The Constructivist Approach to Mathematics Teaching and the Active Learning Strategies used to Enhance Student Understanding

Jodi B. Prideaux
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MS in Mathematics, Science, and Technology Education

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Some mathematics educators take the constructivist approach when it comes to their idea of the perfect classroom. They believe that actively engaging students in learning is the most productive means of teaching. Active learning strategies were incorporated into the Systems of Linear Equations unit in a ninth-grade Math A classroom to show that active learning strategies would motivate and engage students in the learning process, thus resulting in an enhanced understanding of the material. Strategies included a Jigsaw activity, a Carousel activity, tickets-out-the-door and various written expression assignments. Results from an in-class quiz were used as one way to measure student understanding by comparing the results to the previous year, in which active learning strategies were not used. The greatest impact of student understanding was seen through different uses of written expression.
Dedication

To my family – thank you for all of the patience you have given me over the last three years and a special thanks to Jamie for all of those late nights. I could not have done it without you.
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The Constructivist Approach to Mathematics Teaching and the Active Learning Strategies used to Enhance Student Understanding

Many different methods of teaching are used by mathematics educators around the world. One of these methods is constructivism. Constructivism is by no means an innovative teaching method since it dates back to the time of Socrates. For many years the constructivist approach to teaching has appeared in textbooks, curriculum frameworks and literature. The essence of constructivism has been captured through the development of active learning, also known as learning by doing, learning by experience, learning through action, student-centered learning, peer collaboration and cooperative learning.

The following quote expresses what some mathematics educators might identify as a perfect educational system:

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction... The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding... Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (National Council of Teachers of Mathematics, 2000, p. 3)

This quote supports the idea of constructivism and the use of active learning strategies in the classroom.
Educators see the importance of the ideal classroom but are faced with the time constraints and curriculum requirements of the state education department. Educators must decide how they will meet the standards while simultaneously trying to motivate and engage their students in the learning process. Teachers strive to see students succeed. Parents and students might see success as passing the course or getting good grades while most teachers define success as being able to understand and communicate mathematically and think critically.

The intent of this research is to obtain empirical data as well as qualitative data to show that active learning strategies in the classroom will motivate and engage students in the learning process, thus resulting in an enhanced understanding of the material. A Chinese proverb puts this idea into even simpler terms, “I hear, I forget; I see, I remember; I do, I understand” (Rosenthal, 1995, p.108).
Literature Review

A great deal of research has been conducted surrounding the idea of traditional teaching methods versus constructivism. The strong initiative of teachers to develop a more constructivist approach to education is apparent in much of the research literature. The motivation and engagement of students in the learning process has been shown to increase with the use of active learning strategies. The development and implementation of such diverse strategies have both positive and negative affects for the teacher and student.

Many different active learning strategies will be discussed, highlighting research from the literature. The benefits and concerns of using a constructivist approach to learning and the incorporation of active learning strategies will be covered.

*Constructivism/Active Learning*

According to Crawford and Witte (1999) the best word to describe a constructivist classroom is energy. The active engagement of students in the learning process is essential. Obtaining this type of engagement requires a much different classroom from the authoritative and teacher-centered traditional classrooms in which the teacher stands at the front of the room directing the content that is delivered to the students (Polya, 2002). Brooks and Brooks (1999) discuss the need to rethink this traditional classroom and the notion that students will learn on demand and that they will learn the same material at the same pace.

The state and local curriculums address what the students learn and therefore is a guiding tool for educators. The traditional classroom prepares students for standardized tests and clearly does not foster deep learning that students could apply to new situations.
Instead they are trained to mimic learning on the tests. According to Rosenthal (1995), "most mathematicians agree that the best way to learn mathematics is by actively doing mathematics; by discussing it with others; and by synthesizing major ideas" (p. 108) which is typically not seen in a mathematics classroom. A survey conducted by Weiss (1990) indicated that most mathematics lessons at the high school level were still largely didactic, however, some evidence of small groups, hands-on or manipulative materials and the use of computers was available.

In a traditional classroom the teacher's role is to convey knowledge to the students. Over two thousand years ago Socrates conveyed that the teacher should act as a midwife and that ideas should be born into the student's mind by discovering it for themselves. This idea supports the constructivist approach to education in which the central role of learning is placed in the hands of the students (Polya, 2002). In a constructivist classroom knowledge moves in more than one direction. Knowledge moves from teacher to student, student to student and even from student to teacher. A constructivist teacher, according to Brooks and Brooks (1999), would focus on how students learn and what they must learn together as one.

For a classroom to be described as full of laughter, motivation, imagination, engagement, attention, creativity and joy is a great achievement for a teacher and students. Not only is the atmosphere of the classroom important but also the organization and arrangement in the room. The traditional classroom would have desks in rows but in a classroom full of energy desks would be in small groups in order to invite student interaction and the opportunity to build a community of learners (Brooks and Brooks, 1999).
The use of lecturing as the traditional teaching method is not always the most successful approach according to O'Sullivan and Copper (2003). Leonard (2000) highlighted that lectures guarantee that a particular amount of material is covered but does not guarantee that the students have fully understood the material. Learning cannot take place just by reading or listening to lectures (Polya, 2002). In the context of Kieren's 1969 article "activity learning is taken to mean school learning settings in which the learner develops mathematical concepts through active participation" (p. 509). This may involve the manipulation of physical materials, games or experiments with physical objects. Discovery learning actively engages the learner in the process of forming mathematical ideas for himself, a key element of constructivism.

Naturally, there are a variety of methods used by teachers when creating their constructivist classroom. Crawford and Witte (1999) discuss the contextual teaching strategies that should be used when developing an active learning strategy. These strategies focus on the fundamental principle of constructivism - teaching and learning in context. The contextual teaching strategies include relating, experiencing, applying, cooperating, and transferring.

The ability to relate mathematical ideas to the context of a student's life experiences are important. Crawford and Witte (1999) give the example from one teacher's classroom about making fruit punch from frozen concentrate. Through exploration, discovery and invention students can create meaning. The experience that the teacher set up for her students allowed them to discover using their definition of ratio the number of cans of concentrate and the number of cans of water needed to make fruit punch for the entire class.
Another teacher in her algebra class had students collect data by measuring their heights and arm spans to draw a line of best fit. They then used this information to make predictions for the height of their teacher. The use of manipulatives, problem-solving activities and laboratory activities are just a few ways in which students can experience their learning.

The application of open-ended problems or projects provide students with an opportunity to use mathematics in realistic situations. The following example is described by Crawford and Witte (1999) as a typical word problem from a volume lesson: “A hemispherical plastic dome covers an indoor swimming pool. If the diameter of the dome measures 150 feet, find the volume enclosed by the dome in cubic yards” (p. 36). Even though this problem might be real, students may find it difficult to apply to their own life. The example given by another teacher shows how crucial mathematics can be in decision-making situations:

Montgomery is a compounding pharmacist at a pharmaceutical manufacturing plant. He is responsible for selecting the correct capsule sizes for products.
When a compound is prepared, the capsule size determines the dosage. The company uses eight sizes based on the body length \( \ell_b \), cap length \( \ell_c \), and diameter \( d \) of the capsules.

Montgomery must select a capsule size for a 25-milligram dosage of an antidepressant. Each capsule must contain \( 650 \pm 10 \text{ mm}^3 \) of the compound.

Which size should Montgomery select? (p. 36).

Knowing that not all students hope to become a pharmacist, this teacher developed problems that cover diverse situations making them applicable for their current or future
lives. The ability to relate and experience mathematics promotes a deeper desire to learn mathematics.

Boyer (2002) realized that supplementing her lessons with simple activities was only a small part of what she needed to accomplish in her classroom. Building a strong community of learners is just as important in actively engaging students. According to the book Strategies to Inspire Active Learning by Harmin (1998) “dignity, energy, self management, community and awareness” (p. 2) are important aspects of active learning in building a strong community of learners. Boyer incorporated each of these aspects in her pre-algebra classes. She focused on developing character among her classroom by strengthening her students’ confidence and giving them the chance to see value in doing something positive for others.

Educators in the United States are not the only ones faced with being responsible for educating children. Research conducted in the United Kingdom shows that the notion of active learning is widespread. Findings from this particular literature discuss the nature of active learning (Kyriacou, 1992).

