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State Space Analysis and its Connection to the Classroom

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State Space Analysis and its Connection to the Classroom

Abstract

Discrete dynamical systems have been used to theoretically model the complex dynamics of classrooms. While time-series analyses of these models has yielded some insights, state space analyses can yield additional insights; this paper will explore state space analyses and their application to classroom situations. One benefit of state space analysis is that it allows simultaneous exploration of multiple time-series, and so can more easily provide information about divergence and convergence of paths. Additionally, state space analysis, more easily than time-series analysis, can provide information about the existence of multiple paths leading toward a desired state. Further, state space analysis can identify different regimes of behaviors, finding boundaries near which there may be divergent behaviors, and also using those regimes to define a (sometimes) relatively small number of archetypical behaviors. This is particularly useful in tracking behaviors at a microgenetic level, since multiple initial conditions may get to the same (or very close) final states, but in dramatically different ways, and these different routes may have implications for future classroom experiences. Because of these advantages, state space analysis can be used to inform attempts at differentiated instruction in a classroom, assist modelers in identifying appropriate parameter scales, and provide guidance for empirical studies of classroom learning. These ideas will be illustrated through state space analysis of an existing model of teacher-student interactions, identifying four regimes of behaviors, and leading to several implications for classroom practice and research.

Disciplines

Mathematics

Comments

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STATE SPACE ANALYSIS AND ITS CONNECTION TO THE CLASSROOM

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All models are wrong. Some are useful.

- George Box

- Given that useful models are hard to come by, how can we get the most out of them?
  - For teachers?
  - For researchers?
  - For modeling?
- And, how can we introduce better quantitative work in education?

Outline

- Models & Modeling
- Scaffolding
- State Space vs. Time Series
- Implications for Teaching
- Implications for Research
- Implications for Modeling

Bifocal Modeling

- The strength of modeling comes from a bifocal stance (Abrahamson, Blikstein, & Wilenski, 2007):
  - The theoretical model should inform the empirical
  - The empirical should inform the theoretical model
- Note that there must be both
  - careful use of the theoretical models to drive empirical data collection, and,
  - careful analysis of empirical data to drive theoretical model construction
  - Sadly, we usually only get one of these (Byrne & Callaghan, 2013)
- This also addresses some of the issues noted earlier today by Koponen (dislike of modeling)

Model

- van Geert & Steenbeek, 2005 (“vGS”)

$$\Delta L = r_L \cdot L \cdot (1-L/H) \cdot (D-H/L)$$

$$\Delta H = r_H \cdot H \cdot \Delta L \cdot (1-H/G)$$

- Where:
  - L = pupil level
  - H = scaffold level
  - D = optimal scaffolding level
  - G = goal level
  - r_H = adaption rate
  - r_L = learning rate

Model Notes

- Hypothesized scaffolding impact
  - Very reasonable, but other reasonable ones exist
- Original model added constraints to implementation
  - Regimes of applicability; will be addressed in extended approach
- Implementation is trickier with change in levels on both sides of the equation
  - Update L, then update H. This is as it should be – helpers must take their lead from the learners - but sometimes it leads to difficulties with analysis.

Modified Model

- Rewrite vGS

$$L_{n+1} = L_n + r_L \cdot L_n \cdot (1 - L_n/H_n) \cdot f(H_n, L_n)$$

$$H_{n+1} = H_n + r_H \cdot H_n \cdot (P_n - L_n) \cdot (1 - H_n/G)$$

$$P_{n+1} = L_n$$

- Notes:
  - P_n is the learner's previous level
  - vGS uses $f(H_n,L_n) = (D - H_n/L_n)$
    - Others could be used (and were investigated)
  - Nothing has changed except the format, to bring it in line with what is more commonly used

Modified Model

- Changes from a delay-discrete dynamical system (about which no one knows seems to know anything) to a discrete dynamical systems (about which some people know a very little bit)
  - Analysis is less difficult
- Makes sense
  - ZPD is about movement, so teacher needs to see that before acting
- However, it introduces the pre-P problem
  - Must choose the learner's level at the initial time and a previous time, and this choice can cause problems with the numerical modeling
- Now a three dimensional problem
  - Really, it is just on a different surface, so can use projection to L-H plane
  - P is simply L lagged by one time step

