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Abstract

In this paper I report progress on both theoretical and experimental aspects. I describe two approaches to calculating putative effects of gravitational curvature on the Casimir force. The work I describe continues the quest to answer the question: do virtual field excitations follow geodesics?

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Casimir force in Schwarzschild metric: Progress report

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I have been working on this topic for many years. In this paper I report progress on both theoretical and experimental aspects. I describe two approaches to calculating putative effects of gravitational curvature on the Casimir force. I also introduce an innovative technique to improve the sensitivity of the measuring apparatus as well as a method to eliminate island effects on the surfaces. The work I describe continues the quest to answer the question: do virtual field excitations follow geodesics?

Keywords: Casimir force; Quantum fields in curved space; Hanbury-Brown Twiss effect.

PACS numbers: 03.70.+k, 11.10.-z, 04.30.-w, 04.80.Cc, 42.50.Lc, 42.50.-p, 12.20.Fv

1. Introduction

Although quantum fields are well-described in flat space; the results being verified by precision experiments (anomalous moment of electrons, Lamb shift and Casimir force¹), there remains one outstanding issue. This is the puzzle of divergences that inevitably appear in any computation of the effects of quantum fields in flat space. Much effort has been expended to find a solution but none has succeeded. At best some workarounds have been injected, for example by making the divergence logarithmic. Even so the cut-off wave vector is set to an arbitrary value in order to terminate the integral.

In spite of decades of work the issue of renormalization remains unsolved.

The approach I have been advocating is (i) to re-calculate the above-mentioned effects in a curved geometry and (ii) to propose experiments to verify the results of the calculations.² Although in flat space energy is a parameter that can be set to zero at any value and all energies are measured with respect to the chosen zero value, this freedom is not allowed in a curved metric. The reason is that energy gravitates so it creates a curved metric. Infinities that appear in flat space take on physical consequences - these will inevitably appear in experimental results.

For example in the case of the Lamb shift: this has been measured on Earth's surface where the curvature can be neglected. But what if the Lamb shift were measured on the surface of the Sun or on the surface of a white dwarf where the curvatures are significant? Are the measured values different from values on Earth's surface? We don't know because it has not been tried - nor has anyone attempted a

As a topic for experimental investigation instead of the Lamb shift a more practical example would be the Casimir force. The apparatus is simpler; the theory may also be manageable. I will describe an apparatus and method of measurement that will yield results.

I propose using the Casimir force as a tool to measure the effects of the Schwarzschild metric on the Casimir force. The apparatus consists of a pair of Casimir plates. These are arranged orthogonally with a common rotation axis Fig. 1. The apparatus is on a horizontal plane aligned along the East-West axis. The geometry is such that Earth's Ricci tensor couples to the Casimir plates. Depending on the orientation the plates couple either to the $R_{\theta\theta}$ or the R_{rr} components. Rotating the apparatus alternates the Ricci components on the two Casimir plates. The apparatus is sensitive to the difference of the effects of $R_{\theta\theta}$ and R_{rr} on the Casimir force. In the absence of the effect the output is zero. This is a null experiment.

Fig. 2. AC bridge circuit.

Aligning the apparatus along the East-West axis ensures the equality of centripetal acceleration on both sets of plates.

There are constraining forces countering the Casimir forces between the plates. The constraining forces are a measure of the Casimir force.

There is a capacitance between the plates. The plates are part of an AC bridge. The bridge is balanced by adjusting the other arm of the bridge Fig. 2. There is an external AC source exciting the bridge. The imbalance is measured by an AC voltmeter. When the bridge is balanced the voltmeter reads zero. As the apparatus is rotated along the symmetry axis, the plates are exposed to different components of the Ricci tensor. Any change in the forces is manifested as an imbalance of the bridge. The imbalance signal appears at twice the frequency of rotation. This is the signal we are looking for.

2.1. Details of apparatus

Over the years many attempts have been made to measure the Casimir force. These attempts have revealed faults in design. For example it has proved difficult to hold the plates parallel. Furthermore there are patch effects in films deposited on substrates. These issues have limited what gaps and sensitivities that have been achieved.¹

I propose using an off the shelf apparatus to obviate some of the difficulties. For example a Fabry-Perot etalon manufactured by IC Optical Systems³ uses a closed-loop feedback system to hold the pair of plates parallel and the gap fixed to a pre-determined value. The etalon consists of two disks of fused silica of 10cm diameter and 1.8cm thick each polished to $\pm 3nm$ of a flat surface.

