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M. A. Fisher
University of Rochester

Mark F. Bocko
University of Rochester

Linda E. Marchese
University of Rochester

Guizhen Zhang
University of Rochester

Munawar Karim
Saint John Fisher University, mkarim@sjf.edu

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Radio-Frequency Superconducting Parametric Transducer for Gravitational-Wave Antennae

Abstract

We report on the design and testing of an ultrasensitive, electromechanical transducer for use on resonant mass gravitational wave antennae. The transducer is a superconducting, radio frequency resonant bridge circuit operating near 200 MHz. We have minimized several important sources of noise in this transducer system. The Johnson noise of the transducer circuit is reduced through using a superconducting niobium stripline circuit and low-loss insulating materials. At a temperature of 4.2 K we have achieved unloaded electrical quality factors of 200 000. The bridge circuit is balanced by piezoelectric actuators which control the spacing between the proof mass and capacitive segments of the stripline circuit and we have achieved a residual bridge imbalance of 3×10^{-7} . Finally, low noise cryogenically cooled field-effect transistors are used for the first amplifier stage, enabling us to obtain an amplifier noise level which is 5400 times the quantum limit. The transducer, which has a 0.080 kg proof mass, was affixed to the end of a prototype, resonant bar, gravitational wave antenna with a mass of approximately 100 kg. The primary purpose of this small antenna was to evaluate the transducer, which is designed to be mounted on a much more massive antenna. Our theoretical analysis and measurements of the detector agree and indicate a burst noise temperature of 1.8 K using the 100 kg bar. This corresponds to a gravity wave burst sensitivity of $h = 1.1 \times 10^{-16}$, in terms of relative strain amplitude. With no other improvements, if the transducer mechanically resonant frequency were tuned to and installed on a 2000 kg antenna, the antenna would reach a noise temperature of 1.3 mK, which is equivalent to a gravitational wave burst sensitivity, $h \approx 5.7 \times 10^{-19}$.

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Radio frequency superconducting parametric transducer for gravitational wave antennae

M. A. Fisher,^{a)} M. F. Bocko, L. E. Marchese, and G. Zhang
Department of Electrical Engineering, University of Rochester, Rochester, New York 14627

M. Karim
Department of Physics, St. John Fisher College, Rochester, New York 14618

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We report on the design and testing of an ultrasensitive, electromechanical transducer for use on resonant mass gravitational wave antennae. The transducer is a superconducting, radio frequency resonant bridge circuit operating near 200 MHz. We have minimized several important sources of noise in this transducer system. The Johnson noise of the transducer circuit is reduced through using a superconducting niobium stripline circuit and low-loss insulating materials. At a temperature of 4.2 K we have achieved unloaded electrical quality factors of 200 000. The bridge circuit is balanced by piezoelectric actuators which control the spacing between the proof mass and capacitive segments of the stripline circuit and we have achieved a residual bridge imbalance of 3×10^{-7} . Finally, low noise cryogenically cooled field-effect transistors are used for the first amplifier stage, enabling us to obtain an amplifier noise level which is 5400 times the quantum limit. The transducer, which has a 0.080 kg proof mass, was affixed to the end of a prototype, resonant bar, gravitational wave antenna with a mass of approximately 100 kg. The primary purpose of this small antenna was to evaluate the transducer, which is designed to be mounted on a much more massive antenna. Our theoretical analysis and measurements of the detector agree and indicate a burst noise temperature of 1.8 K using the 100 kg bar. This corresponds to a gravity wave burst sensitivity of $h = 1.1 \times 10^{-16}$, in terms of relative strain amplitude. With no other improvements, if the transducer mechanically resonant frequency were tuned to and installed on a 2000 kg antenna, the antenna would reach a noise temperature of 1.3 mK, which is equivalent to a gravitational wave burst sensitivity, $h \approx 5.7 \times 10^{-19}$. © 1995 American Institute of Physics.

I. INTRODUCTION

Gravitational wave antennae will furnish astronomers with unique observational data that will allow the physics and astronomy of strong dynamic gravitational systems to be explored. Stellar core collapse, binary star coalescence, collisions of compact stars, and the excitation of black hole and neutron star vibrations have been predicted to generate gravitational waves strong enough to be observable with terrestrial antennae.¹ Detailed observations of these sources will yield insights into highly nonlinear gravitational field dynamics, the properties of nuclear matter, the structure and birthrate of compact objects such as black holes and neutron stars, and the size of the universe and its large-scale dynamics.

For three decades, efforts to detect the gravitational radiation bursts caused by such cataclysmic, astrophysical events have persevered. The first detectors proposed were resonant mass antennae, invented in the 1960s by Weber.² Research groups around the world have worked to refine both the resonant mass gravitational wave antennae scheme and the almost-free-mass, laser interferometer, gravitational wave antennae. The most sensitive gravitational wave antenna operated to date is at Louisiana State University and consists of a 2300 kg aluminum cylinder cooled to 4 K and instrumented with an inductive transducer and a SQUID (superconducting quantum interference device) amplifier.³ It has

an impulse signal strain sensitivity of $h \approx 6 \times 10^{-19}$ which may be sufficient to detect infrequent gravitational wave bursts from our own galaxy. However, more frequent detection of gravitational wave events will require the construction of more sensitive antennae in order to monitor an effectively larger volume of space. Efforts to increase the sensitivity of resonant mass antennae currently emphasize the construction of massive, omnidirectional, spherical antennae, operating at much lower temperatures, and improving the transducers used to instrument the antennae.