Scaffolding

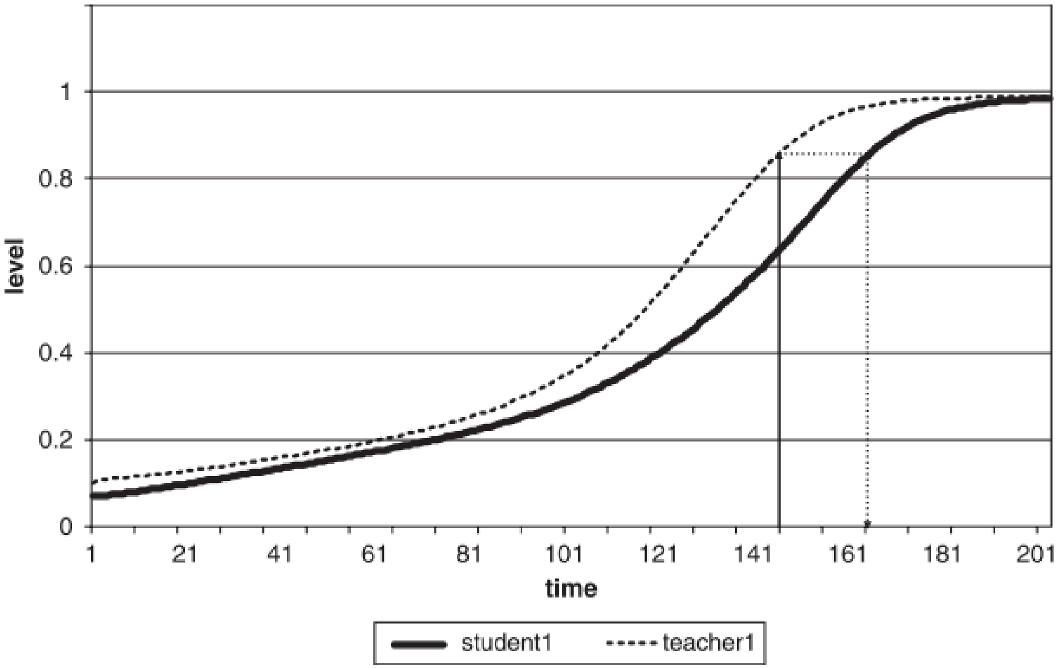
- Wood, Ross, & Bruner (1976)
- Still not clearly understood or conceptualized (van de Pol, Volman, & Beishuizen, 2010)
  - 3 intentions of scaffolding (“what is scaffolded”)
  - 6 means of implementing scaffolding (“how scaffolding is implemented”)
  - That’s 18 possible approaches - Too much!
- Nothing seems to be known with any precision…
  - This analysis can perhaps refine the model by making clearer what is being done by the scaffolder and why

Why This Model?

- For a variety of reasons, wanted to develop a classroom simulation that, to teachers, looked like a classroom.
  - Found that we agree with Opfer (2013, 2014): interactions not well enough understood to do agent-based modeling of this
    - Perhaps only “weirdoes” try to do this?
  - Carlsson (2007) – natural scales don’t exist (D, G, f, r_L, r_H)
- Didn’t have a better model, so worked with this one
  - Good models are hard to find
  - This is not necessarily a criticism of this model, just another step from it
    - Our goal for using the model was a bit different than the original authors

