thead>
<tr>
<th>Garret’s age</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ from Aunt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darla</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$ from</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncle Owen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

b. Which data is linear? (Aunt Darla or Uncle Owen) ________

c. How can you tell?

__________________________________________________________

UNIT TEST FOR LESSON STUDY I, PAGE 2
d. Plot the data for each plan on the coordinate grid below. Be sure to:
   - label your axes
   - make a title for your graph
   - draw a graph model that best fits the points you plotted
   - make a key

![Coordinate Grid]

- Label the x-axis and y-axis appropriately.
- Title the graph appropriately.
- Plot the data points for each plan.
- Draw a graph model that best fits the points you plotted.
- Make a key for the graph model.

e. Write an equation for the linear graph model.

f. Compare the total amounts that Garrett will receive from his aunt and his uncle.
   - How much total will he receive from Aunt Darla?
   - How much total will he receive from Uncle Owen?
2) a. Separate these equations into two groups, linear and nonlinear, by circling the linear equations.

\[
\begin{align*}
&y = 2x \\
&y = x^2 \\
&y = x - 2 \\
&y = \frac{2}{x} \\
&y = 2 + x \\
&y = 2 + x
\end{align*}
\]

b. Which of the linear equations would have a graph model that is decreasing? Explain how you know.

In each graph for questions 3-6, decide whether there appears to be a pattern in the data. If so, draw a line or a curve to model the trend.

3)

Linear or Non-linear: ____________________ Increasing or Decreasing: ____________________
UNIT TEST FOR LESSON STUDY I, PAGE 5
7) Consider these three stories and the following graph models.

Story 1: A parachutist is taken up in a plane. After the jump, the wind blows her off course and she ends up tangled in the branches of a tree.

Story 2: Thomas puts an inheritance in the bank and leaves it there to earn interest for several years. Yesterday, he withdrew half the amount.

Story 3: Gerry orders 30 cubic meters of gravel for his driveway. He is shocked when he sees the enormous pile delivered by the dump truck, but he rents equipment to spread the gravel onto the driveway. On the first day he is enthusiastic and moves half the gravel from the pile to his driveway. On the next day he is tired and moves only half of what is left. On the third day Gerry has less time, so he again moves half of what is left. He continues in this way until the pile has practically disappeared.

- Match each story with one of the graph models below (there will be one graph that won't be used).  
- Label the variables  
- Explain what different parts of the graph mean in terms of the situation you have chosen.

Graph A  
Graph B  
Graph C  
Graph D
b. Make up a story to match the remaining graph. Include the variables and a clear explanation for each part of the graph.

MULTIPLE CHOICE: Place the letter of the correct choice on the line beside each question.

8. Which of the following equations does not represent a linear relationship?
   a. \( y = 8.7x \)
   b. \( y = 100 - 1.08x \)
   c. \( y = 100 (0.5)^x \)
   d. \( y = -4x + 400 \)

9. Based on the following graph model, which type of relationship is being shown?

   a. linear and increasing
   b. linear and decreasing
   c. nonlinear and increasing
   d. nonlinear and decreasing
10. Which table shows a linear relationship between the data?

A. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
1 & 2 \\
2 & 4 \\
3 & 8 \\
4 & 16 \\
\end{array}
\]

B. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
0 & 2 \\
1 & 4 \\
3 & 6 \\
4 & 8 \\
\end{array}
\]

C. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
1 & 1 \\
2 & 3 \\
3 & 6 \\
4 & 10 \\
\end{array}
\]

D. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
1 & 0 \\
2 & 2 \\
3 & 4 \\
4 & 6 \\
\end{array}
\]

Bonus: What is the y-intercept for the linear relationship you found above.

11. Find an equation for the line that passes through the points (2,3) and (3,1). You may use the graph paper below or your algebra skills. Be sure to show all work.
Appendix F

Assessments for Lesson Study II
Say it with Symbols
Quiz

For questions 1-5, state which property is being shown.

1. $7(4x - 2) = 28x - 14$

2. $3(x - 5) = 3(5 - x)$

3. $3(x - 5) = 3(x) - 3(5)$

4. $3(\frac{1}{2}) = 1$

5. $3(x - 5) = (3 + x) + 5$

For questions 6-9, simplify.