A resonant mass antenna absorbs minuscule amounts of energy from a passing gravitational wave; therefore ultrasensitive displacement transducers are needed to measure the correspondingly weak excitation levels of the antenna normal modes. The sensitivity of a resonant mass detector is limited largely by the noise of the displacement transducer used to monitor the antenna so considerable effort has gone into transducer development. In this article we describe our efforts to develop one specific type of displacement transducer, the so-called "parametric bridge" transducer. The parametric transducer concept has been explored and actively developed by several groups and, although specific implementations may differ in the details, the principle of operation is the same. In all parametric transducers an oscillating electromagnetic source, usually in the radio-frequency or microwave band, excites an electrical circuit, a reactive parameter of which is modulated by the motion of a resonant proof mass coupled mechanically to the antenna. The elec-

^{a)}Current address; Naval Research Laboratory, Washington, DC 20375.

FIG. 1. Mechanical model of a resonant bar gravitational wave antenna with a torsional mode transducer. The transducer mass m_2 is supported by a torsion spring located a distance r from its center of mass (c.m.) so that the x motion of m_1 is converted into rotational motion of the transducer mass.

tromagnetic energy in the transducer is modulated by the motion of the resonant proof mass, generating sidebands on the pump signal at the sum and difference of the pump frequency and the proof mass resonant frequency.

Parametric transducers have a number of apparent advantages over other transduction schemes such as the inductive transducer, although at the cost of increased complexity. One of these advantages stems from the fact that the pump frequency is typically much higher than the mechanical proof mass frequency, consequently reducing the effect on the antenna of thermal fluctuations in the transducer circuit, as described in the Manley-Rowe relations.⁴ Thus, parametric transducers relax the stringent requirements on the electrical dissipation in the transducer circuit. Another potential advantage is that parametric transducers may be operated in a back action evasion mode, in which the noise of the amplifier used to sense the transducer circuit may be kept from acting back on the mechanical proof mass. This, in effect, isolates the antenna from the back-acting part of the transducer noise and improves the overall antenna sensitivity. One

FIG. 3. Lumped parameter circuit of the bridge circuit transducer shown in Fig. 2 which includes the coupling to the "pump" source, V_g but excluding the resistive balance controls.

of the drawbacks of the parametric transducer arises from the phase and amplitude fluctuations of the pump. Our solution to this problem is to employ a balanced bridge circuit in which the pump signal, and its noise, is canceled to a large extent at the transducer output.

In this article we discuss several of the key technical issues for parametric transducers, our solutions to the apparent problems presented by this transducer, and our experimental results for the sensitivity of a prototype gravitational wave antenna instrumented with a parametric transducer.

II. MODELING OF THE ANTENNA AND ITS THEORETICAL SENSITIVITY

A. General formalism

In this section we describe the electromechanical model of a gravitational wave antenna instrumented with the parametric transducer, outline our sensitivity analysis, and present the theoretical predictions for the sensitivity of the present and future antenna-transducer configurations. We begin this section with a description of the antenna and transducer.

The prototype antenna is a dumbbell-shaped bar approximately 0.9 m in length with a support flange at its geometric midpoint. This antenna is intended to be a test-bed for resonant transducers designed for use on more massive gravitational wave antennae. Therefore, the dumbbell design was chosen to obtain a resonant frequency around 1 kHz, which is near the frequency of full-size resonant gravitational wave antennae. We model the dumbbell antenna as a mass, m_1 , on a spring; the value of m_1 is approximately equal to the total mass of the antenna, neglecting the mass of the springs, and the spring constant is chosen to agree with the observed antenna resonant frequency. Attached to the bar is a torsional mechanical resonator of mass m_2 , which is able to rotate about an axis perpendicular to the axis of the bar; see Fig. 1. We employed the torsional resonator design because it allowed a planar readout circuit design in which the entire electrical readout circuit is mounted on one side of the transducer resonant mass, which greatly simplifies the transducer design and assembly. Other advantages, such as greater antenna bandwidth of the torsional transducer over common, translational, spring-mass resonant transducers, are discussed in another article from our group.⁵ The resonant frequency of

FIG. 2. The physical design of the radio frequency bridge circuit transducer. The three rectangular capacitor plates are held a small distance from the surface of the transducer mass, the torsional motion of which serves to unbalance the bridge circuit.

the torsional transducer was closely tuned to the bar frequency, which is 973 Hz. Material was removed asymmetrically from the transducer mass so that the torsion spring is offset a distance r , from the transducer center of mass (c.m.) in order to couple the translational motion of the bar's end face to the torsional motion of the transducer.

The electrical readout circuit which converts motion of the transducer into an electrical signal is a radio frequency, resonant bridge. The circuit is composed of three capacitor plates and three inductive stripline segments, as shown in Fig. 2. The three capacitor plates are held a small distance from the surface of the transducer mass so that three capacitors, which share the transducer mass as a common plate, are formed. The circuit consists of two meshes so there are two electrically resonant modes. The values of inductive stripline segments and capacitance are such that the two modes lie in the 150–250 MHz frequency range, depending upon the size of the capacitor gaps. We call the electrical mode in which current flows predominantly around the outer loop of the bridge the pump mode. The other mode, in which current flows through the center branch and splits to return through the two outer branches, is called the idler mode.