Time Series

- vGS gives examples



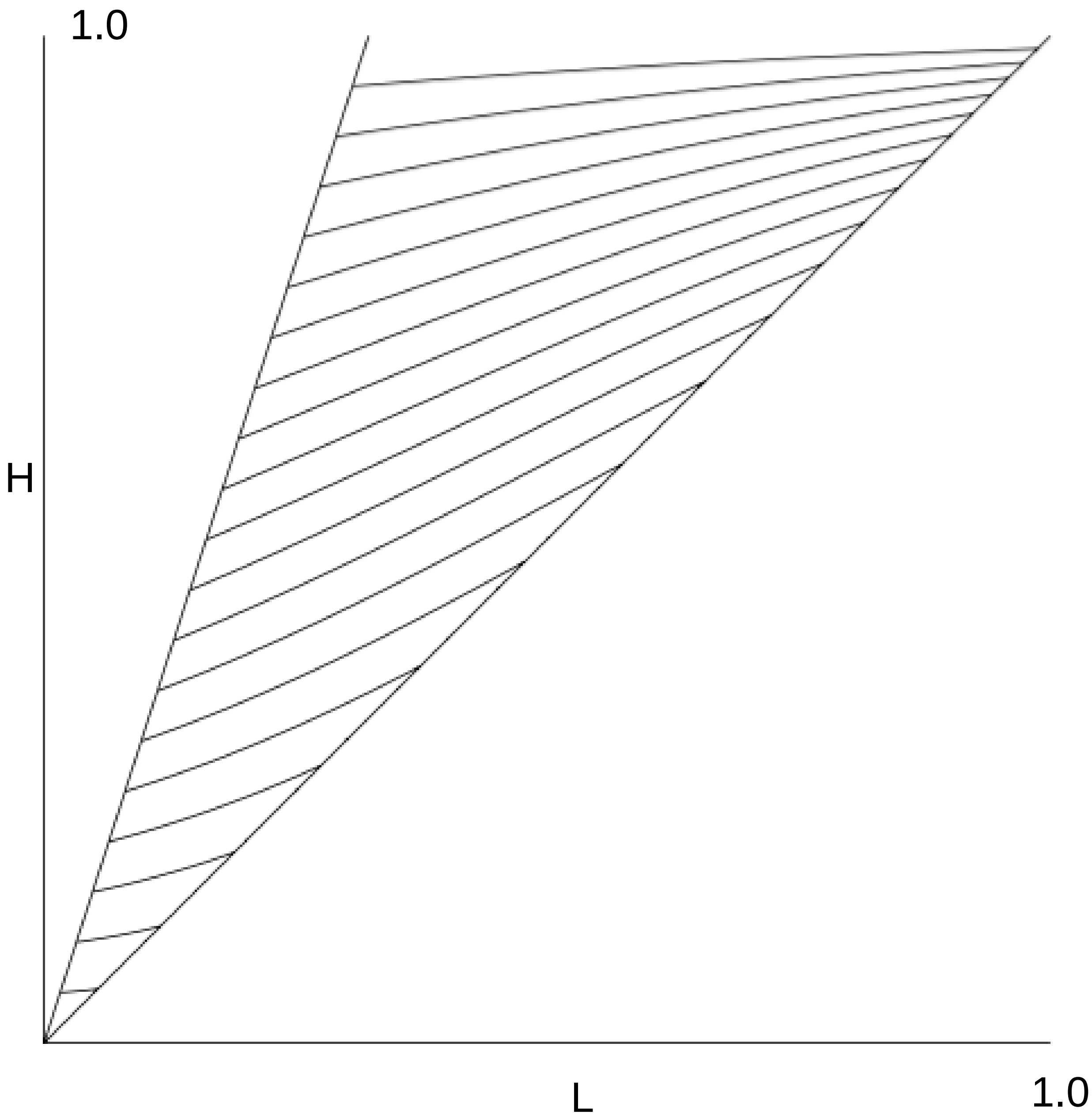
- Can generate more via Excel spreadsheet
- Problem with time series catalogues: Don’t know if all behavior is represented

Better (for us) Analyses

- State Space
  - State variables
    - Here, L & H are the state variables
  - Plot these variables on orthogonal axes
- What is important about this?
  - Fixed points
  - Basins of attraction
  - See time series qualitatively as exemplars of behavior
  - Easier to extend to ensembles (e.g., a classroom of students)

Example

- Several paths, all starting near the left fence (optimal level of scaffolding!)
- $D = 3.1$, $r_H = 2$, $r_L = 0.1$
- Few students get near the goal; the teacher doesn't adjust quickly enough



Notes

- The various parameters (D, G, r_L, and r_H) in the model, as well as the time step, are arbitrary. Further, because there are more of them than there are equations, we can set at least one of them arbitrarily.
- To simplify analysis, we set G = 1

Fixed Points

- No further changes in the behavior. Mathematically:
  - $L_{n+1} = L_n$ and $H_{n+1} = H_n$
  - $P_{n+1} = P_n$ one time step after L stabilizes
- Related to the Markov approach later taken by van Geert, Steenbeek, and colleagues
- In the vGS case, we can solve the system of equations analytically:

$$L_n = P_n = H_n = x$$

$$L_n = P_n = t;\ H_n = DL_n = Dt$$

  - This produces two lines (“fences”) in the H-L plane: a left one with a slope 1/D, and a right one with slope 1
- Fixed points in continuous systems are well understood; discrete systems and fixed lines not so much