The constraining force is provided by a stack of PZT transducers. The gap can be maintained to within an uncertainty of $\pm 10^{-12}m/\sqrt{Hz}$. The disks are made of fused quartz as are the supporting struts. This assures alignments remain unchanged with changing temperatures. The PZT's are held under slight compression in order to allow both up and down movement.

The etalon is a rigid integrated structure with a high degree of immunity to vibration. It is also sealed against the environment.

Fig. 3. Etalon disk showing locations of capacitors and PZT drivers.

Piezoelectric drivers (PZT's) hold the surfaces both at a fixed gap and parallel using a closed-loop feedback system. There are 6 capacitors around the edge of the etalon; the pair of X-capacitors measure changes in the tilt in the X-direction and the pair of Y-capacitors measure the tilt in the orthogonal direction. There is a fifth capacitor Z which detects changes in the separation; the last capacitor is a fixed air space reference against which Z is compared Fig. 3. These pairs are measured in capacitance bridges and any deviation in the user-set position generates an error signal that the servo controller nulls by altering the voltage on the PZT's to maintain a fixed position. This null-servo approach ensures the accuracy of the system as it does not rely on any reference system other than the fixed reference capacitor (which is made of the same materials as the etalon and the other capacitors, and for many configurations is largely self-compensating). The servo system linearizes the movement of the PZT's.

There are three bridges, two for parallelism changes and one for changes in the mean gap. Gaps can be maintained to within $\pm 10^{-12}m/\sqrt{Hz}$; surface parallelism to within $\pm 10^{-11}rads/\sqrt{Hz}$. Reducing the band-width yields a gap uncertainty to within $\pm 3 \times 10^{-15}m$.³

When brought to close proximity ($\approx 1\mu m$) the disks are attracted by the Casimir force. As the gap is reduced the Casimir force increases. Once the gap is selected the feedback servo reacts to keep the gap fixed by applying a compensation force on the PZT drivers until it is equal to the Casimir force. Measuring the Casimir force entails measuring the counter force on the plates by the PZT drivers.

Since PZT's are non-linear it would be inadvisable to use them to measure the constraining (Casimir) force. This force is obtained from the calibrated pressure

transducer. The PZT stack needs to be altered by inserting a calibrated pressure transducer. As the PZT responds to changing Casimir force the output of the pressure transducer measures the force compensating the Casimir force. The signal then is the difference in the outputs between the two plates.

Since depositing films inevitably creates patches because of the incommensurate crystal structure between the substrate and the film. As a result spurious static charges appear that alter the force between the plates. I am proposing a different approach to negate the effect of patches.

Instead of depositing a film I am proposing the use of a silicon substrate. The substrate is made conducting by heavily doping it with a group V element (phosphorus), enough to make the substrate a conductor as good as tungsten.⁴ The surface can be polished and made uniform to within $\pm 2\text{\AA}$. The need for films and the attendant patch effect is obviated.

With this uniformity it is possible to reduce the gap to $< 10\text{nm}$. At these gaps the Casimir force is several hundred pounds - a figure that facilitates calibration.

2.2. Verifying the Casimir force

The first step in the experiment is to verify that the force between the plates is indeed the Casimir force; meaning that the force follows a $1/d^4$ dependence (d is the gap between the plates). The feedback mechanism is turned off for this step. The pressure sensor output is measured for different weights placed on each plate. A graph of the weight vs. sensor output provides the needed calibration.

For the next step the feedback mechanism is turned on. The pressure output is measured while the gap between the plates is changed. The force is obtained from the calibration. The force is plotted against $1/d^4$. A straight line with a slope of $-\pi^2\hbar c/240$ verifies that the force is the Casimir force.

The step is repeated for both sets of plates. Any difference is noted. This would be a measure of the asymmetry between the two sets of plates.

Experience shows that the force does not quite follow the $1/d^4$ dependence since the electric field does not completely go to zero on the conducting surface. Data are sparse for gaps less than 10μ for parallel plates; so the experiment may yield better data over a large range of gaps.