The bridge circuit is operated in the following way: an oscillator tuned to the pump mode resonance provides a current to the stripline transformer, which induces a current in the outer mesh of the circuit. When the gaps of the two, outer capacitors are the same, the bridge circuit is balanced and no current flows through the center branch. If the torsional resonator is rotated slightly, the gap of one of the outer capacitors increases and the other gap decreases, causing a corresponding change in the capacitance values. This unbalances the bridge circuit, forcing some of the pump mode current to be diverted through the center branch of the circuit and thus exciting the idler mode. The idler mode current is sensed by a high input-impedance amplifier connected across a portion of the bridge circuit's central inductor. The output of this amplifier provides a measure of the state of vibration of the transducer and thereby that of the antenna.

To facilitate analysis we represent the radio frequency bridge by a lumped element equivalent circuit, see Fig. 3. In this model we represent losses in the readout circuit by including resistors in the three branches of the bridge circuit. We also include a noise source in the center branch of the circuit to represent the thermal Johnson noise from the idler mode losses plus the back-acting input noise from the high input-impedance amplifier. The upper and lower branches of the bridge circuit are each composed of an inductor, resistor, and capacitor, (L_1, R_1, C_1) and (L_2, R_2, C_2) , respectively. The charge on C_1 is q_1 and on C_2 it is q_2 . We do not include Johnson noise from losses in the outer inductive segments of the bridge in the equations because this contribution would be very small in comparison to the central arm loss contribution when the bridge circuit is nearly balanced.

The parametric transducer is a reciprocal, electromechanical converter, so not only does the motion of the transducer produce a signal in the electrical circuit, but the electrical excitation of the idler mode also produces a back-acting force on the mechanical element. If the torsional resonator is rotated, the outer capacitors become unequal in

TABLE I. Definition of symbols used in Eqs. (1).

Symbol	Definition
q	Idler mode charge: $q = q_1 - q_2$
L	Bridge central arm inductance
L_a	Average outer bridge arm inductance $L_a = (L_1 + L_2)/2$
M_{12}	Mutual inductance between outer arm inductors
R	Central bridge arm resistance
R_a	Average outer bridge arm resistance $R_a = (R_1 + R_2)/2$
C	Central bridge arm capacitance
C_a	Average outer bridge arm capacitance $C_a = (C_1 + C_2)/2$
ν_n	Aggregate voltage noise driving idler mode
q_p	Pump mode charge: $q = (q_1 + q_2)/2$
κ	Mutual inductance imbalance $= 2(K_{2p} - K_{1p})/(K_{2p} + K_{1p})$
δ	Inductive imbalance $(L_2 - L_1)/L_a$
ρ	Resistive imbalance $(R_2 - R_1)/R_a$
ϵ	Capacitive imbalance $(C_2 - C_1)/C_a$
C_{vac}	Nonparasitic part of C_a , i.e., not from stray substrate cap.
r_c	Distance from torsion spring to center of outer cap. plates
d_0	Nominal gap of capacitors
θ	Angular displacement of transducer
m_1	Effective mass of antenna
m_2	Mass of transducer mechanical element
x	Displacement of bar end face
h_1	Translational damping constant of antenna
k_1	Effective translation spring constant of antenna
r	Distance from torsion spring to transducer center of mass
$F(t)$	Signal force acting on antenna
f_L	Langevin noise force acting on antenna
I_2	Transducer moment of inertia
η_2	Torsional damping constant of transducer
κ_2	Torsional spring constant of transducer
ϵ_0	Permittivity of free space
A	Average area of outer capacitor plates $A = (A_1 + A_2)/2$
α	Outer capacitor plate area imbalance $\alpha = (A_2 - A_1)/A$
τ_L	Langevin noise torque acting on torsional resonator
u	Outer capacitor plate location mismatch, $u = r_{c2} - r_{c1}$ where r_{c1} and r_{c2} are the distances from the center of C to the centers of C_1 and C_2 , respectively

value, producing an unequal charge distribution in the capacitors. The resulting imbalance in the electrostatic forces from the capacitors produces a net torque on the mechanical resonator: this is called the “back action” force. Including these electromechanical coupling terms, the following are the equations of motion for the combined transducer electro-mechanical system:

$$\begin{aligned} & \ddot{q}(2L + L_a + M_{12}) + \dot{q}(2R + R_a) + q\left(\frac{2}{C} + \frac{1}{C_a}\right) \\ &= 2\nu_n - \ddot{q}_p \left[L_a \left(\kappa - \frac{\delta}{2} \right) - M_{12} \left(\kappa + \frac{\delta}{2} \right) \right] - \dot{q}_p R_a \\ & \quad \times \left(\kappa + \frac{\delta}{2} - \rho \right) - \frac{q_p}{C_a} \left(\kappa + \frac{\delta}{2} + \epsilon_{stat} \right) - \frac{2C_{vac}r_c}{C_a^2 d_0} q_p \theta, \end{aligned} \quad (1a)$$

$$(m_1 + m_2)\ddot{x} + h_1\dot{x} + k_1x = m_2r\ddot{\theta} + F(t) + f_L, \quad (1b)$$

$$\begin{aligned} & (I_2 + m_2r^2)\ddot{\theta} + \eta_2\dot{\theta} + \kappa_2\theta \\ &= m_2r\ddot{x} - \frac{r_c}{2\epsilon_0A}[(\alpha - u)q_p^2 + 2q_pq] + \tau_L. \end{aligned} \quad (1c)$$

TABLE II. Main physical parameters of the prototype gravitational wave antenna.