The author describes active learning as “the use of learning activities where pupils are given a marked degree of ownership and control over the learning activities used, where the learning experience is open-ended rather than tightly pre-determined, and where the pupil is able to actively participate and shape the learning experience” (Kyriacou, 1992, p. 310). Active learning can be described by the application of any of the following five key concepts to a learning activity:

1. the use of concrete materials and direct learning experience;

2. the use of investigative or problem oriented techniques;
(3) the use of small group work;

(4) pupil ownership of the learning process or task;

(5) personal focus and relevance of the learning process or task (p. 311).

Many studies conducted about active learning have developed from the desire for more student involvement and interest, the need for the communication of mathematical ideas and more meaningful learning (Kyriacou, 1992). Upon giving a postal questionnaire survey to elementary and high school teachers in the United States, Weiss (1990) found that most high school mathematics lessons were largely didactic as stated earlier. Eighty-nine percent of the lessons were based on lecture, discussion and seat work assigned from a textbook. A fair amount of work, 40%, occurred in small groups while the use of hands-on materials was 16% and the use of computers was 8%. Her study along with others showed that the didactic method still dominated at this time in education.

Figure 1 shows a table that was developed from the different sources of data collected from transcripts of classroom teachers that were involved in a series of discussions about active learning. The first learning activity was the traditional method of teaching while the other six were types of active learning strategies. It was possible for more than one of these activities to take place simultaneously. These descriptors were categories identified from the observations and interviews with the mathematics teachers and then used as part of the questionnaire to a random sample of 100 chairs of mathematics departments in comprehensive schools in England aged 11-16 and 11-18. The chairs were asked to “estimate how frequently out of 100 randomly selected
Figure 1. Research conducted in the United Kingdom.

Table 1. Percentage of secondary school mathematics lessons in which each of seven learning activities is estimated to occur (based on data from 52 chairs of mathematics departments)

<table>
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<tr>
<th>Learning Activities</th>
<th>Percentage of lessons in which each learning activity is estimated to occur.</th>
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<tr>
<td>(1) Teacher explains/demonstrates a mathematical process or technique together with oral questioning of pupils to check understanding, followed by pupils undertaking written problems applying the process or technique</td>
<td>Lower school classes: 43 Upper school classes: 63</td>
</tr>
<tr>
<td>(2) Problem-solving or investigational task from which pupils derive mathematical knowledge and understanding</td>
<td>Lower school classes: 27 Upper school classes: 22</td>
</tr>
<tr>
<td>(3) Group discussion and collaboration in which pupils are required to work in pairs or small groups on the task set</td>
<td>Lower school classes: 30 Upper school classes: 22</td>
</tr>
<tr>
<td>(4) Practical simulations using pupils and/or materials to describe or represent mathematical knowledge or processes</td>
<td>Lower school classes: 14 Upper school classes: 10</td>
</tr>
<tr>
<td>(5) Use of structured individualized programs of work such as work cards or booklets</td>
<td>Lower school classes: 41 Upper school classes: 14</td>
</tr>
<tr>
<td>(6) Computer-based activities in mathematics</td>
<td>Lower school classes: 8 Upper school classes: 5</td>
</tr>
<tr>
<td>(7) A mathematical project based on an extended piece of work</td>
<td>Lower school classes: 11 Upper school classes: 13</td>
</tr>
</tbody>
</table>

(Kyriacou, 1992, p. 313)
department this current academic year you would expect to find each of these activities occurring” (Kyriacou, 1992, p. 314). The table shows the average of the estimates.

The mathematics department chairs were also asked “would you say that now compared with five years ago in your school, such activities are used more so now, less so than previously, or about the same?” (Kyriacou, 1992, p. 314). Forty-three of the fifty-two chairs that answered the questionnaire estimated that activity 1 was being used less frequently; consequently activities 2, 3, 6 and 7 were being used more frequently now.

Active learning strategies were introduced into the general chemistry curriculum at the United States Naval Academy. Classroom activities were described using six categories: problem-solving worksheets, creative testing strategies, hands-on learning activities, explain the demo worksheets, student presentations and competitions. Significant improvement in performance was evident with students in the active learning classroom compared to students in a lecture-based course during the first semester of the study. This was evident based on individual exams as well as in the overall course grade (O’Sullivan and Copper, 2003).

Active Learning Strategies

The search for understanding is the driving force to motivate students to learn. A teacher must be able to capitalize on student energy by establishing interest and a need for mathematics (Crawford and Witte, 1999). Mathematical activities must be chosen carefully so that they fully engage the students’ higher mental capacities. Diverse learning activities such as computer-assisted learning, role play exercises, work experience, group discussions, collaborative problem-solving and extended project work
are other forms of active learning (Kyriacou, 1992). Choosing the appropriate level of an activity is important in challenging a student. Smith (1999) explains that each learning activity chosen or constructed must demand mental involvement.

Small groups.

Small group exercises are one form of cooperative learning. According to Good, Reys, Grouws and Mulryan (1989) students are frequently grouped according to achievement, known as achievement groups. These groups complement the students' needs and do not allow for extensive social interaction. Contrary to achievement groups, some teachers may form small groups to promote interaction known as heterogeneous work-groups.

Good et al. (1989) wanted to examine the advantages and disadvantages of using work-groups during mathematics instruction. They conducted 63 observations from the 400 teachers they had polled who reported using groups more than once a week and for half or more than half of a mathematics period. A number of strengths were observed in the 63 lessons. Compared to the achievement-groups more of the students in work-groups exchanged mathematics ideas and were generally more active and constructive learners. Rather than focusing on computational skills the lessons were designed to develop higher-order thinking skills. In general, students were highly motivated to work together to complete the assigned task. The use of work-groups promoted peer interaction which in turn led to more advanced mathematical thinking. For a majority of the work-group lessons the teachers could be credited with developing their own activities as opposed to using lessons from textbooks or teacher's manuals.
Their findings led them to conclude that work-groups consisted of more active learners who were motivated and enthusiastic about mathematics. Lessons did not rely on the rote practice that was evident in the traditional method of teaching. However, “the effectiveness of a work-group depends on students’ mathematical knowledge and their experience in cooperative settings, as well as the teacher’s instructional goals” (Good et al., 1989, p. 61).

Students in the study conducted by Rosenthal (1995) were grouped with three to five students and assigned probability problems. The instructor provided assistance if necessary. In spite of the initial hesitations the small group exercises appeared to be successful. Students worked together discussing their problems, sometimes outside of the classroom. These students gave anonymous feedback about the activity commenting that the exercises were helpful and that they were able to understand the material because of the student input and discussion.

Working in small peer groups allow students to ask questions without feeling threatened or embarrassed. At times they are more willing to explain their understanding of concepts or problem-solving approach. By listening to others, students would be able to reevaluate and reformulate ideas to form their own sense of understanding (Crawford and Witte, 1999).

Mathematics in Context.

The five central tenets of constructivism, identified by Brooks and Brooks (1993) are summarized as follows:

Constructivist teachers

- Discover and value students’ points of view
Constructivist Approach

- Develop lessons that will challenge students' ideas
- Recognize that students want to know the relevance of concepts to the curriculum
- Develop lessons around big ideas, not small pieces of information
- Evaluate student learning in the context of the daily classroom, not as separate events.

These five tenets are manifested in the Core-Plus Mathematics Project described by Coxford and Hirsch (1996). The Project is a standards-based three-year high school mathematics curriculum for all students and includes a fourth-year course continuing to prepare students for college mathematics. The program enables students to think mathematically about problems and situations. Each year of the curriculum features four multiple strands which include “algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics connected by fundamental themes and by habits of mind” (p.23).

The program itself consists of four phases. Phase 1 of the program begins the lesson with a class discussion. Students are introduced to an interesting contextual question illustrating a problem with a strong sense of mathematics. During phase 2 of the program students work in groups on more focused activities to explore the mathematical features of the situation. Groups then organize their thoughts in phase 3 to clarify their ideas and prepare to share their mathematical discoveries with the class. The final phase, phase 4, is where students apply their mathematics to problems in different contexts thus consolidating their learning.
Similar to Crawford and Witte (1999), Coxford and Hirsch (1996) state “putting math instruction and learning in context helps students see that mathematics is part of their world. It also enables them to construct meanings that make sense to them, which, in turn, helps them make sense out of new situations and problems. The project’s emphasis on group work promotes student engagement, mathematical thinking, and better communication” (p. 24). Not all students approach and solve a problem using the same methods. The Core-Plus Mathematics Project curriculum encourages students to explore these different methods.

