Using Strategies to Aid in Mathematical Problem Solving

Jennifer A. Smith
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Using Strategies to Aid in Mathematical Problem Solving

Many students struggle when approached with a mathematical word problem. Although they may be capable of using an algorithm and completing the computations necessary to solve the problem, figuring out which algorithm and computations to use can be difficult. Current statewide mathematics assessments are rich with word problems. While the overall goal of these mathematics assessments is to evaluate students’ ability to solve mathematics problems, if a student struggles with the vocabulary of the question, then the assessments are inevitably evaluating students’ ability to read and solve mathematics problems.

There are many skills necessary to solve a single mathematics word problem. The first skill is reading mathematics. It may seem as though students who learn to read the English language can read mathematics, however, Adams (2003) and Barton and Heidema (2002), discussed mathematics as a complex language. The language of mathematics includes words, numerals, symbols, compounded by abstractions, and technical terminology. As a young student just learning mathematical concepts, it may be very difficult to make the connection between words, numerals and symbols. Thus vocabulary development, along with reading strategies needs to be in place.

Literacy plays a key role in mathematics. Literacy is typically referred to as the ability to read, write, speak, and use language. Mathematical literacy, as it is referred to by Martin (2007), “implies that a person is able to reason, analyze, formulate, and solve problems in a real-world setting” (p. 29). In addition, Adams (2003), Barton and Heidema (2002), Brennan and Dunlap (1985), and Culver (1988) suggested that mathematical literacy is a multi-faceted task that combines print literacy with
mathematics. They further discussed possible reasons for this statement including, the combination of mathematical vocabulary and multiple representations of numerals and symbols.

The literature discussed strategies teachers could use to improve student understanding of the language of mathematics. One of the strategies is the use of a mathematical word wall to improve student understanding of important mathematical terminology and symbols. Another strategy includes using visual or graphic organizers to establish mathematical relationships.

Once students are able to comprehend the language of mathematics, students then need to fully understand the concepts being taught throughout the curriculum. In addition, they must be able to use the concept in real-world contexts. This will allow them connect the material to their real lives. Martin (2007) stated, "when real-world applications are used in the mathematics classroom, student interest is piqued and they are motivated to learn" (p. 31).

Students need the opportunity to experience different strategies, allowing them to perfunctorily choose the one that works best for their learning style. Studies in this area of problem solving include specific research-based strategies, while other research includes a variety of suggested strategies that should be helpful for students. Some studies look at specific strategies and others analyze their effect on students with different abilities. Studies by Case, Harris, and Graham (1992), Jitendra, DiPipi, and Perron-Jones (2006), and Jitendra, Hoff, and Beck, (1999), discussed how below average achieving students and/or students with learning disabilities responded to different instructional or problem solving strategies. More specifically, the research (Jitendra et al., 1999; Jitendra
et al., 2002; Xin, Jitendra, & Deatling-Buchman, 2005), suggested using a schema-based approach to solving word problems. This is a representational strategy that focuses on schemata, which is a problem pattern or structure. A distinguishing feature of schema-based instruction is the use of schemata diagrams to map important information related to a particular problem type, this helps to determine the semantic relationships.

Studies by De Corte, Verschaffel, and De Win (1985), Parmar, Cawley, and Frazita (1995), Xin (2007), and Xin, Jitendra, and Deatling-Buchman (2005), explain that ineffective instructional strategies could relate to students’ poor problem solving performances. Specifically using the key word strategy where students are taught to look for cue words to determine what operation is necessary, for example, altogether means add.

Research by Barton and Heidema (2002), Braselton and Decker (1994), Johanning (2007), Palincsar and Brown (1985), and van Garderen (2007), suggested the use of many different strategies, such as guess and check, reciprocal teaching and the use of graphic organizers to guide students through Polya’s (1957) four-step problem solving method of read, plan, solve and look back.

Regardless of the strategy used, problem solving is and should be an integral part of curriculums. As Van de Walle (2004) stated that “the process of solving problems is now completely interwoven with the learning; children are learning mathematics by doing mathematics” (p. 37).

The goal of this study is to explore the role of literacy in mathematics throughout the years. An investigation of different strategies for teaching mathematical literacy will
take place. The study will help to determine the effect of incorporating literacy strategies into the mathematics curriculum on students' understanding of mathematics content.
Mathematical Problem Solving

In a traditional mathematics classroom we could expect to see a curriculum that was linear and based on the scope and sequence of skills and algorithms (Martin, 2007, p.29). This is, in part, the case in today's classrooms although now students are also responsible for more than just skills and algorithms. According to the New York State Learning Standards for Mathematics (2005), students in today's classrooms need to understand and be proficient with mathematical concepts and skills, be able to communicate and reason mathematically, as well as become problem solvers. Teaching and learning mathematics is a multi-faced task. Teachers need to prepare students for the content they need to learn, as well as provide meaningful opportunities for students to communicate mathematically and problem solve.