Parameter	Value	Description
f_i	233 MHz	Idler mode resonant frequency
f_+	977.2 Hz	Resonant frequency of mechanical mode
Q_i	3.9×10^4	Q of idler mode
Q_+	1.75×10^5	Q of mechanical mode
L_a	11.8 nH	Nominal value of one bridge inductor
r_c	17.9 mm	Distance between fulcrum and center of one outer capacitor plate
C_a	39.9 pF	Nominal value of a single bridge capacitor
A	3.6 cm^2	Area of one capacitor plate
d_0	$79.9 \text{ }\mu\text{m}$	Nominal capacitor gap
m_1	107 kg	Mass of bar and transducer
m_2	0.080 kg	Mass of mechanical resonator of transducer
I_2	$1.64 \times 10^{-5} \text{ kg m}^2$	Moment of inertia of transducer about its center of mass
r	2.65 mm	Distance between fulcrum and center of mass of transducer

The definitions of the symbols used in Eqs. (1) are given in Table I.

Experimentally the normal mode amplitudes of the coupled mechanical system are the most readily measurable quantities so we will express the effective noise and sensi-

tivity in terms of the normal mode variables which we define as follows:

$$x_{\pm}(t) \equiv x(t) - \left(\frac{U_{\pm} \kappa_2}{m_2 r} \right) \left(\frac{m_1 + m_2}{k_1} \right) \theta(t), \quad (2)$$

where

$$U_{\pm} = \frac{\left\{ \frac{I_2 + m_2 r^2}{m_2 r^2} - \frac{\kappa_2}{k_1 r^2} \frac{m_1 + m_2}{m_2} \pm \left[\left(\frac{\kappa_2}{k_1 r^2} \frac{m_1 + m_2}{m_2} - \frac{I_2 + m_2 r^2}{m_2 r^2} \right)^2 + 4 \frac{\kappa_2}{k_1 r^2} \right]^{1/2} \right\}}{2 \frac{\kappa_2}{k_1 r^2} \frac{m_1 + m_2}{m_2}}. \quad (3)$$

To determine the theoretical sensitivity of the system, the complete electromechanical system equations [Eqs. (1)] were numerically solved. The noise spectrum of the output observable, q , was computed along with the signal transfer function, i.e., the ratio of the output charge q to the input force $F(t)$. Using these quantities, the noise equivalent impulse p_0 was computed. This is the strength of a gravitational wave impulse which would give a signal-to-noise ratio of unity, assuming the output of the system was optimally filtered to detect impulses.⁶ It is conventional to express detector sensitivity as a noise temperature, which is given by⁷

$$T_n = \frac{1}{k_B} \left(\frac{p_0^2}{2m_1} \right), \quad (4)$$

where k_B = Boltzmann's constant = 1.38×10^{-23} J/K.

B. Predicted sensitivity of the prototype antenna

In our prototype antenna system, only the higher frequency normal mechanical mode, with a displacement represented by the term x_+ , had a high mechanical quality factor; the other mode was unintentionally heavily damped. Therefore, in our analysis we only consider the x_+ normal mode. The thermal Brownian and Johnson noise terms were computed, based on the resonant Q 's stated in Table II and the operating temperature of the detector, which was measured

to be 5.9 K. Phase noise calculations were based on the measured noise level at 1 kHz from the carrier for the synthesizer used to excite the bridge circuit. This level was -107.6 dBc/ $\sqrt{\text{Hz}}$ (decibels below the carrier).

For the prototype system parameters, shown in Table II, the impulse noise temperature was computed to be 1.81 K. This value is obtained for a pump level of -20.7 dBm at the bridge input transformer, which corresponded to the experimental pump level in our sensitivity measurements. Expressed in terms of its spectral density, the theoretical, broadband noise displacement in the mechanical mode was found to be

$$S_{x_+}^{1/2} = 1.19 \times 10^{-16} \text{ m}/\sqrt{\text{Hz}}.$$

The dominant noise sources in this total were the phase noise of the pump and the Johnson noise of the idler mode, which together account for about 70% of the overall noise.

We compute the effective gravitational wave strain from the noise temperature using the following relation, based on results by Dewey.⁸

$$h_0 = \frac{\sqrt{32}}{\tau_g \omega_a^2 l_a} \left(\frac{k_B T_n}{m_a} \right)^{1/2}, \quad (5)$$

where m_a is the effective mass of the antenna mode, (103 kg including the transducer support mass), l_a is the effective length of the antenna taken to be the distance between the centers of mass of the two thick cylindrical segments of the dumbbell antenna, $l_a = 0.648$ m, ω_a is the angular resonant frequency of the antenna, and τ_g is the duration of the gravitational wave burst. Starting from Eq. (5) we find that the theoretical minimum detectable burst amplitude, h_0 , for our present antenna is 1.1×10^{-16} .