2.3. The experiment

The plates are mounted on a shaft, placed on a horizontal plane and aligned East-West. The plates form two arms of an AC-bridge. The output of the bridge is connected to a phase-sensitive detector Fig. 2. The other two capacitors in the arms are adjusted to null the output. An external oscillator (@ 1 kHz) excites the bridge.

The plate assembly is rotated @ 2 Hz; this is also the reference oscillator for the phase-sensitive detector.

The signal, if there is one, will appear at twice the frequency of modulation - 4 Hz. If there is an imbalance or asymmetry in the plates this will show up at 2 Hz.

Hanbury-Brown Twiss noise suppression scheme

Fig. 4. Noise suppression using Hanbury-Brown, Twiss effect.

2.4. Sensitivity

Since we don't know before hand what will be the signal amplitude we will try to make the apparatus as sensitive as possible. The sensitivity will be limited by transistor noise which is typically $\pm 5nV/\sqrt{Hz}$. If the integration spans one month the noise level is reduced to $\pm 5 \times 10^{-12}V$.

2.5. Noise suppression

Noise can be further suppressed. Using the theory of partial coherence (Hanbury-Brown Twiss effect⁵) the amplifier output $\Delta V(t) = V(t) - \langle V(t) \rangle$ is fed into a circuit shown in Fig. 4. The signal is sent through a band-pass filter that is centered at the frequency of the signal (4 Hz). The output is $\Delta V(t)$. It is then split into two branches one of which is delayed by τ with respect to the other. The two signals are then multiplied - $\Delta V(t) \Delta V(t + \tau)$ in a circuit. A following circuit integrates the product over a time interval T . The output is the mutual coherence function $\langle \Delta V(t) \Delta V(t + \tau) \rangle_T = |\Gamma_{12}(\tau)|^2$. As the time delay τ is varied, because of partial coherence, the output $|\Gamma_{12}(\tau)|^2$ goes to zero. Under this condition the noise approaches zero limited only by quantum fluctuations in the external oscillator. Quantum fluctuations can be calculated from

$$\frac{\Delta \bar{n}}{\bar{n}} = \sqrt{\frac{1}{\bar{n}}}. \quad (1)$$

If the fluctuations need be $< 10^{-6}$ @ $1kHz$ then $\bar{n} = 10^{12}$. This sets the power requirement for the external oscillator. Thus the net sensitivity is limited by noise to a few parts in 10^{18} .

2.6. Systematic uncertainties

With changing orientation the etalon chassés may also be affected by the Ricci tensor. Keeping the angle fixed and measuring the Casimir force isolates this effect. Plotting the force at each angle against $1/d^4$ and extrapolating to $1/d^4 \rightarrow 0$ distinguishes the Casimir force from the tidal force acting on the etalons. Or one can fit the data to a polynomial in powers of $1/d$ and extrapolate the results in the limit $1/d \rightarrow 0$. The latter option covers the possibility that the Casimir force may not follow a $1/d^4$ dependence under the influence of curvature. The electronic drift of the etalons. is $\pm 50 \text{ pm}/^\circ\text{C}$, with careful control of temperature the temperature stability can be 1 part in 10^{10} . Other issues are discussed in Ref. 7.

3. Theoretical estimates

Although it is not known if virtual field excitations follow geodesics, we can speculate what the effect would be *if* they do. I will use an expression for the expectation value of the stress tensor (renormalized by subtracting out infinities)⁶

$$\langle T^{\mu\nu} \rangle = \frac{\pi^2 \hbar c}{180 d^4} \left[\frac{1}{4} \eta^{\mu\nu} - \hat{x}^\mu \hat{x}^\nu \right], \quad (2)$$

$\eta^{\mu\nu}$ is the Minkowski metric, $\hat{x}^\mu \hat{x}^\nu$ are area normals of the plates. $\langle T^{\mu\nu} \rangle$ being a traceless tensor when it is inserted in the Einstein equation it looks like this

$$R^{\mu\nu} = -\frac{8\pi G}{c^4} \langle T^{\mu\nu} \rangle. \quad (3)$$

When the right hand side is zero the solution is the Schwarzschild metric. Since $\langle T^{\mu\nu} \rangle$ is very small we can use the Schwarzschild metric as a first order solution. The second order term is a small correction. Thus