6. $x(x - 5)$

7. $-5(2x - 3)$

8. $-3(2x + 5) - (5x - 2)$

9. $3(x - 4) - 4(2x + 5)$
For questions 11-12, \textit{Factor}.

10 \( x^2 - 7x \)

11 \( 6m^2 - 15m \)

For questions 12-14, \textit{Evaluate}.

12 \( 3(x + 4) - 4(2x - 5) \) when \( x = 6 \)

13 \( -x^2 - x - 6 \) when \( x = -2 \)

14 \( -3(x + 2) + 3x - 5(x - 9) \) when \( x = 2 \)

For questions 15-17, \textit{Multiply the Binomials}.

15 \((m - 5)(3m - 6)\)
16. \((2n - 4)(3n - 7)\)

17. \((5 - n)(3 + 2n)\)

18. Jordan, the class treasurer, is responsible for keeping track of the expenses for the fall dance. While it is not important to make a profit, Jordan would like to at least break even. He made the following estimates of expenses and income. Notice that some of the expenses and income are fixed and some depend on the number of people who attend.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound equipment rental: $200</td>
<td>Ticket for 9th grade student: $3</td>
</tr>
<tr>
<td>Decorations: $50</td>
<td>Ticket for 8th grade student: $1.50</td>
</tr>
<tr>
<td>Refreshments: 50 cents per person</td>
<td>Donation from student council: $100</td>
</tr>
</tbody>
</table>

Suppose that \(x\) represents 8th graders and \(y\) represents 9th graders. Jordan wrote the following expression for expenses: \(200 + 50 - 0.50x - 0.50y\) and the following expression for income: \(3.00y + 1.50x + 100\).

A. Write an equation for profit.

B. Simplify your equation.

C. Evaluate one of your equations for 150 8th graders and 100 9th graders.
Say it with Symbols Quiz

For questions 1-5, state which property is being shown. Use each word once.

<table>
<thead>
<tr>
<th>Identity</th>
<th>Inverse</th>
<th>Distributive</th>
<th>Associative</th>
<th>Commutative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (7(4x - 2) = 28x + 14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. (x + 5 - 5 + x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. (24 + 0 = 24)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. (3(\frac{1}{3}) = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. (3 + (x + 5) = (3 + x) = 5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 6-7, **Distribute**

6. \(x(x - 5)\)

   [Blank for answer]

7. \(-8(2x - 3)\)

   [Blank for answer]

For questions 6-7, **Distribute then Simplify**

8. \(-3(2x + 5) - 4x\)

9. \(3(x + 4) - (2x - 5)\)
For questions 11-12, *Factor*.

10. $5x - 20$

\[
\begin{array}{c}
5x \\
20
\end{array}
\]

\[ (\_ + \_ + \_ + \_ + \_ + \_) \]

11. $6n^2 + 15n$

\[
\begin{array}{c}
6n^2 \\
15n
\end{array}
\]

\[ (\_ + \_ + \_ + \_ + \_ + \_) \]

For questions 12-14, *Evaluate*.

12. $3(x + 4) - 4(2x + 5)$ when $x = 6$

13. $-2x + 6$ when $x = -2$

14. $-5(x + 9)$ when $x = 3$
15. Jordan, the class treasurer, is responsible for keeping track of the expenses for the fall dance. While it is not important to make a profit, Jordan would like to at least break even. He made the following estimates of expenses and income. Notice that some of the expenses and income are fixed and some depend on the number of people who attend.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound equipment rental: $200</td>
<td>Ticket for 9th grade student: $3.00</td>
</tr>
<tr>
<td>Decorations: $50</td>
<td>Ticket for 8th grade student: $1.50</td>
</tr>
<tr>
<td>Refreshments: 50 cents per person</td>
<td>Donation from student council: $100</td>
</tr>
</tbody>
</table>

Suppose that x represents 8th graders and y represents 9th graders.
Jordan wrote the following expression for expenses: $(200 + 50 + 0.50x + 0.50y)$
and the following expression for income: $(3.00y + 1.50x + 100)$

A. Write an equation for profit. Income - Expenses = profit

B. Simplify your equation

C. Evaluate one of your equations for 150 8th graders and 100 9th graders

**Bonus:** For questions 15, *Multiply the Binomials.*  Hint:

$(n + 5)(3n + 6)$
Reflection Assignment for Lesson Study II, Page 1

For each question, you are to have clear, correct explanations written in complete sentences and examples that you either make up or take from the work we have done in class. (You might want to do the examples first, and then explain your example.)