III. DESCRIPTION OF TRANSDUCER AND PROTOTYPE ANTENNA

A. Mechanical system

In this section we describe the design and construction of the prototype antenna and the parametric transducer. The dumbbell-shaped bar with the transducer mounted on it has a total mass of 107 kg and a frequency of oscillation in the first longitudinal mode of 959 Hz. The mechanical resonant frequency of the transducer is intended to match that of the antenna, although in our experiments the value for the transducer frequency was slightly detuned to 977 Hz. The torsional resonator is a rectangular prism of dimensions 2.54 cm $\times$ 5.08 cm $\times$ 1.00 cm. In order to shift the center of mass of the rectangular prism away from the torsion spring location, pockets with an area of 1.0 $\times$ 1.0 cm and a depth of 0.89 cm were machined into one side of the prism. Experimental measurements of the coupled, mechanical modes indicate that the actual location of the prism center of mass is 0.265 cm from the fulcrum. The total mass of the prism was 80 g and its moment of inertia about its center of mass was 1.64×10^{-5} kg m². The torsional resonator was machined from a large, reaction mass which is affixed to the antenna. The large reaction mass helps to isolate the resonator from losses at the attachment point of the transducer to the bar. The vibrating mass, springs, and reaction mass were all machined from a single piece of niobium, used because of its extremely low mechanical and electrical loss parameters at liquid helium temperature. Niobium has a lower coefficient of thermal contraction than aluminum and we used this fact to mount the transducer to the antenna by inserting the transducer into a pocket machined into the end of the bar; when the antenna is cooled down to its operating temperature the bar tightly clamps the transducer.

B. Electrical bridge circuit

The lumped element model of the circuit is illustrated in detail in Fig. 3. Microwave modeling tools were used to determine the geometry and dimensions of the striplines that form the circuit; see Fig. 2. Bulk niobium sheet 1 mm thick was used to form the stripline circuit which was held in place by electrically insulated, niobium clamps. All of the substrate sections and insulators were fabricated from Teflon to reduce the chance of breakage due to thermal contraction, except for the top clamp over the center part of the bridge (see Fig. 2), which is made from sapphire in order to conduct heat away

FIG. 4. Exploded cross-sectional view of the bridge circuit readout and the transducer torsional resonator showing the capacitive balance control via push rods which bend the stripline circuit slightly and the location of the first stage cryogenic amplifier.

from the circuit. All materials in the bridge are composed of either superconducting niobium or low-loss insulators in order to preserve the electrical Q 's of the circuit and to minimize Johnson noise.

Each of the three capacitor plates is of area 24 $\times$ 15 mm and the gap between these plates and the surface of the mechanical element was adjustable via piezoelectric actuators over a distance ranging from 38 to 80 μ m, depending on the desired frequency of operation (approximately 160 MHz at 38 μ m and 230 MHz at 80 μ m). The nominal resonant frequency of the idler resonance was designed to match the pump mode resonant frequency to within a few kHz, although tuning was possible by adjusting the capacitor plate gaps. The inductance of each of the three arms of the bridge circuit was chosen to be 11.8 nH and the length and width of the inductive sections were adjusted accordingly. A detailed microwave analysis based upon stripline models was carried out using LIBRA®, a software package for analyzing microwave networks.⁹ This simulation tool permitted us to refine our designs of the inductor geometry and the input coupling transformer.

It was anticipated that there would be different amounts of resistive loss in the outer branches of the circuit so to tune out this resistive imbalance we inductively coupled a variable resistor to each side of the bridge circuit (refer to Fig. 2 once more). The small, additional resistance is produced by inductively coupling to the channel of a Sony 3SK166 GaAs field-effect transistor (FET), the resistance of which is controlled by varying the gate-source bias voltage. Increasing the conductance of the drain-source current path has the same effect as adding resistance in series with the transformer primary inductor. If the inductive coupling to the GaAs FET is weak enough, the shift in bridge inductance is negligible. The resistive balance circuit is designed to tune out a 2:1 resistive imbalance in the bridge (much worse than we actually encountered).

Figure 4 shows a cut-away view of most of the major components of the transducer. The stripline circuit pieces are clamped to the circuit mounting block, which is a niobium disk, 2.54 cm thick, and slightly less than 15.24 cm in diameter. The first-stage amplifier is bolted onto the other side of the mounting block. A hole was drilled in the mounting block

TABLE III. Measured and inferred sensitivities of various gravitational wave antennae configurations with the parametric transducer.

Parameter	Existing prototype	Refined prototype	Higher idler Q	2000 kg antenna
Temperature (K)	5.9	5.05	5.05	4.2
Effective mass (kg)	107	107	107	1000
Uncoupled mechanical frequency difference (Hz)	18	0.5	0.5	0.5
Transducer mechanical Q	3.3×10^5	10^7	10^7	10^7
Bar mechanical Q	6.1×10^3	1.4×10^7	1.4×10^7	1.4×10^7
Idler mode Q	3.9×10^4	3.9×10^4	5.0×10^5	5.0×10^5
Electromechanical coupling factor, β	2.0×10^{-4}	2.0×10^{-2}	2.0×10^{-2}	2.0×10^{-2}
Second-stage amplifier noise compared to Johnson noise	2	~ 0	~ 0	~ 0
Pump oscillator phase noise (dBc/ $\sqrt{\text{Hz}}$ at 1 kHz offset)	-107.6	-150	-150	-150
Antenna noise temperature (mK)	1810	2.86	1.17	1.27
Minimum detectable burst strength, h_0	1.1×10^{-16}	4.5×10^{-18}	2.9×10^{-18}	5.7×10^{-19}
Quantum limited burst sensitivity for this antenna	1.8×10^{-20}	1.8×10^{-20}	1.8×10^{-20}	3.3×10^{-21}

to allow a Teflon-insulated, niobium foil strip to carry the output signal from the bridge to the amplifier. One end of the foil strip was spot welded to the output port of the bridge circuit and the other end was spot welded to a niobium-core wire with a copper jacket to allow it to be soldered to the amplifier circuit. We found it easier to spot weld niobium foil to a copper clad niobium-core wire than to a solid, copper wire.