Students in this program were found to be better at the end of the year at reasoning and applying mathematical concepts than those enrolled in traditional mathematics classes. This instructional model engaged students in important mathematics while providing support for struggling students. In support of Crawford and Witte (1999), group work allowed students to clarify their understanding by discussing mathematical ideas with each other.

Written Expression.

A good way to encourage students to think about the information they are learning is to create written assignments. Rosenthal (1995) asked students to write a five page essay clearly explaining some particular aspect of the course material. Students had mixed reviews on the assignment but the assignment forced the students to put together several ideas and explain them clearly. Rosenthal (1995) stressed the importance of communication as a skill needed by all students. Neide (2000) also expressed that writing can be used to help students explore their own understanding about concepts they are to understand.
A second essay was given to the Rosenthal’s (1995) students but this time they were required to exchange their essays with two classmates. The students were expected to fill out a review of the essay highlighting strengths, weaknesses and suggestions for their classmates’ work. They later discussed in small groups their essays. Students commented that the reviews helped them to improve their essays. They also felt that the essay writing assignments helped them to have a deeper understanding of their topic. The overall idea of Rosenthal’s research was that lectures could be enhanced by using various techniques that encourage active learning.

According to Martinez (2001) “meaningful words problems are more effective than traditional exercises at engaging students in comprehensive and active learning. They encourage students to think mathematically and to develop reasoned problem solving strategies rather than rely on memorized procedures” (p. 248). Two major factors Martinez mentions in regards to overcoming negative views of word problems include the “need to engage students’ imaginations with creative, thought-provoking problems and involve the students more directly in evaluating their own word-problem-solving strategies by having them think and write descriptively and critically about their mathematical thinking” (p. 248).

Martinez (2001) gave students a thinking and writing exercise using Lancelot Hogben’s adaptation of Zeno’s famous paradox “Achilles and the Tortoise:”

Achilles runs a race with the tortoise. He runs ten times as fast as the tortoise. The tortoise has 100 yards’ start. Now, say Zeno, Achilles runs 100 yards and reaches the place where the tortoise started. Meanwhile the tortoise has gone a tenth as far as Achilles, and is therefore 10 yards ahead of Achilles. Achilles runs
this 10 yards and is therefore 1 yard in front of him. Achilles runs this 1 yard. Meanwhile the tortoise has run a tenth of a yard, and is therefore a tenth of a yard in front of Achilles. Achilles runs this tenth of a yard. Meanwhile the tortoise goes a tenth of a tenth of a yard. He is now a hundredth of a yard in front of Achilles. When Achilles has caught up this hundredth of a yard, the tortoise is a thousandth of a yard in front. So, argued Zeno, Achilles is always getting nearer the tortoise, but can never quite catch up.

(Hogben 1993, p. 11)

Hogben made the philosophical dimension of the paradox a word problem that could be solved at the high school level. Martinez (2001) used the paradox as an in-class activity asking the students to address the problem in two stages. In the first stage Martinez stated: “Keeping in mind that Zeno’s tale is a paradox, do you think that Achilles ever catches the tortoise?” (p. 249) and the second stage he stated: “If not, why not? If so, at what point does Achilles catch the tortoise?” (p. 249). Students used written expression to convey their understanding of the problem.

The students were broken into small groups to discuss and brainstorm about the problem while Martinez’s role switched to questioner. The problem itself engaged the students’ imaginations. The use of the thinking and writing exercise increased the students’ involvement in the process. “Doing this taught me more than I ever thought possible. I didn’t think I could enjoy solving a math problem” (Martinez, 2001, p. 251) remarked one student. “All students were more confident that, with practice, they would be able to accomplish both tasks – solving word problems and describing what they are doing to solve word problems – more effectively” (p. 251).
According to Hamden (2005), writing allows students to understand concepts by coordinating between new and old concepts. Writing allows for dialogue between the student and teacher and provides a record of the students’ development over a period of time. The use of journals could be used as a record as well as a means of fostering student reflection (Neide, 2000).

*Desktop Teaching.*

Desktop teaching is another active learning strategy that gives students the opportunity to prepare a lesson about a particular topic and share the lesson with the rest of the class (Draper, 1997). Students take on the role of teacher and are responsible for developing a lesson that will motivate and make their students want to learn. Desktop teaching engages students in both the discussing and learning of mathematics. Not only are they responsible for doing the work but making sure their work is at an acceptable level for others to learn.

*Reflection in Mathematics.*

Creating constructivist lessons promote an energetic classroom exposing students to different ways of learning traditional lessons. The use of self-assessments are an important reflection tool for students to manage what they learned, their progress and the goals they would like to have for work they would have to do in the future. Boyer (2002) created a classroom of learners by having each student create a mural of how they used mathematics in their daily lives. Students then shared with each other their personal connection to mathematics. Finally, making students aware that the information they were learning could be related to something in their personal life allowed them to be mindful and attentive.
An informal comparison to her previous years of teaching, Boyer (2002) noticed an increase in academic achievement. Overall class averages were five percent higher than before she had incorporated in her lessons Harmin’s five teaching strategies mentioned earlier. She had given her students a survey that supported her ideas that student motivation and performance increases with using active learning. Her students were “achieving at a higher rate, smiling more often and were interested in seeing how mathematics can be used every day” (p. 51).

*Technology Integration.*

An increase in technology over the years has provided teachers with innovative ways to actively involve students in the learning process. Students no longer have to be passive recipients of information. The use of technology as a teaching tool allows for student interactivity (Brown, 2004). A webquest is an activity in which students utilize World Wide Web resources to obtain information that is then used in a group project. The integration of technology can be used in games like Jeopardy. Students could play on their own, in groups or as a class. Technology has become a natural part of society and the depth and quality of its use is largely determined by the teacher (Brown, 2004).

*Other Strategies.*

Research suggests that peer learning often helps students learn. Neide (2000) discusses many other active learning strategies that could be used in any classroom. TV commercial refers to students acting out a film or designing a poster to showcase a particular theme or objective of a lesson. Students could also get involved in their own learning by creating a list of questions to answer, words to define or people to identify. Students could then question each other trying to find the answers.
An activity designed to stimulate immediate interest in the subject matter is Go to Your Post. Students can begin the class actively moving to the part of the room that has a topic that interests the student. Students at the same post can discuss the topic and generate ideas to share with others.

Some strategies help students learn curricular content as well as allow the teacher to assess the student's understanding of the material. Fast Facts is a strategy in which students take notes during a lecture-based class and are immediately given an open-note quiz. Students may work with a partner or in a small group to share information. Each student is responsible for the knowledge thus allowing the teacher to randomly choose a student to answer the question.

Muddy waters is similar to Fast Facts since it allows the teacher to get immediate feedback about what the students have comprehended. Three by five cards are given to the students to write down anything they did not understand or need clarified from the lesson. This gives the students a non-threatening way to ask questions.

The Jigsaw activity requires students to be responsible for one mathematics problem or concept. The students work in a group to fully understand the problem. They then are broken into a second group of students. Each student has their own problem to share with the new group. By the time all students have shared the information, all members of the group should be able to explain each problem. An open note quiz could be given at the end of the activity to ensure the knowledge was conveyed appropriately and that students stayed on task (Neide, 2000).
Concerns of Constructivism/Active Learning

There are many concerns that educators have when it comes to designing and implementing learning activities. “Mathematical activities alone are not enough to achieve learning by themselves; they need to be carried out with a consideration of aspects of presentation, the nature of the pupil’s mental activity, the need to ensure pupil reflection and the achievement of socialization of the learning” (Smith, 1999, p. 110).

The study conducted by Good et al. (1989), mentioned earlier, observed that students had to develop communication skills that they didn’t already have. They also needed to become accustomed to working cooperatively since the norm of the classroom setting had been independent work.

Many educators as well as Good et al. (1989) were concerned that group work would increase the amount of time spent on drill and practice and increase the opportunity to converse with one another about non-mathematical material. Rosenthal (1995) found that some mathematics students were not skilled to work with other people.

Development of learning strategies takes time. Time is a major concern for many educators around the world. Not only does it take time to create activities but it also takes time to implement or facilitate these activities into the daily classroom routine (Neide, 2000). One of the concerns raised on the comment section of the questionnaire discussed earlier by Kyriacou (1992) was the pressure of time on the staff. Greater involvement in teaching and assessing the work produced and the lessening of the number of periods scheduled for mathematics contributed to this pressure.