1) What steps do you follow to evaluate an expression?

Example(s):

2) Describe the distributive property, and explain how it can be used to simplify an expression.

Example(s):
3) When can we use the process of FOIL? Describe how FOIL can be used to simplify an expression.

Example(s)

4) What does it mean to factor an expression?

Example(s)

5) What is the difference between the associative property and the commutative property?

Example(s)
For each question, you are to have clear, correct explanations written in complete sentences, and examples that you either make up or take from the work we have done in class. (You might want to do the examples first, and then explain your example.)

1) What steps do you follow to evaluate an expression?

To begin, you have to follow to evaluate an expression in order. You have to do the operation in order: first multiplication or division (which ever comes first) and lastly addition or subtraction (which ever comes last). Before you do PEMDAS, you have to plug (substitute) in any variables. Then you can do the PEMDAS:

Example(s):

\[
\begin{align*}
\text{Step} & \quad x^2 + 10 - 7 \quad \text{Plug } x = 5 \\
 & \quad 25 + 0 - 7 \\
 & = 18
\end{align*}
\]

2) Describe the distributive property, and explain how it can be used to simplify an expression.

The distributive property to multiply any number inside the parenthesis with an addition or subtraction. You multiply by the number inside the parenthesis. You will then simplify because you break down the problem:

Example(s):

\[
\begin{align*}
10(x+2) & - 5(x-2) \\
10x + 20 - 5x + 10 & \\
10x + 30 & \\
12x + 30
\end{align*}
\]

\[
\begin{align*}
3(x+2) & = (x-12) \\
3x + 6 - x + 12 & \\
2x + 18 & \\
-2x + 30
\end{align*}
\]
3) When can we use the process of FOIL? Describe how FOIL can be used to simplify an expression.

We use FOIL when there is no number/variable outside the parentheses. To find you do the first number in the first parenthesis times the two numbers/variables in the second parenthesis. You follow this step in the second number/variable in the first parenthesis. Then you multiply or simplify.

Example(s):
\[
(x + 2)(x - 4) \quad 7x^2 - 4x + 14x - 8 \\
3x^2 + 7x + 8
\]

4) What does it mean to factor an expression?

To factor an expression means to write the reduced form of the expanded form. The factors are the numbers that make the number a product.

Example(s):
\[
11x + 3x = 11(x + 3) \\
20x - 90 = 10(2x - 9)
\]

5) What is the difference between the associative property and the commutative property?

The difference between the associative and commutative is in associative you have the parenthesis around different numbers and commutative you switch the order of the numbers.

Example(s):
\[
A = (2 - 5)10 = 3(5 - 10) \\
8(7 + x) = (8 - 9) \cdot x \\
C = 2 + 3 + 5 = 8 + 3 + 5 \\
x + 10 + 5 = 5x + 10
\]
Appendix G

Sample Observation Protocol
Lesson Study Observation Protocol

Pre-Lesson
Background Information:
- Teacher: M. Weaver
- Observer: 
- Date of Observation: 10/20/05
- Lesson title: Exploring Graphs (continued)
- Subject/Grade: Math 8

Demographics:
- # of students: 18
- # of male students: 11
- # of female students: 7

Lesson Focus (circle one):
- Engage
- Explore
- Explain
- Extend
- Evaluate

Lesson Emphasis (check all that applies):

**Engage**
- ✔ Providing "hook" for lesson introduction
- ✔ Demonstrating a discrepant event
- ✔ Uncovering misconceptions
- ✔ Assessing prior knowledge
- □ Demonstrating a principle or phenomenon

**Explore**
- ✔ Providing an opened-ended investigation
- □ Designing student investigations
- ✔ Recording data-collecting evidence
- □ Following prescribed steps of a laboratory

**Explain**
- □ Introducing new concepts
- □ Learning new vocabulary/facts
- □ Presenting background content information

D. Llewellyn/SJFC/Lesson Study/Observation Protocol
Elaborate
- Providing problem-solving activity
- Completing an extended investigation
- Following prescribed steps of a laboratory
- Applying exploration to real-world situation

