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What's in a Name? The Matrix as an Introduction to Mathematics

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What's in a Name? The Matrix as an Introduction to Mathematics

Abstract

In lieu of an abstract, here is the article's first paragraph:

In my classes on the nature of scientific thought, I have often used the movie *The Matrix* to illustrate the nature of evidence and how it shapes the reality we perceive (or think we perceive). As a mathematician, I usually field questions related to the movie whenever the subject of linear algebra arises, since this field is the study of matrices and their properties. So it is natural to ask, why does the movie title reference a mathematical object?

Disciplines

Mathematics

Comments

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What's in a name? *The Matrix* as an introduction to mathematics

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In my classes on the nature of scientific thought, I have often used the movie *The Matrix* to illustrate the nature of evidence and how it shapes the reality we perceive (or think we perceive). As a mathematician, I usually field questions related to the movie whenever the subject of linear algebra arises, since this field is the study of matrices and their properties. So it is natural to ask, why does the movie title reference a mathematical object?

Of course, there are many possible explanations for this, each of which probably contributed a little to the naming decision. First off, it sounds cool and mysterious. That much is clear, and it may be that this reason is the most heavily weighted of them all. A quick look at the definitions of the word reveals deeper meaning for movie's title. Consider the following definitions related to different fields of study taken from Wikipedia on April 23, 2006:

- Matrix (mathematics), a rectangular array of scalars
- Matrix (biology), the material between animal or plant cells or the material in the inner membrane of a mitochondria
- Matrix (geology), the fine grains between larger grains in igneous or sedimentary rocks
- Matrix (chemistry), a continuous solid phase in which particles (atoms, molecules, ions, etc.) are embedded

All of these point to an essential commonality – a matrix is an underlying structure in which other objects are embedded. A mathematical matrix has scalar quantities (basically numbers) embedded in it. A biological matrix has cell components embedded in it. A geological matrix has grains of rock embedded in it. And so on. So, a second reason for the cool name is that we are talking, in the movie, about a computer system generating a virtual reality in which human beings are embedded (literally, since they are lying down in pods). Thus, the computer program forms a literal matrix.

However, there are other explanations for the naming of the movie that operate on a higher level and are explicitly relevant to the mathematical definition of a matrix as well as the events in the trilogy of movies. These are related to computer graphics, Markov chains, and network theory and will each be explored in turn.

Formally in mathematics, a *matrix* is an arrangement of information, usually numerical, into rows and columns. Such arrangements can be added to each other, multiplied, and inverted. They have many special properties that help characterize the information contained in the matrix, and they relate to many different physical and social phenomena. One of their most common applications today is in computer graphics, and the reason is very simple. To represent an object on a computer screen (or a movie screen, or on the mind of an enslaved human embedded in *the matrix*) we define the coordinates of each point on the object. However, in the case of simulating a three-dimensional object, different viewpoints will result in the object taking on different appearances in terms of shape, lighting, texture and so forth. A matrix provides the necessary tool for determining how the shape of an object varies with different viewpoints.

The essential idea can be understood from a two-dimensional problem. Suppose we have a triangle with its vertices (labeled A, B, and C) at the coordinates (-1, -2), (0, 6) and (3, 4) respectively and we wish to know the coordinates of the shape after it is rotated 30 degrees around the origin (0, 0). If we write each point on the triangle as a vertical column of numbers – a matrix with two rows and one column – we can perform a matrix calculation that outputs the coordinates of the point after the rotation. Take point C. As a matrix, point C looks like

$$C = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

To compute the new coordinates of point C (let's label these new coordinates C') we multiply by a rotation matrix, R_θ . In the notation below, θ represents the angle through which the object is rotated, in this case, 30 degrees.

$$R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} = \begin{pmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{pmatrix}$$

Multiplying this matrix by the coordinates of point C give the coordinates of C'.

$$C' = R_\theta C = \begin{pmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0.866(3) - 0.5(4) \\ 0.5(3) + 0.866(4) \end{pmatrix} = \begin{pmatrix} 0.598 \\ 4.964 \end{pmatrix}$$

If we proceed through each point of the object, the new coordinates can be determined and the object redrawn to appear as it would from another perspective. This is surely an important function of the computer program in the movie, since it must continually construct a shared reality among many different viewpoints and allow each to perceive the virtual world correctly as they interact with it.

But it is the use of matrices to represent an object called a Markov Chain that allows us to see how the *Matrix* trilogy uses the idea of a mathematical matrix in order to deal with the ages-old debate of freewill vs. predestination. In other words, the mathematical concept of a matrix can help us understand how, in a computer-generated world such as depicted in the *Matrix* trilogy, a human mind – or any “mind” – can possibly know the future.