The difficulty of encouraging some staff to adopt new approaches and strategies to teaching and learning was another issue (Kyriacou, 1992). Many educators felt that
moving toward an active learning approach would result in less material being covered in the class. Teachers also found that adopting new approaches would require the need for more curriculum materials to use in work-groups (Good et al., 1989).

Many educators and researchers discussed the motivation factor for students as another important concern while developing lessons. Organizing a constructivist classroom is not only difficult work for teachers but requires the rigorous intellectual commitment and perseverance of the students. The shift from making sure that all students learn the same concepts must take place. Teachers must then carefully analyze student’s understandings in order to customize the teaching approaches developed (Brooks and Brooks, 1999).

Summary

Traditional teaching methods focus on the student as a passive recipient of information. In contrast, constructivism puts students at the forefront of their learning. The teacher is responsible for creating a learning environment that will allow for students to obtain a deeper understanding of the material by actively involving students to “talk, listen, write, read, reflect, and then apply what they have learned to real-life problems” (Neide, 2000, p. 29).

Active learning strategies may require more work on the part of the teacher but once the strategies have been implemented the extra time and effort will not feel like an encumbrance. These strategies are often fun for students which in turn motivate and engage them in the learning process. The strategies and activities developed can be used to achieve the stated learning goals.
Methodology

Students worked together to develop an understanding of the concepts in the Systems of Linear Equations unit rather than being passive recipients of the information. While portions of the unit were lecture-based, active-learning strategies were incorporated into the daily classroom activities and homework assignments.

Participants

Participants in the research consisted of approximately 65 students in a rural high school in upstate New York. Dispersed throughout four Math A classes the majority of the students were freshman but included four sophomores retaking the course. Each mathematics period varied in the number and gender of students. Period 1 consisted of 14 students—8 females and 6 males, Period 2 consisted of 14 students—6 females and 8 males, Period 5 consisted of 20 students—14 females and 6 males, and Period 9 consisted of 17 students—4 females and 13 males.

Student desks were grouped in twos in an amphitheater arrangement with a central focus towards the SmartBoard during the lecture-based portion of the lessons. Alternate desk arrangements were made for the different learning activities that took place and are discussed in the sections forthcoming.

Instruments and Materials

As part of the usual classroom routine students received a guided note packet for Solving Systems of Linear Equations. As usual, the note packet included a cover page with the daily objectives for the unit listed, important vocabulary, as well as the homework assignments attached at the end of the packet.
Procedures

The first day of the unit students received their note packet. The objectives for the first lesson were to determine whether a system of linear equations has zero, one or infinitely many solutions and to solve systems of equations by graphing. The first example was a real-life application problem and was completed as a class. A brief discussion took place on the number of solutions to a particular graph of a system.

The next three questions students were divided into Jigsaw groups predetermined by the teacher. Each expert group was given one of the three problems to solve. Students spent the remainder of the class period solving their problem. The activity continued the following day where students began by breaking into another set of pre-arranged groups in which each student had a different problem. Each student in the group acted as the teacher for their particular question. Students were responsible for taking turns to explain how they graphed their system of equations and how they determined the solution to the problem.

At the end of each lesson students completed a How-to worksheet in addition to their daily homework (Appendix A). The worksheet required students to communicate mathematically using written expression on how to solve a system of equations using the indicated method. The How-to worksheets were collected at the beginning of the next class in order for the teacher to assess student understanding and provide feedback to the students. The students compiled their How-to homework assignments to help them create a brochure on How to Solve a System of Linear Equations. Guidelines and a rubric were provided to the students the same day that the first How-to homework was assigned (Appendix B and Appendix C).
The Carousel active-learning strategy was used for the lesson objective to solve word problems using systems of equations. The desks were arranged in twos around the room to allow for students to move from one set of desks to the next. Predetermined by the teacher students sat with their partner as they entered the room. An odd number of students in the class resulted in one group of three. At each set of desks was a word problem and a different colored marker. An explanation of the activity was clearly outlined on the SmartBoard for the students to see (Appendix D).

As the activity began students first had to read the problem and determine what the problem was asking them to find out. Once they had determined this information they were responsible for choosing a variable or variables and explain what they represented. The students rotated to the next problem after 2-3 minutes taking with them their colored marker. At the second problem students had to read the problem and look at the work completed by the previous group. Students then had to decide if the group before them had decided on an appropriate variable or variables and then write a system of equations that could be used to solve the problem.

Upon the third rotation students once again had to check the previous work and begin to solve the system of equations. A fourth rotation required students to find the solutions to the problem and a fifth and final rotation required the solution(s) of the problem to be checked.

Data Collection

As part of the usual classroom activities tickets-out-the-door were used to obtain anonymous student reflections and assessments of student understanding. Observations made during classroom activities provided the teacher with the knowledge of the amount
of understanding shown by the students. Information provided on the How-to homework assignments used for the student's brochures were also used as a way to gauge student understanding.

The results from the in-class quiz were compared to the results of the quiz taken by Math A students from the previous school year (2005-2006). Active-learning strategies were not used during this unit in 2005-2006 and consequently all lessons were lecture-based.
Results

The various active learning strategies incorporated throughout the unit provided the teacher with information as to the engagement of the students and their understanding of the material. Qualitative data was collected during the Jigsaw and Carousel activities as well as from tickets-out-the-door and the How-to homework assignments. A comparison of the results of the unit quiz to the results of the unit quiz taken by students the previous year was used to obtain quantitative data.

The implementation of the Jigsaw activity promoted student interaction. Students worked together to solve their given problem once the introductory example was completed as a class. The majority of students were observed willingly explaining to those students struggling with the problem. However, others needed to be encouraged by the teacher to help students that did not understand. A few students were reluctant to attempt the problem but eventually completed the problem with the help of their group members. Many of the groups required little assistance or direction from the teacher and were successful in finding the solution to their problem.

As the students regrouped the next day they took on the role of being the teacher. Each student explained their problem and found in some cases they needed to explain the problem in a different way in order for all group members to understand. Students in the groups were observed asking questions about why the teacher solved the problem in that particular way. The teachers were also asked to explain more specifically the steps used to solve the system of equations graphically.

In one of the groups a lower achieving student wanted assurance from the classroom teacher that he was explaining the problem correctly. The classroom teacher
acted as an observer during the student’s explanation but did encourage the student to trust his explanation. Another group during the same class had difficulty role-playing the teacher as two of the three students were reluctant to explain to the third student how to solve each problem. While most student groups were on-task during the activity there were groups that needed redirection from the classroom teacher.

During both days of the activity complications with student absences forced the classroom teacher to rearrange groups. Despite some of the obstacles faced each student left the classroom with all three problems complete and correct.

Teacher feedback was given to the students on their How-to homework assignments in order for the brochures to be accurately completed and to give the teacher an idea of which students understood the solution process. It was found that only a handful of students were able to explain in specific and accurate detail of how to solve a system of linear equations graphically. Students struggled with the mathematical terminology used in their explanations as well as providing a detailed explanation of the process. The teacher identified two crucial concepts unidentified by students – how many equations must be graphed and how does one find the solution to the problem? Consequently, discussion was held the following day about these two questions.

Solving a system of equations by substitution was the second How-to homework completed by students. An improvement in the use of mathematical terminology was evident in the description of this particular method. Students were able to identify the need to replace a variable when using substitution as well as find the value of both variables before the problem was finished.
Similar results were found in the remaining How-to homework assignments involving elimination and elimination with multiplication. The use of the correct mathematical terminology increased. Students were using words like additive inverses, eliminate, solution, substitute and variable throughout their explanations. In addition, the length and descriptiveness of the explanations increased. One misconception was recognized while reading the explanation of solving a system of equations using the elimination with multiplication method. Some students stated the need to multiply one of the equations by a number in order to create additive inverses amongst the two given equations. They neglected to state that some systems may require both equations to be multiplied in order to create additive inverses.

Students continued expressing their understanding through writing. A ticket-out-the-door asked students to state which method would be the best to solve the given system of equations (Appendix E). They then had to explain why they chose that particular method. Students were able to correctly identify the best method to use when solving each system of equations. In addition the majority of students were able to clearly explain the reason for their choice.

The Carousel Activity produced a lot of interaction amongst students. Some students had difficulty with the initial writing of the two equations from the word problem and quickly became frustrated. Seeking out teacher assistance occurred throughout all classes for certain word problems. Some groups finished working on their problem rather quickly while others struggled with the time limit set at each station. Many of the time issues arose due to an incorrect step performed by a previous group. Time adjustments were made throughout the different classes depending on how the
groups were progressing. Students remained focused on the portion of the problem they were responsible for completing. At times throughout the activity groups were interacting with other groups to clarify a step they had previously completed. The final step of the activity allowed for students to see whether the problem had been solved correctly.