The substrate insulating the bridge from the mounting block was fabricated from Teflon, as opposed to sapphire, because the large number of holes required in the piece would have made it susceptible to cracking. The insulator between the circuit and the clamp has been machined from sapphire in order to provide a heat sink for the circuit. It is a geometrically simpler piece than the other substrate so there was less risk of breakage. All of the clamps are composed of niobium and are held in place with niobium screws.

Three titanium push rods are located in holes in the circuit mounting block to allow tuning of the bridge capacitors. The push rods are insulated from the circuit by small, sapphire blocks and electrically shielded with rectangles of niobium foil to prevent the titanium from introducing extra electrical loss in the circuit. Pressure is applied to the upper ends of the push rods with piezoelectric actuator driven lever systems. The fully assembled circuit mounting block was bolted to the reaction mass of the torsional resonator using titanium bolts.

IV. EXPERIMENTAL RESULTS

A. Mechanical system

The bridge circuit assembly with its support block, with a total mass of approximately 4 kg, was bolted onto the niobium reaction mass containing the torsional resonator and the complete transducer was installed on the bar as described in Sec. III. The entire transducer assembly added about 10 kg

to the antenna mass so the total effective mass of the antenna increased to 107 kg, thus reducing the antenna resonant frequency to 950 Hz at room temperature. At liquid helium temperature, the antenna had a resonant frequency of 958 Hz. This mode has a relatively low Q , approximately 6.2×10^3 , possibly due to the large number of additional wires and other parts connected to the bar. The piezoelectric tuning systems, for example, were found to have heavily damped resonances, ranging in frequency from 1012 to 1178 Hz and ranging in Q from 1300 to 2700, which could have significantly contributed to the mechanical loss of the bar. The transducer mode at 977 Hz had a Q of 1.75×10^5 . The 19 Hz difference in the two mechanical frequencies resulted in the bar mode and the transducer mode being relatively weakly coupled. Table III shows the measured values of mechanical Q .

FIG. 5. The forward transfer scattering parameter S_{21} and the input reflection S -parameter S_{11} are plotted for the cryogenic bridge circuit readout in the balanced state. In the absence of a mechanical signal the pump is suppressed at the output but the pump mode of the bridge circuit is visible in reflection.

FIG. 6. A plot of the forward transfer scattering parameter S_{21} similar to that in Fig. 5 for the bridge circuit readout after intentionally imbalancing the circuit. The bridge circuit idler mode resonance is clearly visible.

B. Electrical bridge circuit: Static properties (no mechanical signal)

The output signal from the transducer mounted first stage amplifier was sent to a second cryogenic amplifier mounted on the cold plate of the cryostat and the output of this amplifier was transmitted to additional room-temperature amplifiers. The frequency response of the electrical bridge circuit was determined at liquid helium temperature by using a vector network analyzer to measure the scattering parameters of the two-port network consisting of the pump input port (port 1) and the output of the room-temperature amplifier stage (port 2). Figure 5 displays S_{21} and S_{11} (the forward transfer and the input reflection coefficients, respectively) for the case in which the bridge was balanced as well as possible and the pump and idler modes were tuned to the same frequency. The 10 μ u background line in the S_{21} trace is the result of direct cross talk between the input and output cables. Figure 6 shows the magnitude of S_{21} for the case of a slight imbalance of the bridge circuit. In this case, the vertical scale is ten times larger than that of Fig. 5.

Based on our lowest attainable residual imbalance of ~ 10 μ u, illustrated in Fig. 5, we concluded that the capacitor gaps can be balanced to within 4.84×10^{-11} m, which corresponds to a fractional displacement of 6.1×10^{-7} for the 79.9 μ m gap of the capacitor plates in this circuit. This value represents the total, combined displacement of the centers of both of the outer capacitor plates. This is equivalent to each side of the bridge capacitance being unbalanced by 2.42×10^{-11} m, which would be a fractional displacement of 3.11×10^{-7} .

To determine the intrinsic Q 's of the readout circuit we measured the pump mode electrical Q and corrected for the loss caused by the input coupling, which is calculated from the geometrical layout of the input coupling transformer. The result is that the intrinsic Q of the readout circuit is nearly 200 000. The idler mode is not directly coupled to the input loop and its Q is not significantly affected by the input coupling strength, however the idler mode Q does not approach its intrinsic value because of the coupling to the amplifier, which adds damping. The maximum observed idler Q is 4.04×10^4 , although the useful Q is reduced slightly to 3.90×10^4 by the small amount of added loss required to operate the bridge in a resistively balanced state.