Evaluate
- Answering textbook short and/or open-ended questions
- Reflecting on readings and problems
- Writing reflections in a journal or notebook
- Preparing a oral or written presentation of evidence
- Completing homework sheets
- Completing performance assessments
- Making entries to a portfolio

Classroom Instruction (Check all that applies):
Indicate major materials/resources used during the lesson:
- Print materials - commercial textbook
- Print materials - teacher-made
- Print materials - trade books, magazines, etc.
- Hands-on materials - commercial kits
- Hands-on materials - district-produced kits
- Hands-on materials - general laboratory supplies
- Hands-on materials - models
- Technology resources - computers
- Technology resources - calculators
- Technology resources - maps, charts, etc.

Structure of student work:
- Whole group
- Small group
- Pairs
- Individual

Student Engagement:
- Entire class is engaged in the same activity at the same time
- Groups of students are engaged in different activities at the same time

Class Discussion:
- Whole group lead by teacher
- Whole group lead by student(s)
- Small groups

D. Llewellyn/SJFC/Lesson Study/Observation Protocol
During the Lesson

Comments: Record the time and observation throughout the lesson. Capture the salient interactions between the teacher and the students and among students as they work in groups.

<table>
<thead>
<tr>
<th>TIME</th>
<th>OBSERVATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:07</td>
<td>Class Starts, Review Previous Day, Explain Handouts</td>
</tr>
<tr>
<td>8:09</td>
<td>Explain Warm-Up → students predict whether graphs are linear or non-linear</td>
</tr>
<tr>
<td>8:14</td>
<td>Explain Activity → how to change windows on graphing calculators</td>
</tr>
<tr>
<td>8:18</td>
<td>Hand-Out calculators / check HW</td>
</tr>
<tr>
<td></td>
<td>Students explore all 12 graphs using graphing calculators</td>
</tr>
<tr>
<td></td>
<td>*Awesome job of displaying windows for students → got right to the math and objectives for the day!</td>
</tr>
<tr>
<td></td>
<td>*Gave students &quot;back pocket&quot; questions to extend thinking</td>
</tr>
<tr>
<td></td>
<td>*Awesome discourse between students (Tons of Math Talk)</td>
</tr>
<tr>
<td>8:40</td>
<td>Whole Class Summary</td>
</tr>
<tr>
<td></td>
<td>Students generate responses</td>
</tr>
<tr>
<td></td>
<td>- good questioning techniques</td>
</tr>
<tr>
<td></td>
<td>- explanations from students</td>
</tr>
<tr>
<td></td>
<td>- great connection to yesterday</td>
</tr>
</tbody>
</table>

OBSERVATION PROTOCOL FOR DAY 2 OF LESSON STUDY I, PAGE 3
Appendix H

Lesson Plans
### Planning to Teach CMP

**Unit Mathematical Models, Activity 4.4, Problem 4.3, Explore Graphs**

#### Mathematical Goals

Gain an understanding of the similarities and differences between linear, non-linear, increasing, and decreasing relationships such as exponential, linear, and inverse, and how these unique characteristics are represented in their equations.

<table>
<thead>
<tr>
<th>Launch</th>
<th>Materials</th>
<th>Vocabulary and Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-Up: Students individually predict which of the 16 equations on the warm-up sheet fall into each of the 4 categories matching the categories on the investigation sheet (linear, exponential, inverse, and parabolic)</td>
<td>Warm-up sheet</td>
<td>Chart paper</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Explore</th>
<th>Materials</th>
<th>Vocabulary and Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students use graphing calculators to explore each of the 4 graph types</td>
<td>Graphing calculators</td>
<td></td>
</tr>
<tr>
<td>They record their results in the 4 sections of the classroom sheet</td>
<td>Classwork sheet with 4 categories</td>
<td></td>
</tr>
</tbody>
</table>

| Summarize | |
|-----------| |
| Extensive summary discussion in which students re-analyze their predictions recorded on the chart paper and on their warm-up sheets based on what they learned from the explore phase using the graphing calculators. Connections should be made to previous units on linear, exponential, and inverse functions | |

**Homework**

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**LESSON PLAN FOR DAY 1 OF LESSON STUDY I**
### Thinking with Mathematical Goals

Gain an understanding of the similarities and differences between linear, nonlinear, increasing, and decreasing relationships among exponential, linear, and quadratic functions, which are expressed in their equations and graph models.