Feedback about the activity was collected anonymously through a ticket-out-the-door the following class period (Appendix F). Student responses were both positive and negative. There were a handful of negative comments that recurred on multiple student papers but the number of positive comments overwhelmingly exceeded the negative ones (Appendix G). Many students commented that they liked the activity. One student commented "I liked this activity because it was a hands-on, interacting activity and it made it fun to participate in." Comments like this one amongst others, as well as classroom observations showed that students were actively engaged and motivated to learn.

Students were given a quiz at the completion of the unit identical to the quiz used the previous year (Appendix H). The grading of the quiz and point deductions were kept constant for both years. The teacher compared the grades that students received on the quiz in 2006-2007 to those students that took the quiz in 2005-2006. The grade range was broken down to compare the number of students scoring below a 65, between 65 and 69, 70-79, 80-89 and 90-100. Seventy-three students took the quiz in 2005-2006 compared to sixty-five students in 2006-2007. Since the total number of students each year was different, quiz results were compared based on the percentage of the students that scored in each grade range.
Figure 2 shows a graph comparing the quiz results from 2005-2006 to 2006-2007. Although the percentage of students scoring between 80-89 and 90-100 did increase slightly it was not a significant change from the previous year. However, the percentage of students scoring in the remaining grade ranges showed significant changes from the previous year. The percentage of students scoring 70-79 increased from 8.2% to 16.9%. The percentage of students scoring 65-69 decreased from 6.8% to 3.1% and the percentage of students scoring below 65 also decreased from 11.0% to 4.6%.
Figure 2. Quiz Results.
Discussion and Conclusion

The intent of the research conducted was to show that active learning strategies in the classroom would motivate and engage students in the learning process, thus resulting in an enhanced understanding of the material. Active learning strategies were implemented into the Systems of Linear Equations unit, contrary to the previous year in which the unit had been strictly lecture-based.

The Jigsaw and Carousel activities provoked student interaction in the learning process while the How-to homework assignments forced students to understand and convey in words the process of solving a system of equations using various methods. Student feedback about the activities supported the fact that students were motivated and engaged during activities throughout the unit. Results from the unit quiz showed a decrease in the percentage of students scoring below a grade of 69, thus supporting an increase in student understanding of the material.

The Jigsaw activity supports Brooks and Brooks (1999) and their notion that knowledge moves in more than one direction. The activity began with a teacher to student flow of knowledge about how to solve a system of equations graphically and then became a student to student flow of knowledge by the conclusion of the activity. During the activity most of the students were fully engaged and active in the learning process. However, even with the teacher's best effort it was difficult to get every student hooked and participating during the Jigsaw activity. One student did not feel as though it was their responsibility to teach and explain the problem to another student in the group. This particular student often requires a lot of teacher direction to help get started and stay focused on a task. School is not seen as important to this student and thus lacks any
motivation to complete assignments on time or work successfully with certain students in the classroom. This student often admits to being lazy and not caring about whether or not their school work gets done. Therefore, the group members did not have a lot of patience with this student and did not want to teach their problems and learn this students’ problem. A different grouping of students in this class would have worked much better for this unmotivated student.

Another challenge faced during day one of the activity was trying to prevent students from working ahead on the other two problems. Some of the higher-achieving students work ahead in their note packets and were forced during this activity to complete only their assigned problem. They were then encouraged to make sure every group member felt comfortable teaching their problem to a new group the next day before moving on to start their homework.

The engagement of students in discovering solutions to the problems in both the Jigsaw and Carousel activity relates to Kieren’s 1969 article, which stated students should be active participants in developing mathematical concepts. Students were able to work together in small groups to take ownership of their learning, supporting two of the five key concepts of a learning activity described in Kyriacou’s article (1992). The group members in both activities were predetermined by the teacher and were heterogeneously grouped in groups of three or four during the Jigsaw activity and groups of two or three during the Carousel activity. The heterogeneous grouping of students thought by Good et al. (1989) would promote student interaction and lead to more advanced mathematical thinking. The interaction among students in both activities was thought-provoking.
Students asked each other questions and even reached out to other groups while rotating through the Carousel activity.

Many students provided feedback about group work during the Carousel activity (Appendix G). Even though not all students liked to work in a small group or with a partner, the majority of students had positive things to say about working with others. Students made comments like “we had to work in pairs so we got other peoples’ thoughts about the problem instead of just the teacher’s” or “we were able to express our opinion in a small group.” These two comments emphasize that students have the desire to be involved in their own learning and be active participants in smaller groups. Crawford and Witte (1999) discussed a similar matter in regards to communication amongst students in a smaller group. Through their research they found students were more comfortable communicating with each other than with the teacher. This allowed students to give opinions and ask questions in order to better understand the problem.

In addition to these comments there were many other highlights. Many of the students noted that the activity was more exciting and much better than taking notes. They enjoyed doing a hands-on activity. The majority of them also liked being able to move around the room to complete the problems saying “it was fun” and that “rotating helped because what we didn’t know other groups helped.”

The Carousel activity was designed with the purpose that all groups would complete each step of the process used to solve a system of equations word problem even though they were rotating to a different problem every few minutes. Some students liked not having to solve a complete word problem while others became frustrated because “we didn’t get to start over each time and we had to just jump into the problem” or “we only
had to do certain parts of the problem so I didn’t understand it all.” Similar observations were made by the classroom teacher during this activity. Some students had difficulty changing problems and picking up where other groups left off. After rotating from one problem to the next students did not necessarily read the problem in front of them before working on the step they needed to complete. There were groups in each of the classes that wrote down an incorrect system of equations. Groups that did not read the problem and check the previous work completed continued to solve the problem resulting in a solution that did not work for the given problem. Many of the groups did not like to fix other groups’ mistakes or trying to figure out where the previous groups had made a mistake. This was a valuable experience for students – mistakes happen but the ability to find and correct the mistakes is an even more valuable experience.

In each of the four classes groups that had word problems that involved the use of the substitution method needed assistance to write the system of equations. Problems involving the elimination with multiplication method were much easier for students to write. By the end of the period most of the classes had finished the problems and their solutions had checked. Groups that found their solutions did not check did not have enough time to go back to find out where the mistakes had been made. The following day an answer key to the Carousel word problems was handed out to students. Appendix I shows a completed word problem by one of the groups during the activity.

The ability to discuss with others and synthesize major ideas was agreed upon by most mathematicians to be seen as the best way to learn mathematics (Rosenthal, 1995). Creating a brochure using the How-to homework assignments forced students to look at the various methods used to solve a system of equations. The ability to identify the best
method and explain how to apply the method to find the solution forced students to have a deeper understanding. Students were expected to complete and turn in their How-to homework assignments on time in order to allow the teacher to give feedback about their explanations and return them to the students the following day.

The first How-to homework assignment was given to students on day three of the unit and after two homework assignments requiring students to solve a system of equations graphically. Appendix J shows two sample student explanations for the graphing method. The first sample is an incomplete explanation and was common among the majority of students. The student explains how to graph a line but makes no mention for the need to graph a second line or how to find the solution to the system. In addition, some students had difficulty using the correct mathematical terminology. The second sample shows a thorough explanation and an example to go along with the explanation.

After returning the first How-to homework assignments these misconceptions were pointed out before the day’s lesson. Students were also told that this was a rough copy of their explanation that would be used for their brochure. They were encouraged to read the comments made and seek help if needed to correct or improve their explanations.

Many students struggled to express the mathematical concepts in words but were urged by the teacher to pretend they needed to explain to a little brother or sister how to solve a given system of equations. It was also explained to students that if they could explain how to solve a system of equations then they would in fact be able to solve the system of equations. Neide (2000) and Rosenthal (1995) both stressed the importance for students to be able to communicate their understanding in writing. Being able to follow a
list of steps to solve a problem is one thing but understanding the how and why of the
procedure is another thing.