$$g^{\mu\nu} = g_S^{\mu\nu} + \gamma^{\mu\nu}, \quad (4)$$

$\gamma^{\mu\nu}$ is a small time-independent perturbation due to the presence of the stress tensor and is expected to be zero far from the plates. $g_S^{\mu\nu}$ is the Schwarzschild metric. I will use the perturbed metric to calculate $\gamma^{\mu\nu}$ using the known form of the stress tensor keeping terms to first order in $\gamma^{\mu\nu}$ and also in m/r . $\gamma^{\mu\nu}$ is generated by the stress tensor. $\gamma^{\mu\nu}$ is then inserted in the unperturbed stress tensor. To first order the solution for $\gamma^{\mu\nu}$ is obtained from the Poisson equation

$$\varphi = -\frac{4\pi G}{c^2} \int \frac{\rho(x')}{|x - x'|} dv', \quad (5)$$

$$\gamma^{\mu\nu} = -\frac{4\pi G}{c^4} \frac{\langle T^{\mu\nu} \rangle v}{r}, \quad (6)$$

$v = 7.85 \times 10^{-9} \text{ m}^3$ which is the approximate volume of the cavity between the plates. The perturbed metric is thus

$$g^{\mu\nu} = g_S^{\mu\nu} - \frac{4\pi G}{c^4} \frac{\langle T^{\mu\nu} \rangle v}{r}. \quad (7)$$

Fig. 5. Casimir force balanced by constraining force modeled as a spring-mass system.

The stress tensor is altered to

$$\langle T^{\mu\nu} \rangle_{\Gamma} = \frac{\pi^2 \hbar c}{180 d^4} \left[\frac{1}{4} \left(g^{\mu\nu} - \frac{4\pi G}{c^4} \frac{\langle T^{\mu\nu} \rangle v}{r} \right) - \hat{x}^{\mu} \hat{x}^{\nu} \right]. \quad (8)$$

This then is the altered Casimir force.⁷ Since this expression is coordinate-dependent it will need a transformation before it can describe measurable quantities..

Another approach is to model the Casimir plates and the constraining force (compensating force exerted by the PZT stack) as a spring-mass system Fig. 5. The equivalent restoring force is k_{β}^{α} and the plates have a mass m . When this system is inserted in a gravitational field with a metric described by $g^{\mu\nu}$ (Schwarzschild metric) one can use a Lagrangian such as

$$L = \frac{1}{2} m g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} - \frac{1}{2} k^{\alpha\beta} g_{\alpha\mu} g_{\beta\nu} x^{\mu} x^{\nu} - \frac{1}{2} m R_{0\gamma 0}^{\sigma} g_{\sigma\mu} \delta_{\nu}^{\gamma} x^{\mu} x^{\nu}, \quad (9)$$

$$k^{\alpha\beta} g_{\alpha\mu} g_{\beta\nu} = k_{\mu\nu}, \quad (10)$$

$$L = \frac{1}{2} m g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} - \frac{1}{2} k_{\mu\nu} x^{\mu} x^{\nu} - \frac{1}{2} m R_{0\gamma 0}^{\sigma} g_{\sigma\mu} \delta_{\nu}^{\gamma} x^{\mu} x^{\nu}, \quad (11)$$

$$\frac{2L}{m} = g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} - \frac{k_{\mu\nu}}{m} x^{\mu} x^{\nu} - R_{0\gamma 0}^{\sigma} g_{\sigma\mu} \delta_{\nu}^{\gamma} x^{\mu} x^{\nu}, \quad (12)$$

$g_{\mu\nu}$ is diagonal so

$$\frac{2L}{m} = g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} - \frac{k_{\mu\nu}}{m} x^{\mu} x^{\nu} - R_{0l0}^k g_{k\mu} x^{\mu} x^{\nu}. \quad (13)$$

The spring-mass system has a resonant frequency (kilohertz) which is

$$\omega_0^2 \equiv \frac{k_{\mu\nu}}{m}. \quad (14)$$

Thus

$$\frac{2L}{m} = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \omega_0^2 x^\mu x^\nu - R_{0l0}^k g_{k\mu} x^\mu x^\nu. \quad (15)$$

In this expression M = mass of Earth and R its radius. The apparatus is on Earth's surface.