The key lies in thinking about the future as a series of events, each with its own probability of occurring. But it goes even further. If you think about an event that occurs as defining the current “state” of the future, the chance of transitioning into a different state may be different depending on your starting state. For example, if you go to use a photocopier at the office, it will be in one of two states, working (W) or broken (B). Now, past experience might tell you that when you need to copy something and have no prior information, you expect the copier to be in state W about 80% of the time and state B 20% of the time. But, once it is in state B, you are in a new state. The chance of a broken copier remaining broken the next day might be 55%, while the chance that it is really fixed (and thus back to state W) is 45%. Thus, the future is not known, because we must have the specific sequence of events leading up to the present and their

probabilities in order to compute the likelihoods of future events. This chain of events determined by transition probabilities is called a Markov Chain, and the transition probabilities from state number i to state number j can be written as the elements of a matrix, the transition matrix T . By raising T to the n^{th} power we can study the likelihood of the system (in this case the copier) being in a particular state after n time steps.

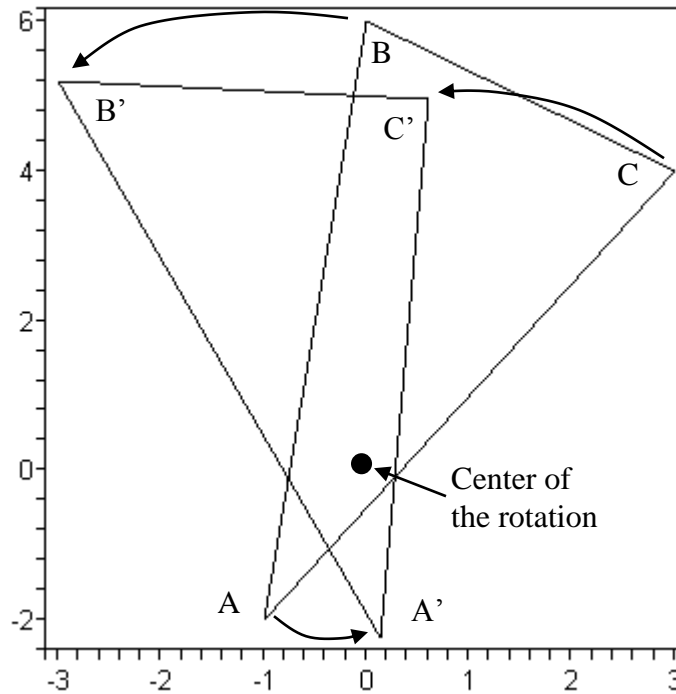


Figure 1. The original triangle ABC and the rotated triangle A'B'C'.

Thus, the use of a matrix in a Markov Chain process can help us determine the probabilities for future events, but it cannot tell us for certain (a) how we get there (the chain of states we pass through) or (b) in which state we will definitely be. This is physically illustrated in the final scenes of the second matrix movie (*The Matrix Reloaded*) as Neo sits in front of the Architect discussing the reality of the matrix. He is surrounded by hundreds of television screens, each of which depicts different possible reactions Neo might have to the information he is receiving. As Neo starts to make a decision and act, more and more of the screens depict the same resulting action, neatly illustrating the increased probability of transitioning from his current state to a particular future action.

This idea of the Markov Chain also explains the Oracle's insistence that Neo already knows the future and has made his choice but does not understand it. Essentially, this means that the set of states and the transition probabilities he has been using to make decisions is about to come to an end. Until he understands the radical events surrounding him and until he constructs a new mental picture of the possible future states, he cannot predict future events, since his list of current states and transitions is not up to the task. The copier example illustrates this further. Suppose after a week, we have gone through a succession of states like W-W-B-W-W-B-X and arrive at a new state X that means the copier is broken and not able to be fixed. This new state

was not part of our original list of states. It is not accounted for in our transition matrix, thus we could never have predicted this outcome. Until we revise our set of states from {W, B} to {W, B, X} we can never represent the chain of possible futures. Philosophically, this means that our assumptions about the copier were wrong; we assumed that it could always be fixed. Neo's assumptions about the future fall apart after Trinity's near death experience as a result of the new information he has received. Until he processes this and understands it, his mind cannot make further predictions about the future.

A final connection between the movies and linear algebra comes from the emerging science of network theory. This is the study of how interconnected agents (in this case, we are referring to the people trapped in the matrix, not the Agents in the movie) share and pass information along through the network. One can represent the interconnections with a matrix and by exploring the properties of the matrix come to understand how various phenomena (usually called "epiphenomena" or "emergent phenomena") come about purely as a result of these interactions. A typical example is the way a field of crickets will initially chirp independently, but as a result of the feedback from hearing each other, will quickly synchronize their chirps. The emergent phenomena – synchronized chirping – is not the result of a central dictatorial cricket demanding harmony, but rather an emergent property of the way each individual cricket responds to its surroundings. For more details on this field of study – albeit without the technical features – one should consult Duncan Watts' volume *Six Degrees* or *Nexus* by Mark Buchanan.