In spite of the misconceptions and incorrect terminology with the first How-to
homework there was an improvement in the remaining How-to homework assignments.
More students began to use the correct mathematical terminology and become more
detailed in their explanations. Some students spent their study halls working together
with other students in the presence of their teacher in order to ask questions if needed.
The teacher would act as the little brother or sister asking questions in order for the
student to really think about the process. The assignments forced students to think and
write mathematically, thus increasing their involvement in their own learning. Similar to
the students of Martinez (2001), by the end of the unit students were not only able to
solve a system of linear equations but also describe how to solve the system of equations.
Students worked hard to create their How-to brochures. Even the unmotivated student
put time and effort into the brochure accurately explaining and solving the systems of
equations. This student spent time during a study hall to have directions and expectations
explained to him a day prior to the deadline. Appendix K provides three student samples.
Each brochure is unique in its composition since students were given the flexibility in
their design.

The final ticket-out-the-door given to the students asked them to state the best
method that could be used to solve the given system of equations and explain why they
chose that particular method. No method could be used more than once since four
systems of equations were provided. The ticket-out-the-door allowed the students to
communicate their understanding of the four methods to solve a system of equations
mathematically, therefore giving the teacher another means to check their understanding (see Appendix L for a student sample).

The results of the unit quiz did show an improvement in student understanding from the previous year's results. However, the results shown in the graph do not accurately represent the true understanding that students had upon the completion of the unit. The majority of students showed a clear understanding of when to use the different methods and how to apply those methods to solve the given system of equations. Students lost points on the quiz for computational errors or conceptual errors unrelated to how to solve a system of linear equations. One example of a conceptual error made was subtracting a term on the same side of the equal sign when they should have combined like terms.

The ability for students to communicate mathematically throughout the unit was a key component for the improved quiz grades. The percentage of high-achieving students did not increase significantly but a significant improvement was made amongst the lower achieving students. More students scored in the 70-79 grade range resulting in less students scoring below 69. The written assignments forced the lower-achieving students to think about and understand the concepts in the unit. Some of these students received a lot of feedback on their How-to homework assignments and were encouraged to seek extra help during a study hall or after school. Many students took advantage of this, knowing they would have to create a brochure that would be graded. The incorporation of active learning strategies helped these students improve their knowledge of the material as well as their ability to communicate with others. While the Carousel and
Jigsaw activities were key factors in motivating and engaging students, the different uses of written expression had the greatest impact to enhance student understanding.

The engagement and motivation of students during this unit as well as the effort put into creating a brochure was a success. "It was different and FUN" said one student in regard to the Carousel activity. Another student liked "being active while learning."

These comments support the need to incorporate active learning strategies into the daily classroom routine. In the years to come this unit will be taught in a similar fashion taking into consideration the comments made by students. The Jigsaw activity will be conducted again with the addition of a ticket-out-the-door or a warm-up activity the following day. This would be used to ensure that students know how to solve a system of linear equations graphically.

Improvements would need to be made to the Carousel activity. The integration of multiple types of word problems were difficult for students to handle. It might be better to solve word problems involving elimination on one day and solve word problems involving substitution on a different day to allow for more practice. This would help to improve some of the time constraint issues. The extension over a two or three day period would allow for students to take their time on the problems and check the other students work. In addition time would be spent as a class looking at the problems completed and discussing the different solutions.

Time was also an issue for the teacher in regards to reading and giving feedback to over 50 students for four different How-to homework assignments. As worthwhile as the assignments were it became difficult to provide students with all the feedback they needed by the following day. Active learning strategies and the implementation of these
strategies take time and require more of the teacher’s time. Time is a common concern amongst educators and was discussed throughout much of the literature. Neide (2000), Kyriacou (1992) and Good et al (1989) all discussed concerns surrounding time.

Enhancing and improving units from one year to the next is an essential part of teaching. The active learning strategies used in the Systems of Linear Equations unit did motivate and engage students in the learning process. The use of written expression pushed students to understand and be able to explain mathematical concepts. Students were successful in being able to explain the four methods used to solve a system of linear equations and know when to use each method. The success of this unit confirms the fact that more units need to incorporate active learning strategies and allow students to have more ownership over their learning. Implementation of different activities involving written expression into other mathematical units of study would be a recommendation for future research.
References


Educational Research, 14, 139-155.
Appendix A
How – To Homework

How-To Homework
Due: Monday Jan. 8th
Name ____________________________
Date ________________ Period ______

Explain in words how to solve a system of equations by Graphing. Use the examples in your notes to help with your explanation.

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Use the space below to show an example if you choose.
How-To Homework

Due: Tuesday Jan. 9th

Name __________________________
Date ___________ Period ______

Explain in words how to solve a system of equations by Substitution. Use the examples in your notes to help with your explanation.

________________________________________________________________________

________________________________________________________________________

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Use the space below to show an example if you choose.

KEEP THIS PAPER TO HELP YOU CREATE YOUR BROCHURE
How-To Homework

Due: Wednesday Jan. 10th

Explain in words how to solve a system of equations by **Elimination**. Use the examples in your notes to help with your explanation.

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________________________________________________________________________

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________________________________________________________________________

Use the space below to show an example if you choose.

________________________________________________________________________
How-To Homework

Due: Thursday Jan. 11th

Name ____________________________

Date ______________ Period ______

Explain in words how to solve a system of equations by Elimination with multiplication. Use the examples in your notes to help with your explanation.

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________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Use the space below to show an example if you choose.
Appendix B

Brochure Directions

Systems of Equations

How-To Brochure

Create a “How – To” brochure or pamphlet for solving a system of equations using the four methods described throughout the unit. The methods you will describe include:

(1) Graphing
(2) Substitution
(3) Elimination
(4) Elimination with multiplication

The following is a bank of systems of equations to be used as examples for your brochure.

\[
\begin{array}{cccc}
-3x + y = -7 & 4x + 3y = 12 & y = 4x - 7 & 3y = -6x - 3 \\
y = -x + 1 & 2x - 5y = -20 & -2x + y = 9 & y = 3x - 16 \\
8x + 2y = -2 & y + 6 = 2x & 4x + y = 8 & 2x + 5y = 20 \\
y = -5x + 1 & 4x - 10y = 4 & -3x - y = 0 & 3x - 10y = 37 \\
& & x + y = 10 & 3x + 2y = -19 \\
& & -x - 2y = -14 & x - 12y = 19
\end{array}
\]

Your brochure/pamphlet will be graded using the attached rubric.

**IMPORTANT DUE DATES:**

Rough Draft: FRIDAY, January 12\textsuperscript{th}

Final Project: FRIDAY, January 19\textsuperscript{th}
## Appendix C

### Grading Rubric for Brochure

<table>
<thead>
<tr>
<th>Brochure Mechanics</th>
<th>CATEGORY</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing - Organization</td>
<td>Each section in the brochure has a clear beginning, middle, and end.</td>
<td>Almost all sections of the brochure have a clear beginning, middle and end.</td>
<td>Most sections of the brochure have a clear beginning, middle and end.</td>
<td>Less than half of the sections of the brochure have a clear beginning, middle and end.</td>
<td></td>
</tr>
<tr>
<td>Writing - Grammar</td>
<td>There are no grammatical mistakes in the brochure.</td>
<td>There are no grammatical mistakes in the brochure after feedback from an adult.</td>
<td>There are 1-2 grammatical mistakes in the brochure even after feedback from an adult.</td>
<td>There are several grammatical mistakes in the brochure even after feedback from an adult.</td>
<td></td>
</tr>
<tr>
<td>Attractiveness &amp; Organization</td>
<td>The brochure has exceptionally attractive formatting and well-organized information. The work is presented in a neat, clear, organized fashion that is easy to read.</td>
<td>The brochure has attractive formatting and well-organized information. The work is presented in a neat and organized fashion that is usually easy to read.</td>
<td>The brochure has well-organized information. The work is presented in an organized fashion but may be hard to read at times.</td>
<td>The brochure's formatting and organization of material are confusing to the reader. The work appears sloppy and unorganized. It is hard to know what information goes together.</td>
<td></td>
</tr>
<tr>
<td>Mathematical Terminology and Notation</td>
<td>Correct terminology and notation are always used, making it easy to understand what was done.</td>
<td>Correct terminology and notation are usually used, making it fairly easy to understand what was done.</td>
<td>Correct terminology and notation are used, but it is sometimes not easy to understand what was done.</td>
<td>There is little use, or a lot of inappropriate use, of terminology and notation.</td>
<td></td>
</tr>
<tr>
<td>Mathematical Concepts</td>
<td>Explanations show complete understanding of the mathematical concepts used to solve the problem(s).</td>
<td>Explanations show substantial understanding of the mathematical concepts used to solve the problem(s).</td>
<td>Explanations show some understanding of the mathematical concepts needed to solve the problem(s).</td>
<td>Explanations show very limited understanding of the underlying concepts needed to solve the problem(s). or is not written.</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Mathematical Errors</td>
<td>90-100% of the steps and solutions have no mathematical errors.</td>
<td>Almost all (85-89%) of the steps and solutions have no mathematical errors.</td>
<td>Most (75-84%) of the steps and solutions have no mathematical errors.</td>
<td>More than 75% of the steps and solutions have mathematical errors.</td>
<td></td>
</tr>
<tr>
<td>Diagrams, Sketches, and/or Examples</td>
<td>Diagrams, sketches and/or examples are clear and easy to understand.</td>
<td>Diagrams, sketches and/or examples are somewhat difficult to understand.</td>
<td>Diagrams, sketches and/or examples are difficult to understand or are not used.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

"How-To" homework: _____ /4

Rough Draft: _____ /4

Brochure Mechanics _____ + _____ + _____ + _____ = _____ /16

Brochure Mathematics

Graphing _____ + _____ + _____ = _____ /12

Substitution _____ + _____ + _____ = _____ /12

Elimination _____ + _____ + _____ = _____ /12

Elimination with Multiplication _____ + _____ + _____ = _____ /12

Project Total: _____ /72
Carousel Activity

Each group will have one color marker. You will use the same color marker throughout the entire activity.