Stability of Fixed Points

- Simple dynamical systems: Fixed points can be stable, unstable, neither, or a combination; the resulting properties of fixed points can be used to divide a plane into regions which include, for stable fixed points, *basins of attraction*.
- The fences have regions of stability (where they attract nearby points) and instability (where they repel nearby points)
  - The structure of these regions depends upon the various parameters.
    - Example, for $0 < r_L(D-1) < 1$, and $0 < r_H < 4$, the left fence is entirely unstable and the right fence is entirely stable
- Such things are insufficient to give us the entire picture

Basins of Attraction

- For several values of the parameters, the regions of the fences implies that the domain of applicability (between the fences) is also divided into multiple regions, known as *basins of attraction*.
- Analytic determination of these is a very difficult problem, however, and not solvable in general.
- Hence, we turn to numerical simulations:
  - Start at an arbitrary point
  - Trace the path until it no longer changes (fixed point) or reaches the goal or moves outside domain of applicability
  - Mark the starting point depending upon the ending point
  - Repeat for more points (try for a fine mesh of points)

Characteristic Basin Maps

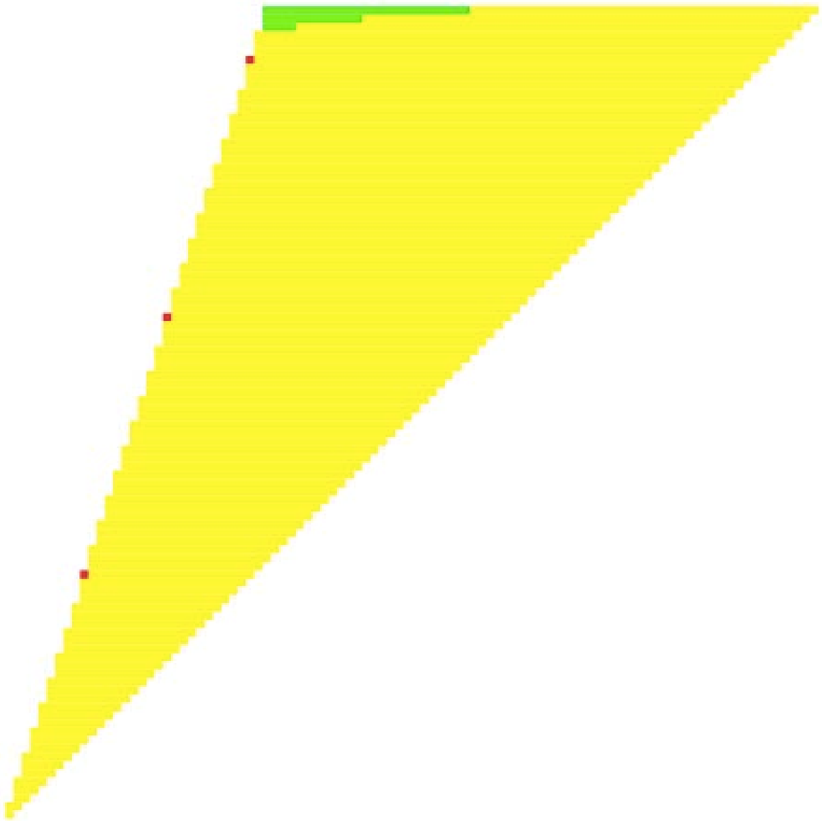
- Based on the analysis of the lines of fixed points, it was indicated that there are 7 qualitatively different maps.
- Regions depend upon only on $r_L(D-1)$ and r_H
  - Why these two quantities, I don’t know…