C. Sensitivity measurements

The system sensitivity was determined by driving one of the mechanical modes, x_+ , with a strain gauge mounted on

FIG. 7. A RF spectrum of the output of the excited bridge circuit transducer. A small residual imbalance leads to the central peak labeled "static imbalance" and the mechanical excitation of the transducer generates two sidebands offset from the pump by ~ 1 kHz. The upper sideband falls on the central frequency of the idler mode. The lower sideband is roughly equal in amplitude because the idler mode peak is several kilohertz in width.

the antenna and comparing the output of the bridge readout circuit to the bridge output signal from a known, static imbalance. To calibrate the static imbalance of the transducer, we measured the bridge output signal at the pump frequency and the bridge circuit resonant frequency shifts resulting from changing the voltages on the piezoelectric tuning drivers.¹⁰ Figure 7 shows an example in which the imbalance level is 11 μ u and the mechanical mode is being driven with 50 mV at the strain gauge mounted on the antenna. The equivalent static displacement, causing the center peak, is 5.32×10^{-11} m and the 977 Hz displacement amplitude, measured from the sideband, is 5.19×10^{-12} m. After the calibration procedure was complete, the bridge was balanced as well as possible. The rf signal from the known mechanical excitation was mixed down to audio frequency and sent into a fast Fourier transform (FFT) spectrum analyzer. By comparing the noise floor of the audio-frequency FFT plot to the mechanical signal peak on the same plot, the background noise level could be determined. This result is not exactly the ultimate sensitivity of the detector, since it ignores the back-action excitation of the mechanical mode, however, it does provide an estimate of the antenna in the relatively weak coupling limit, see Fig. 8. Figure 9 shows the transducer

FIG. 8. Audio frequency spectrum of the output of the pumped bridge circuit transducer with no intentional mechanical excitation. To obtain this spectrum the bridge circuit RF output is mixed with a local oscillator at the pump frequency. The high Q mode of the prototype antenna mode dominates the spectrum. The higher frequency modes are those of the bridge circuit capacitive balance mechanism.

FIG. 9. Audio frequency spectrum of bridge circuit transducer with broadband mechanical excitation. Both antenna modes X_+ and X_- are clearly displayed.

output with both mechanical modes excited.¹¹

By comparing the background noise level shown in Fig. 8 to the magnitude of the mechanical signal peak we determined that the background noise spectral density has a value of 1.7×10^{-15} m/ $\sqrt{\text{Hz}}$. Using Eq. (2), this value yields the background noise level expressed as an equivalent x_+ displacement. The ratio of transducer displacement to x_+ mode displacement is 11.2 for the current system. We find that the equivalent, background noise level in the x_+ mode is therefore 1.52×10^{-16} m/ $\sqrt{\text{Hz}}$. The value for the background noise level of the x_+ mode predicted from the theoretical analysis of Sec. II was 1.19×10^{-16} m/ $\sqrt{\text{Hz}}$, 27% less than the measured value.

The Brownian motion of the transducer resonant mass was obscured by the antenna excitation from the large level of background vibrations passing through the antenna support. The thermally driven Brownian fluctuation was predicted from theory to have a rms amplitude of 1.84×10^{-16} m, and its direct measurement will require improvement of the prototype antenna vibration isolation. The rms mechanical excitation from stray vibrations was measured to be 4.2×10^{-13} m in the transducer. This corresponds to 3.75×10^{-14} m in the x_+ mode, which is 200 times larger than the amplitude of the Brownian motion.

Figure 10 is a plot of the broadband noise, referred to the x_+ mode, as a function of the bridge circuit imbalance. The

FIG. 10. Plot of the noise equivalent displacement of the antenna X_+ mode as a function of the bridge circuit imbalance. The minimum equivalent noise obtains for the minimum bridge imbalance.

lowest noise is obtained when the bridge circuit imbalance is minimized, which as mentioned earlier, gives a broadband noise effective x_+ displacement of 1.52×10^{-16} m/ $\sqrt{\text{Hz}}$. The data in Fig. 10 were taken with a pump drive of approximately -20.7 dBm at the bridge input transformer which corresponded to a dimensionless electromechanical coupling constant¹², β , of 2.0×10^{-4} .

We observed that the bridge balance and tuning slowly drifted over time which we believe is due to creep in the piezoelectric actuators used to balance the circuit. We attribute the asymmetry of the data in Fig. 10 to this drifting. To improve the sensitivity measurements in the future, it will be necessary to use an active, feedback control system to regulate the balance of the bridge and the frequencies of the idler and pump modes.

The largest pump amplitude that could be applied with our current setup was 10 dBm into the input cable. At this pump level the loaded mechanical Q of the x_+ mode was 1.2×10^3 . Based on the mechanical mode equations, this observed reduction in mechanical Q corresponds to an electromechanical coupling factor¹³ of $\beta = 2.0 \times 10^{-2}$. The incident power at the bridge input transformer required to give this result is -0.7 dBm, which indicates that there is approximately 10.7 dB of attenuation from the input cable to the bridge. This is reasonable for the several feet of lossy, stainless steel coaxial cable and the large number of connectors through which the pump signal was transmitted.

D. Potential improvements

Although limitations in our prototype antenna vibration isolation prevents determination of the practical noise temperature of this detector, we have made wideband noise measurements and found reasonable agreement with the theory. For the prototype detector, the theoretical noise temperature is 1.81 K and the corresponding, minimum, detectable, gravity wave burst amplitude is 1.1×10^{-16} .