1st rotation - read problem
- fill in call out boxes
- write 2 let statements

2nd rotation - write 2 equations
- determine which method to use - then write it down

3rd rotation - solve for one variable

4th rotation - solve for the other variable
- write the solution and label the answers

5th rotation - check the solution
Appendix E

Ticket-out-the-door (Choose the Best Method)

State which method would be the best to solve the given system of equations. Then explain why you chose that particular method. Do not solve the system of equations.

1. \[3x + 4y = -25\]
   \[2x - 3y = 6\]

2. \[y = -x + 5\]
   \[y = x - 3\]

3. \[-5y + 3x = -16\]
   \[5y + 2x = 31\]

4. \[a = 3b + 1\]
   \[5b - 2a = 1\]
Appendix F

Ticket-out-the-door (Anonymous Student Feedback)

What did you like about the word problem activity on Friday?

What did you dislike about the word problem activity on Friday? What did you find frustrating?
Appendix G

Anonymous Student Feedback

(Student comments have been arranged into categories by the instructor)

What did you like about the word problem activity on Friday?

Notes vs. Activity
- “We didn’t take notes and did more of a hands on activity”
- “We didn’t have to take notes and we actually got to do a ‘hands on’ project”
- “I liked how we did the activity instead of just taking notes”
- “The activity was more exciting than taking notes”
- “It was easy and better than notes”

Mobility and Group-work
- “It was a way to get out of our seats”
- “We got to move around and that we got to do different parts of different problems”
- “Rotating helped because what we didn’t know other groups helped”
- “I like that the activity gave us more practice. It was fun getting up and moving around”
- “I got to move around and interact with others”
- “I like how we got to move around and get a taste of each problem. Also that we got to work in groups and traveled around seeing other peoples opinions and checked if they had their problems right or wrong”
- “It gave us good help and more experience with solving the equations”
- “We had to work in pairs so we got other peoples’ thoughts about the problem instead of just the teacher’s”
- “It was a way to use partner working skills”
- “Being in groups and working step by step”
- “I liked that we got to see all the different problems and that we got to see how other people handled the problem. It could help us later.”
- “How we all worked together and helped each other”
- “I like how we had a smaller group to work with”
- “We were able to express our opinion in a small group”

Miscellaneous
- “It was different and it was FUN”
- “Being active while learning”
- “I liked this activity because it was a hands on, interacting activity and it made it fun to participate in”
- “This activity was a great way to explain this lesson to a visual learner like me”
- “We broke up the problem’s and learned how to solve them”
- “Using markers”
- “visuals”
- “I liked to solve the problems that people started”
- “It was a way to study and make sure we know it”
What did you dislike about the word problem activity on Friday?
What did you find frustrating?

Time Constraints
- "I didn’t like how we had a lot of extra time at each station"
- "We didn’t have enough time at each station"
- "Moving from different problems and having to switch and focus that fast"

Problem Constraints
- "It was frustrating because we didn’t get to start over each time and we had to just jump into the problem"
- "We only had to do certain parts of the problem so I didn’t understand it all, so that was frustrating"
- "I didn’t get to solve the whole problem"
- "Doing different problems each time"
- "I didn’t like that we couldn’t finish our problems when we were close to the end"

Group Constraints
- "If one person messes up, the rest of the problem is wrong"
- "I didn’t like trying to figure out what the last group did for their step"
- "Having to fix the prior groups mistakes"
- "Having to finish others work"
- "People didn’t show all of their work"
- "I didn’t really like working with partners. I’d much rather work in a bigger group or by myself."

Miscellaneous
- "That we didn’t write notes about the problems"
Appendix H
System of Equations Unit Quiz

Quiz 6-1 to 6-6

Part I

Solve the system of equations graphically. Check your work algebraically.

1. \[ y = -x + 3 \]
   \[ y = x - 3 \]

Solve each system of equations. Check your solutions.
(You may use any of the methods we have learned in class)

2. \[ y = 3x \]
   \[ x + y = 4 \]

3. \[ x + 4y = -8 \]
   \[ x - 4y = -8 \]
4. \[5x - y = 10 \]
\[7x - 2y = 11\]

5. \[2x + 3y = 6\]
\[3x + 5y = 15\]

Match each word with the appropriate description, definition or example. NOT every letter will be used.

_____ 6. proportion

_____ 7. simplify

_____ 8. commutative property

_____ 9. product

_____ 10. expression

A. \[a + b = b + a\]

B. A mathematical statement that does not contain an equal sign.

C. \[a + 0 = a\]

D. A comparison of 2 numbers by division.

E. An equation that states that 2 ratios are equal.

F. Removing parentheses and combining like terms

G. The result of a multiplication problem.
Write and solve a system of equations. Check your solution.

11. Tommy and Ryan had lunch at the mall. Tommy ordered 3 slices of pizza and 2 cokes. Ryan ordered 2 slices of pizza and 3 cokes. Tommy’s bill was $6.00 and Ryan’s bill was $5.25. What was the price of one slice of pizza? What was the price of one coke?
Appendix I

Carousel Student Sample

Claudia and Zella went shopping at Price Buster. Claudia bought two jumbo rolls of aluminum foil and three packages of AA batteries for a total cost of $26.50. Zella bought five identical jumbo rolls of aluminum foil and two identical packages of AA batteries for a total cost of $25. Find the cost of one jumbo roll of aluminum foil and one package of AA batteries.

Claudia's purchase:
- 2 rolls of aluminum foil
- 3 packages of AA batteries
- Total cost: $26.50

Zella's purchase:
- 5 rolls of aluminum foil
- 2 packages of AA batteries
- Total cost: $25

Let A = the price of aluminum foil
Let B = the price of AA batteries

Solution:

- For Claudia's purchase:
  - $10A + 15B = 105$
  - $10A + 15(3) = 105$
  - $10A + 75 = 105$
  - $10A = 30$
  - $A = 3$

- For Zella's purchase:
  - $5(2A + 3B) = 21$
  - $10A + 15B = 105$

Elimination method:
- Multiply Claudia's equation by 5:
  - $50A + 75B = 525$
- Subtract from Zella's equation:
  - $-25A - 4B = -50$
  - $A = 4$

Substitute $A = 4$ into Claudia's equation:
- $10(4) + 15B = 105$
- $40 + 15B = 105$
- $15B = 65$
- $B = 4$

Therefore, the price of one jumbo roll of aluminum foil is $4 and the price of one package of AA batteries is $4.50.
Incomplete Student Explanation

Explain in words how to solve a system of equations by GRAPHING. Use the examples in your notes to help with your explanation.

First you must isolate the variable that is not already alone. Next you need to find the y-intercept of the problem. After that plot it on the graph. Next you find the slope of the line. For example, y = -7x + 5.

If y is already isolated so the 1st step is done and then you find the x-intercept which is 5 after this you need to turn the equation into y = mx + b. To find the slope which is \( \frac{5}{1} \). After this you count down seven and right one, right of it is your slope. Then you must draw line through these points with ones on the ends and label it.
Complete Student Explanation

Explain in words how to solve a system of equations by **graphing**. Use the examples in your notes to help with your explanation.

The first thing you do when graphing equations is to make sure $y$ is by itself.