Solutions of the Euler-Lagrange equations along the r - and θ - variables, keeping only leading terms, are

$$\ddot{r} + (\omega_0^2 + R_{010}^1) r = 0, \quad (16)$$

and

$$\ddot{\theta} + (\omega_0^2 + R_{020}^2) \theta = 0. \quad (17)$$

Since I am only interested in the excursion between the r - and θ - components I am ignoring all off-diagonal terms ($l \neq k$). This also simplifies the calculations.

The metric alters the forces and thus resonant frequencies are changed to

$$\omega_r^2 = \omega_0^2 + R_{010}^1 = \omega_0^2 - \frac{2GM}{R^3} \left(1 - \frac{2MG}{c^2 R}\right), \quad (18)$$

and

$$\omega_\theta^2 = \omega_0^2 + R_{020}^2 = \omega_0^2 + \frac{GM}{R^3} \left(1 - \frac{2MG}{c^2 R}\right). \quad (19)$$

Since the second term is very small one can approximate as follows

$$\omega_r \equiv \omega_0 \left[1 - \frac{GM}{\omega_0^2 R^3} \left(1 - \frac{2MG}{c^2 R}\right)\right], \quad (20)$$

$$\omega_\theta = \omega_0 \left[1 + \frac{GM}{2\omega_0^2 R^3} \left(1 - \frac{2MG}{c^2 R}\right)\right]. \quad (21)$$

Taking the difference

$$\frac{\omega_r - \omega_\theta}{\omega_0} \approx -\frac{3}{2} \frac{GM}{\omega_0^2 R^3} \left(1 - \frac{2MG}{c^2 R}\right) \approx -\frac{3}{2} \frac{GM}{\omega_0^2 R^3}, \quad (22)$$

$$\frac{\omega_r - \omega_\theta}{\omega_0} \approx -\frac{10^{-12}}{\omega_0^2}. \quad (23)$$

This is the final result. It is coordinate independent. It favors a small value for ω_0 . The angular dependence is $P_2(\cos \theta)$. Practical considerations (isolation from terrestrial vibrations) demand a minimum value of $\omega_0 \approx 10^3 s^{-1}$. Thus

$$\frac{\omega_r - \omega_\theta}{\omega_0} \approx -10^{-18}. \quad (24)$$

This maybe within experimental capabilities.

It should be evident that the apparatus measures the difference between the Casimir forces in the two pairs of plates; it does not measure the absolute force in either plate. As such the requirement on the precision is less demanding.

4. Conclusion

If the measured value is consistent with the theoretical estimate we can conclude that gravitationally induced metric curvature affects the Casimir force. That virtual vacuum excitations do follow geodesics. As a result the vacuum field near an event horizon will be polarized; black holes will evaporate, thus confirming the existence of Hawking evaporation. Furthermore, along with infalling matter, the vacuum field would also fall freely through the event horizon. The vacuum field can be expected to be increasingly stressed during its free fall. The stressed vacuum would be expected to decay by emitting radiation of decreasing wavelength. When the stress energy exceeds the pair-production value, one can expect $e^+ - e^-$ emission followed by emission of increasing massive particles. The rate of energy loss through radiation or particle emission can be either less than, equal to or greater than the rate of increase in the vacuum stress energy.

References

1. H. G. B. Casimir, *Proc. Ned. Aka. Wetenschappen*, Amsterdam **60**, 793, (1948); M. J. Sparnaay, *Physica* **24**, 751 (1958); G. Bressi *et al.*, *Physical Review Letters* **88**, 041804 (2002); J. Zou *et al.*, *Nature Comm.* **4**, 1845 (2013).
2. M. Karim *et al.*, *Frontier Tests of QED and Physics of the Vacuum*, eds., E. Zavattini, D. Bakalov and C. Rizzo, (Heron Press, 1998), p. 419; *Phys. Lett.* **A171**, 1992, p. 7; *Coherence & Quantum Optics VI*, eds., J. Eberly, L. Mandel and E. Wolf (Plenum, NY, 1990), p. 583.
3. IC Optical Systems Ltd., Beckenham, Kent, BR3 4TW, UK.
4. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U.P., 1995), p. 160.
5. T. Hossain, Cerium Laboratories, Austin, TX 78741, private communication.
6. L. S. Brown and G. J. Maclay, *Phys. Rev.* **184**, 1272 (1969).
7. M. Karim and A. H. Bokhari, *EPJ Web of Conferences* **75**, 05001-05001, (2014), p. 10.