A few modest improvements to the detector would be extremely beneficial to its performance. In this section we discuss these improvements and conclude with an estimate of the potential performance improvements for our transducer and antenna. First, we consider potential improvements to the mechanical elements. We found that it is possible to tune the uncoupled mechanical frequencies of the antenna and transducer to within 0.5 Hz of each other by trimming material from the torsional springs of the transducer so closer matching of the antenna and transducer frequencies should be straightforward. Further, the niobium torsional resonator was not annealed after it was machined; this heat-treating step should improve the transducer mechanical Q . Annealed niobium resonators have been shown to exhibit Q 's greater than 10^7 at liquid helium temperature.¹⁴ Currently, the mechanical losses appear to be dominated by the large number of components and wires attached to the antenna, in particular the capacitive tuning system. These factors reduce the bar Q from its intrinsic value of 1.4×10^7 to 6.2×10^3 . A simplified capacitive tuning system and wiring scheme as well as improved wire vibration isolation should reduce these damping factors.

The operating temperature of the current detector is 5.9 K, not 4.2 K. This is due to heat dissipation of the first-stage amplifier and the higher than optimal thermal resistance between the antenna and the cryostat cold plate which is dominated by the thermal resistance of the flexible portions of the heat sink, composed of thin copper foil to prevent the transmission of vibrations. These flexible, foil sections could easily be doubled in thickness without significantly compromising the vibration isolation. If this were done, the temperature drop across the heat sink would be reduced from 1.7 to 0.85 K and the antenna operating temperature would be 5.05 K, reducing the thermal noise from both the mechanical and electrical components.

Several improvements to the electrical system are possible. One obvious improvement would be to use a more stable pump source at larger pumping amplitudes. We have a quartz crystal oscillator with phase noise specifications of -150 dBc at 1 kHz from the 160 MHz carrier, which is a phase noise reduction of 35 dBm over the oscillator used in the experiments reported above. Sustained operation of the transducer at large pump amplitudes is problematic in the present system. It would be optimum to operate the bridge at a pump level of 1.06 dBm at the input of the bridge circuit, which corresponds to an electromechanical coupling factor, β , of 2.0×10^{-2} . This value of β has already been achieved for short times in the present system, however in the high coupling strength regime there is an increased risk of driving the mechanical system into oscillation due to shifts in the bridge resonances which cause the transducer to operate in the parametric amplifier mode. Operation in the strong coupling limit, $\beta \sim 0.01$ or greater, will require reliable, automated control of the bridge resonant frequencies in order to avoid this unwanted effect.

Additionally, the noise impedance of the first-stage amplifier, i.e., the ratio between the additive noise voltage and the back-action noise current, could be reduced from the present value to obtain a lower system noise temperature. This can be done with a transformer at the input of the amplifier or, equivalently, by changing the location of the point where the amplifier is connected to the central inductor of the bridge circuit. Finally, the effect of additive noise from the currently used, second-stage amplifier could be reduced by improving the gain of the first amplifier. With the present system, the noise of the second-stage amplifier dominates the amplifier noise, as evidenced by the fact that the idler mode resonance peak is not visible in the noise spectrum of the unpumped bridge. In earlier tests of the amplifier, we were able to see the noise peak of a resonator of comparable Q with no externally applied signal. The peak results from a combination of Johnson noise and amplifier back-action noise driving the resonator. The absence of this feature is most likely due to cable attenuation between the first and second stages. Although reduction of the effect of second-stage additive noise could be achieved by reducing the noise temperature of the second stage, it would be easier to simply increase the gain of the first-stage amplifier.

With these improvements, a noise temperature of 2.86 mK should be possible for our current system, corresponding to a minimum detectable burst amplitude of 4.53×10^{-18} .

Further steps could be taken to decrease the Johnson noise of the readout circuit by reducing dissipation in the idler mode. We have found experimentally that unloaded electrical Q 's over 5×10^5 can be obtained with the materials we used to construct our transducer bridge circuit. The present idler Q is primarily limited by the damping required to stabilize the amplifier. If a stable amplifier can be developed with low enough input conductance so that there is not a significant reduction of the idler mode Q from its intrinsic value, the noise temperature of the detector could be reduced from 2.86 to 1.17 mK, corresponding to a burst sensitivity of 2.90×10^{-18} . Throughout we have assumed that β equals 2.0×10^{-2} , the same value as was used in the previous example.

It is intended that ultimately the improved detector, with an idler mode Q of 5×10^5 will be mounted on a cryogenic 2000 kg cylindrical gravitational wave antenna, with an effective mass of 1000 kg and a physical length of 2.75 m resulting in a fundamental resonance at 1 kHz. An advantage inherent for a more massive antenna is that the vibration isolation is stiffer for a given level of isolation. This allows for the use of a much thicker heat sink, with a correspondingly lower temperature drop from the antenna to the cold bath. For a full size antenna, we should achieve an operating temperature very close to the boiling point of liquid helium, 4.2 K. The resulting detector, assuming that β equals 2.0×10^{-2} , would have a noise temperature of 1.27 mK and the minimum detectable gravitational wave burst amplitude would be 5.7×10^{-19} which, according to estimates of gravitational wave source strength,¹⁵ approaches an interesting sensitivity. Table III summarizes the theoretical sensitivity of antennae employing this transducer under various conditions.

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