If it is, you're good to go. Next thing you do is trace your paper, create a

coordinate plane and label. The 2nd thing you would do would be to find the

$y$-intercept and graph it on the coordinate plane. You would then look at

your slope to see if its negative or positive. Now you take your slope if

it's not a fraction put your number over 1. If you can do in $y = mx + b$,

and just plot it and then

take your slope and go rise over run. When you get off the grid

go back to your $y$-intercept. And instead of rise over run, your going

to want to go across and then go your $y$ in the opposite direction

of the $x$-intercept. Draw a line from your points. As you increase or decrease

and keep your line. For the second equation you would do everything from

the first. Then you find where the 2 intersect. You would then check

your solution by plugging in the point and solving the equation. If the 2 equations

come from your answer is correct. Write solution: ( )

and put your answer in the parentheses.
Appendix K

Sample Brochure: Student 1

Graphing

First, add 3x to -9. Now, you should have the results y=2x. Now find the intercepts of the equation which is -3. Then, plot a point on -7 and use the rise over run method. Draw 3 and go to the right 1. Plot the points each time until you run out of room on the graph. For the next equation, use the rise over run method. Plot the point positive 1, go down 1 and to the right 1. Plot the points each time. Now, find the point of intersection of these two lines and the coordinates will be your answer. Check these equations and re-solve them by also inserting the answers for x and y, they should both come out correctly.

Elimination

First, line up the equations and cross out positive and negative y. Now, add 4x to -3x and 0 to 0. You should receive the answer x=3. The next step should be taken to find out what the variable equals. Take either equation and substitute 0 for the variable x. Solve the equation and you should get the answer y=-24. Your solution should be x=3 and y=-24. Check these equations and re-solve them. Also re-solve the answers for x and y, they should both come out correctly.

Elimination With Multiplication

First, line the equations up and observe that you cannot cancel out y by and 2y because they are not added except. Next, multiply the equation 2x+3y=-19 by 2 and distribute 6 to all numbers. The solution should be (18y+12y=-114). Line up this new equation with x+2y=19 and cancel out positive and negative 12y. Add the other numbers together and you will get 19x=-95. Divide 19 from -95 and x will equal -5. Now, find the value of y equals, solve either equation and substitute -5 for x. Solve this equation and you will find that y=2. Your solution is x=-5 and y=2. Check these equations and re-solve them by also inserting the answers for x and y, they should both come out correctly.

Substitution

First, substitute -6+1 in the equation 3x+2y=-2 for the variable y. Now, the problem should read 3x+2(-6+1)=-2. Next, distribute 2 to the numbers in parentheses and the results are 3x+10x=-3. The next step should combine 8x and -10x. Solve this like any other equation and you should receive the answer x=2. Then, rewrite the y equation y=8x-1 and substitute x for 2. Solve this equation and the answer is y=-8. Your final solution should be x=2 and y=-8. This is how we use this method substitution for solving a system of equations. Your last step is to check your work. Re-write both equations and substitute the variables for the numbers which they equal. Solve these problems and they should come out correct.
Sample Brochure: Student 2

**System of equations**

- **Graphing**
- **Elimination**
- **Elimination with Multiplication**
- **Substitution**

---

Answers:

1. First, you have to solve for $y$.
2. Once you get $y$, you make a point and graph the point. From that point you go up or down depending on the slope. Then you move either how many units based on the equation number under the slope. You repeat these steps for the next equations.
3. Always remember the development of a line. The point should come out to be a solution.
4. Then, solve the original equation on the lines then find your points of intersection to make your solution.

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Graphing

Elimination

Elimination with Multiplication

Substitution
Constructivist Approach

First, when solving systems of equations with elimination, you add the two equations. This should eliminate one variable. You then solve for the remaining variable and plug it into one of equations to solve. Find your second variable. This will then complete your solution. Then you check by plugging both variables into both equations and solve to check.

Elimination

Elimination with Multiplication

Substitution

When solving an equation with elimination with multiplication, you first have to add the equations. If neither variables in the system will eliminate you must multiply one or both of the equations so when you add the equation, you will eliminate one of the variables. After doing so, you then have to solve for that variable. Once you get your first variable, you then choose one of the equations and plug your solved variable to find your remaining variable to get your solution. Then check the solution by then plugging both variables into both equations to check.

Elimination with Multiplication

Substitution

\[
\begin{align*}
4x + 3y &= 12 \\
-2(2x - 5y) &= -20
\end{align*}
\]

\[
\begin{align*}
4x + 3y &= 12 \\
4x + 3(4) &= 12
\end{align*}
\]

\[
\begin{align*}
4x + 3y &= 12 \\
4x + 12 &= 12
\end{align*}
\]

\[
\begin{align*}
4x &= 0 \\
x &= 0
\end{align*}
\]

Solution:
\[
\begin{align*}
x &= 0 \\
y &= 4
\end{align*}
\]
To solve an equation using substitution, you must first replace the variable from one of the equations with the value of the other variable. Once that is done, you must solve for that variable, then plug it back into the other equation and solve for the second variable. This gives you your two variables for your solution. Then check by plugging them into both equations.

Example:
\[
\begin{align*}
y &= x + 1 \\
y &= 3x - 7
\end{align*}
\]
\[
\begin{align*}
x &= -11 + 16 \\
y &= 3(-11) + 7
\end{align*}
\]
Solution:
\[
\begin{align*}
x &= 5 \\
y &= -2
\end{align*}
\]
First, you start out with at least two equations. Use the $y = mx + b$ formula to graph the equations. The "m" is the slope which is rise over run. The "b" is the y-intercept. Look at the y-intercept and graph it on the y-axis. Using the slope, go up however many the top number is. If the slope is negative, you go up and to the left. If the slope is positive, you go up and to the right. Do this for all your equations, then draw the lines. Look to see where your equations intersect. The point where they all come together is the solution. To see if it's correct, use the solution to plug into your formula for the $x$ and the $y$. 

Graphing

System Of Equations

Example

$\begin{align*}
    y - 3x &= -7 \\
    y &= -x + 1 \\
    y + 3x &= 7 \\
    y &= 3x - 7
\end{align*}$
Substitution

Start out with two equations. One of the equations has to be an isolated variable, equal to an expression on the right or left of the equals sign. To substitute, put the expression that is equal to the variable, in place of it in the other equation. Find what the new variable equals, then replace it in the first equation you started out with. This will determine what number is in place of the variable that was alone. Then you get the solution. Replace the numbers from the solution into your equation to see if they check.

Elimination

Start out with two equations. Both equations have the same two variables. The only exception is that one equation has a term that is positive and one that is negative, also known as additive inverses. When you add the two equations together, one of the variables cancels out, leaving you to solve for the one that is left. After finding the sum of the equations and solving, chose one of the equations and replace the variable that you just solved for, with what it equals. Start solving for the variable that cancelled out in the beginning. After you find the number for that variable, replace it in the equation to check.
When solving a system of equations by elimination with multiplication, you use the same process as elimination. One exception is that the signs are the same or the numbers in front of the variable are different. When this happens, you multiply one or both equations so that adding the equations will eliminate one of the variables. Solve for the variable that is left; replace it in one of the expressions to solve for the variable that was eliminated. After you find both variables, replace them in both equations to check.

Example

\[2(x+y=10) \rightarrow 2x + 2y = 20\]
\[-x - 2y = -14 \rightarrow -x - 2y = -14\]
\[x=6\]

Now find \(y\):
\[x+y=10\]
\[6+y=10\]
\[y=4\]

Solution: \(x=6\)
\(y=4\)
Appendix L

Choose the Best Method Student Sample

State which method would be the best to solve the given system of equations. Then explain why you chose that particular method. Do not solve the system of equations.

1. \[\begin{align*}
2(3x+4y &= -2) \quad \text{Elimination} \\
-3(2x-3y &= 6) \quad \text{w/mult.}
\end{align*}\]

For this problem I would use elim. w/mult. bc then the variables would cancel out.

3. \[\begin{align*}
-5y + 3x &= -76 \\
5y + 2x &= 31
\end{align*}\]

I would use elim. for this problem because -5y and 5y can cancel out so we don’t have to multiply.

2. \[\begin{align*}
y &= -x+5 \\
y &= x-3
\end{align*}\]

I would use graphing bc the problem is already set up for graphing.

4. \[\begin{align*}
a &= 3b + 1 \\
a &= 5b - 2a = 1
\end{align*}\]

For this problem I would just plug a = 3b + 1 into 5b - 2a = 1 because the variable is all alone.