<table>
<thead>
<tr>
<th>Launch</th>
<th>Explore</th>
<th>Summarize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up activity: Students predict linearity or nonlinearity of the 12 equations on their classwork sheet. Teacher explains how to change windows on the graphing calculator.</td>
<td>Students explore all 12 graphs using individual graphing calculators. They refer to recommended windows on overhead projector. Extension (&quot;back pocket&quot;) questions given to students who finish early.</td>
<td>Which class summary and discussion of how to classify equations as linear or nonlinear, increasing or decreasing, and the equation forms that fall into each category.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials</th>
<th>Vocabulary and Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classwork sheet</td>
<td></td>
</tr>
</tbody>
</table>

**Homework**

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**LESSON PLAN FOR DAY 2 OF LESSON STUDY I**
### Technology in Mathematics Education

#### LESSON PLAN FOR DAY 1 OF LESSON STUDY II

**Planning to Teach CMP**

<table>
<thead>
<tr>
<th>Unit</th>
<th>11 With Symbolic Investigation 3 Problem Review Day</th>
<th>1/30/06</th>
</tr>
</thead>
</table>

**Mathematical Goals**

- Review and solidify student understanding of: properties, evaluating expressions, distributing, multiplying binomials, and factoring.

**SPPF Goals:** To actively engage students through a repertoire of instructional strategies in order to promote varied and complex thinking.

<table>
<thead>
<tr>
<th>Launch</th>
<th>Materials</th>
<th>Vocabulary and Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain how Jeopardy game on lap-top computers (wireless lab) will be used for review.</td>
<td>Wireless laptop</td>
<td></td>
</tr>
<tr>
<td>Emphasize responsibility with computers.</td>
<td>Computer Info</td>
<td></td>
</tr>
<tr>
<td>Explain rules of game.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demonstrate one problem on TV screen.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Explore | |
|--------|-----------------
| Students play the Jeopardy game in teams of two or three. Within each homogenous group, they support and help each other. | Lap-top computers |
| They use scrap paper to figure out problems and a piece of chalk to keep track of their scores. | Scissors and chape paper |

| Summarize | |
|-----------|-----------------
| The homework sheet serves as a summary. | Handout |
| Students extend their understanding from the class activity by completing homework involving a profit, income, and expense problem scenario as well as problems from the 5 categories included in the Jeopardy game. | |

**Homework:**

Handout described above

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### Planning to Teach CMP

**Unit:** 12, Int. F, with L. Investigation 3, Problem Review, Day 2

**Mathematical Goals**
- Review and solidify student understanding of properties of expressions, evaluating, distributing, multiplying binomials, and factoring.

<table>
<thead>
<tr>
<th>Launch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand out score sheet and scrap paper</td>
</tr>
<tr>
<td>Re-visit directions for Jeopardy game on laptop computers</td>
</tr>
<tr>
<td>Model computer start-up process</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wireless Computer Lab</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Explore</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students play the Jeopardy game in the same homogeneous groups as the day before.</td>
</tr>
<tr>
<td>They keep track of their points earned each day and their total points.</td>
</tr>
<tr>
<td>The teacher circulates to answer student questions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vocabulary and Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laptop computers</td>
</tr>
<tr>
<td>Score sheet</td>
</tr>
<tr>
<td>Scrap paper</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summarize</th>
</tr>
</thead>
<tbody>
<tr>
<td>The homework sheet again serves as a summary. Students again extend their understanding from class using the homework review sheet, which includes another profit problem. The homework sheet also helps them summarize and solidify concepts from the Jeopardy game since it includes all 5 problem types.</td>
</tr>
</tbody>
</table>

| Class Discussion (Student Feedback): The last 10 to 15 minutes of class, students share thoughts about the effectiveness of the computer game in helping them understand algebra concepts, as well as suggestions for improvement. |

<table>
<thead>
<tr>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handout review sheet described above</td>
</tr>
</tbody>
</table>

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**LESSON PLAN FOR DAY 2 OF LESSON STUDY II**