Characteristic Basin Maps

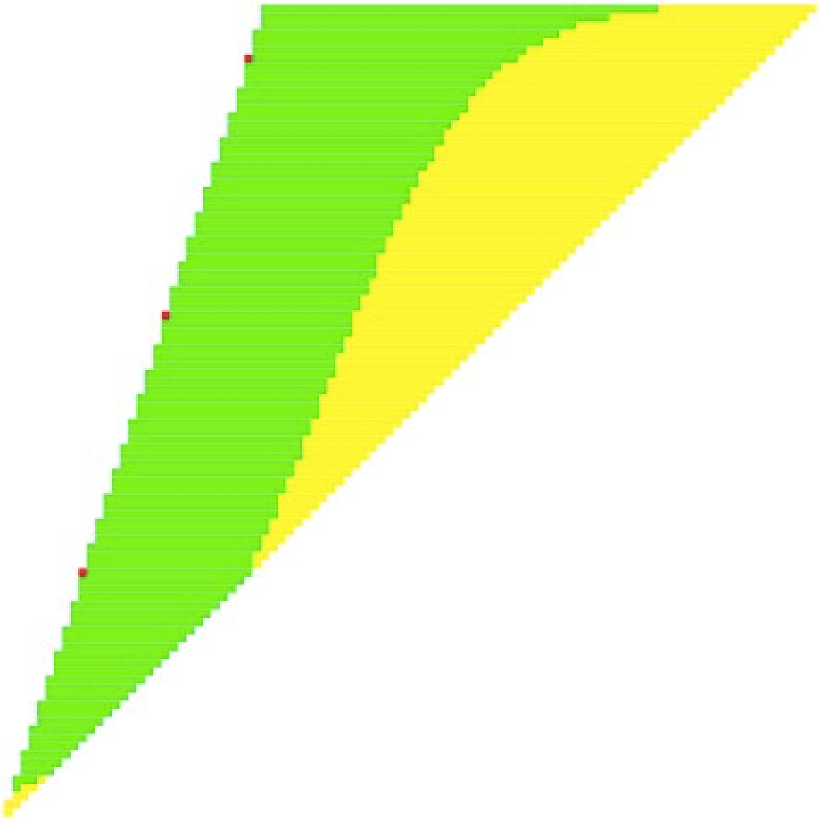
- Briefly will look at each of these; we can look at many more, but these capture the important characteristics of the model.
- Will show *heat maps* where the color indicates the ending position of a starting point:
  - Green – reach the goal (L = 1)
  - Yellow – reach the right fence, but not the goal
  - Orange – goes to the left *fence*
  - Red – goes somewhere else
- Man details aren't important for what follows, so we'll look only briefly at a few of them to get a sense of how to use these to draw conclusions

Increase r_H (D=3.1, r_L=0.3)