Basically, in a network model, one can represent the connections as a graph or in the form of one of several different matrices, one of which is called the "adjacency matrix." Visualizing a matrix as a two-dimensional array of numbers, the adjacency matrix involves simply listing all the individual components down the left side of the matrix and above the top row of the matrix. If two components of the network are connected, then there is a 1 in the corresponding entry of the matrix. If they are not connected, there is a 0 in that entry. Using this approach, one-way connections can also be represented. For example, if our network of interest is a food web, the individual components (agents, nodes, or whatever term is your favorite) are the types of organisms in the food web. The connections might represent who eats whom. Thus, in the matrix entry representing the interaction between the row containing a shark and the column containing a small fish, there would be a 1. However, since the fish does not prey upon the shark, the entry going the other direction, from the row containing the fish to the column containing the shark, would contain a 0. Such matrices lack the property of symmetry, which implies that $T_{ij} \neq T_{ji}$ where T_{ij} indicates the element in row i and column j of the matrix T . In this way, one can study various properties of the food web by computing quantities related to the adjacency matrix representing this web.

A critical component of making the network aspect function, though, is that each agent in the system (i.e. the people in the pods) can influence each other and cause real changes in their behavior. This is illustrated at several points in the trilogy. One of the most graphic illustrations of this is the use of food. The Merovingian claims to have written a program – in the form of a piece of cake – which influences a young woman's behavior. The Oracle gives Neo a cookie she baked herself, claiming that he will feel "right as rain" when he finishes it. And at their second meeting, Neo comes away full of candy. After each of these food-related encounters, Neo undergoes a significant change. Without this ability to explicitly affect each other, information

could not pass through the connected network of individuals, as it would exist only within the computer-generated construct and not within the individual minds connected to the system.

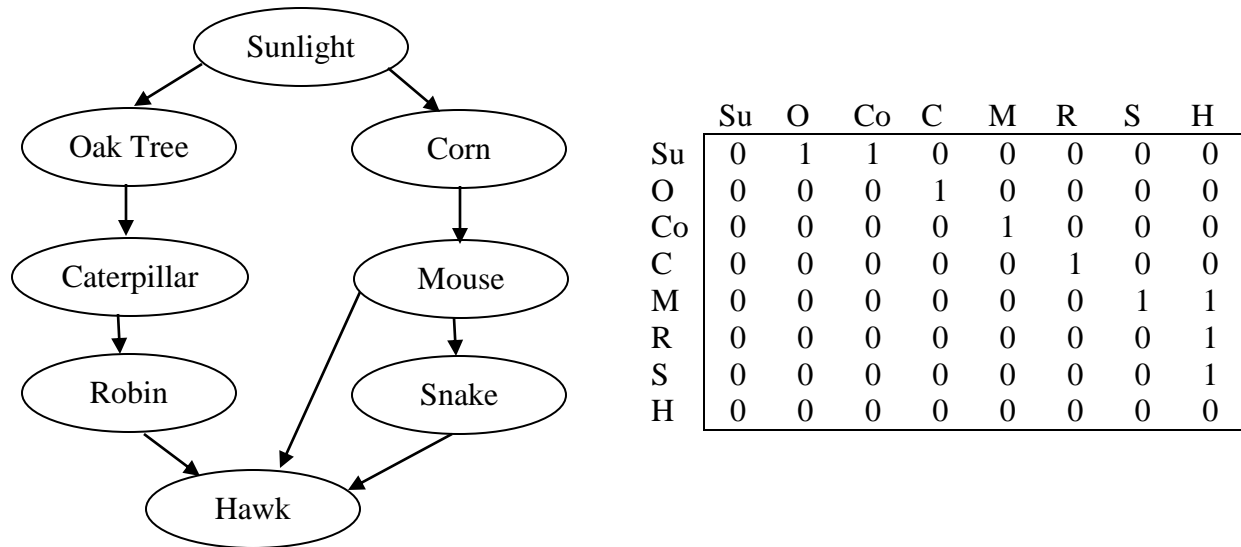


Figure 2. A simple ecosystem and its adjacency matrix. Prey/resources are located at the tails of the arrows and predators/consumers are located at the heads of arrows. Each organism is represented in the matrix by its first letter, except for Sunlight (Su) and Corn (Co). Ecosystem taken from fourth grade TIMSS science test.

On a related note, the movies take this to the next logical step, illustrated in the way the newly released Agent Smith is able to write himself over other programs in the system. Going further, the Wachowski brothers have Smith copy himself into the brain of an individual who has been freed from the system, thereby releasing himself into the real world. Although this may seem impossible, it is merely an extension of the mechanisms already at work in the computer system. For example, the rebels can easily download information from cartridges into a person’s mind in order to teach them Kung Fu or how to fly a helicopter. This surely involves changes in the real brain of the individual, for they must remember, at the very least, that they have this knowledge in the computer world in order to make use of it the next time they “plug in.” Thus, programming in the computer world can have effects in the real world. Without these reciprocal effects, information – about people and places and abilities – would never pass through the system of interconnected people, and the rebellion would be doomed to failure.

In the end, we are left with not only popular reasons for the movie title, but also mathematical reasons. Without matrices, the computer graphics used to project the images into the minds of the embedded people would not be possible. Matrices also form a vital tool for analyzing the possible paths of future events through the use of Markov chains. And matrices are common tools for representing the connections in a network or agents that share information.