$r_H = 5$

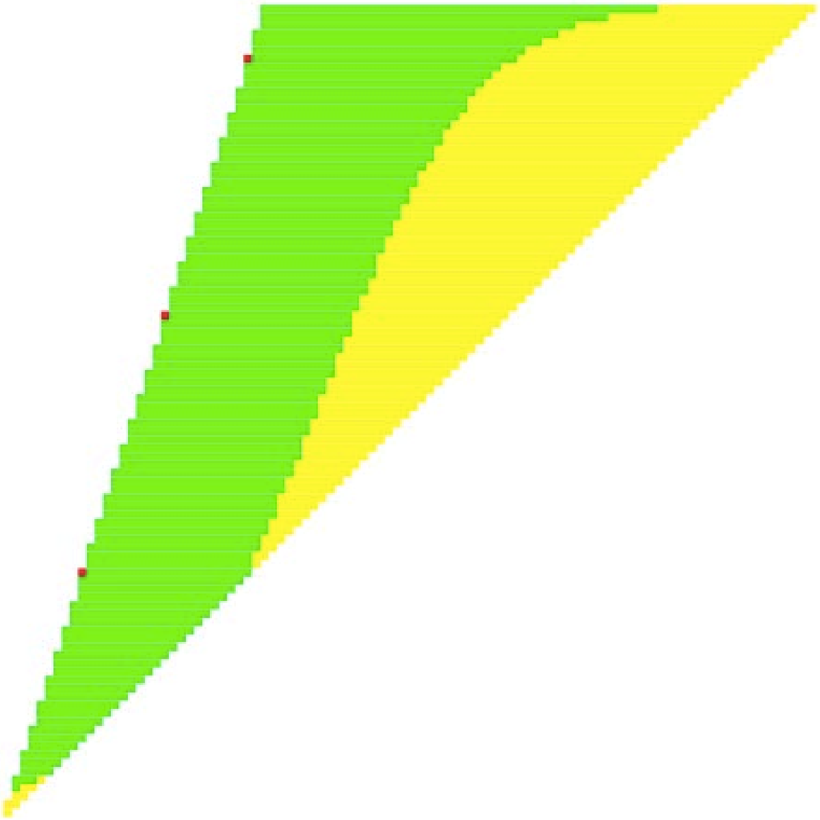


$r_H = 10$

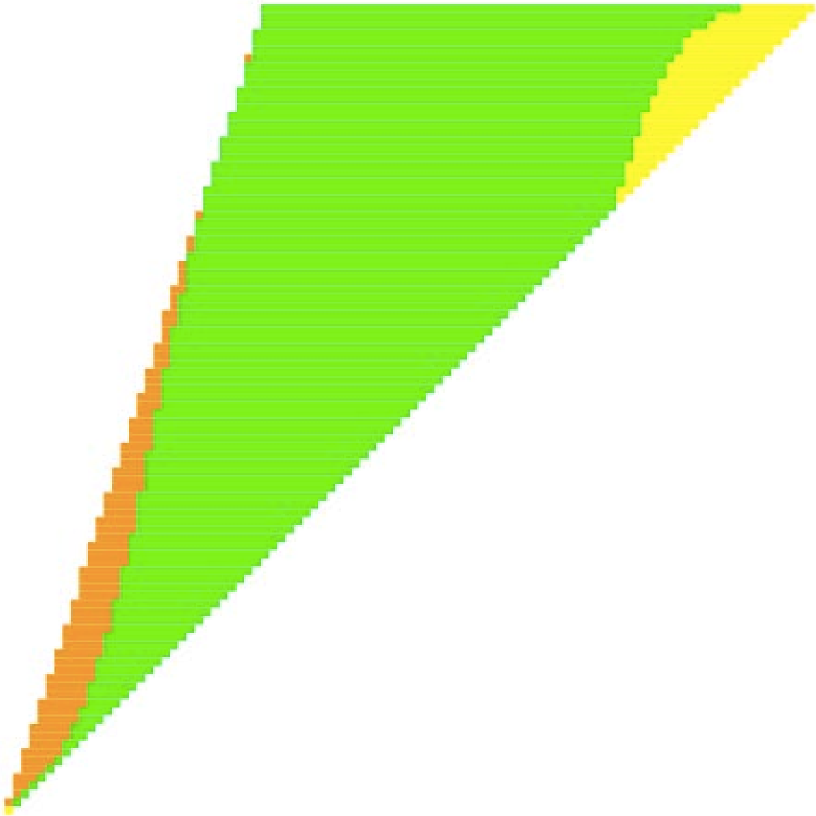


Increase r_H too much ($D=3.1$, $r_L=0.3$)

$r_H = 10$



$r_H = 20$



Implications: Teaching

- Four types of classrooms
  I. Slow teacher (r_H too low)
  II. Desirable (almost all students reach goal)
  III. Overshooting (teacher goes up and back)
  IV. Fast learners (outside domain)
- Consider a distribution (horizontal line) of learners
  - Do all students get what they need?
- Get teachers to recognize they may need multiple dynamics
  - Optimal ratio of r_H/r_L depends (somewhat sensitively) upon D

Implications: Research

- No natural scale
  - What do we measure?
    - Educational research isn't used to this type of measurement, whatever it is
    - Probably not a single number anyway
- Learning Progressions & Trajectories
  - Issues with LP/LT
    - Not just one, but many
    - May remove wrong scaffolding
  - Affordances of LP/LT
    - Gathering data: May allow for tuning of model (but see scale above)
    - May give insight into D, as that quantity is important

Implications: Modeling

- Always the danger that what is seen is built into the model rather than a result of the model. Such artifacts include:
  - The initial value of P
  - Crossing behavior: Is it real, or not?
  - Some left fence behaviors
- Can only push so far with any model
  - Care must be taken with over-tuning parameters
  - But the model indicates what we need more information about

Conclusions

- State space allows for additional analysis/insight
- Moves from single student to ensemble
- Potentially provides additional guidance for teachers and researchers

- Questions?

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Complicity

- *Complicity: An International Journal of Complexity and Education*
- http://bit.ly/ComplexityEducation
  - You should read it!
  - Matt & Dimitrios to develop a special issue from today's sessions
  - I would be happy to discuss with any of you about publication
    - Yeah, I'm the Editor-in-Chief, so this is partially self-promotion…
    - …but probably I'm also the person who can best assist you