This review of literature will discuss what factors may cause students to perform poorly on word problem solving tasks, and then further investigating what aspects of literacy tie into mathematics. The review will also include strategies that have been previously researched and yielded positive results. Finally, the review of literature will discuss other problem solving strategies that have been presented throughout the literature but have not been discussed as a positive strategy or a negative strategy.

Literacy in Mathematics

Problem solving is typically not a favorite topic for students to learn in mathematics. Questions incorporating problem solving skills also involve word problems. In order to solve the problem, a student must first decide what the problem is asking. Reading word problems could be a daunting task for students, especially below-average achieving students and/or students with learning disabilities. This was supported
in a study by Helwig, Rozek-Tedesco, Tindal, Heath, and Almond (1999) which concluded that students were more likely to perform better on standardized problem solving problems when they were presented to them orally (p.123). Despite this fact, Schurter (2002) states, “readers of mathematical problems must be able to translate the English phrases and sentences into mathematical expressions and equations” (p. 23). In order to help students improve in problem solving, they need to be taught how to understand the language of mathematics.

The language of mathematics.

According to the research, (Adams, 2003; Barton & Heidema, 2002; Brennan &Dunlap, 1985; Culyer, 1988) there are many complexities in the language of mathematics. All studies distinguished that mathematics text presents more concepts per word, per sentence and per paragraph than any other content-area text. Barton and Heidema (2002) further discussed that there exists “a potentially confusing overlap of mathematics vocabulary with that used in everyday conversations and with vocabulary used in other content areas” (p. 14). Words like difference, odd, and similar have very specific definitions in mathematics; however their meaning in language arts may be a bit more ambiguous.

Yet another facet, which adds to the complexity of the language of mathematics, is the use of symbols. According to Adams (2003), Barton and Heidema (2002), and Brennan and Dunlap (1985), weakness in students mathematics ability is often due in part to the obstacles they face in focusing on these symbols, in attempt to read the language of mathematics. When a student comes across a difficult word or term in written language, they can use decoding strategies to eventually determine the meaning. In mathematics,
when a difficult symbol arises, the decoding strategies may not work; therefore, students must learn the meanings of the symbols. Barton and Heidema (2002) and Brennan and Dunlap (1985) state that learning the meaning of mathematical symbols in the language of mathematics is equivalent to learning sight words in the English language. However, even if a student does learn the meaning of a mathematical symbol they may again become discouraged when they start to read a combination of symbols. In any other content area we consistently read left to right; this is only sometimes true for mathematics. Barton and Heidema (2002) discuss symbols for division, for example $27 \div 3$ and $3)27$ convey the same idea, but the order of the symbols is reversed, potentially fostering misconceptions of the division concept.

Studies by Adams (2003) and Martin (2007) stated that exposing students to a variety of mathematics print will give them multiple opportunities to read mathematics. This can be done through different books, a word wall, or other strategies. Brennan and Dunlap (1985) and Braselton and Decker (1994), however, caution implementing student independent reading from mathematics textbooks. Upon their examination of several mathematics textbooks, Brennan and Dunlap (1985) discovered that “the math concepts presented may be appropriate to the grade level for which the books are designed; however, the reading level of the text is often one, two, or even three years above the population for which the text is intended” (p. 158).

Thus, issues such as multiple meanings of words, the use of symbols and combinations of symbols, as well as difficult reading levels of textbooks, can hinder a student from really understanding the underlying concepts in mathematics.
_Strategies for teaching mathematical literacy._

Teachers can help students overcome some of these issues inhibiting them from reading and understanding the language of mathematics. Culyer (1988) states that educators need to recognize that some effective strategies in reading can be appropriate in mathematics, even though the content and cognitive demands of mathematics are sufficiently different from reading. Culyer (1998) further pointed out structural similarities between mathematics and reading, as well as similarities in comprehending reading and mathematics.

Some daily classroom activities that are suggested by Adams (2003), Barton and Heidema (2002), and Martin (2007), include developing sight words through the use of a word wall or flash cards. Barton and Heidema (2002) further discuss how this could be worked into a fun review activity where students are given small clues about the sight word or symbol. Students also need time to practice reading charts and graphs that tie into their real-life experiences. Martin (2007) describes the importance of making connections to other disciplines using meaningful real-world problems. Weidmann (1995) elaborated that, "problems should reflect situations with which students are familiar; for example, students in urban schools may not be interested in problems that require them to design barns for animals" (p.16). Providing multiple representations of the vocabulary or symbols can help students to form a complete understanding. Adams (2003) supported this by stating; "it is helpful to make connections between children’s prior understandings of the word and the mathematical meaning of the word so that children can develop definitions from their own experiences" (p.788). Adams also
described having students make a chart to help them make the connection between the word, mathematical meaning and the everyday meaning (2003).

Another meaningful strategy recommended by both Braselton and Decker (1994) and Clarke (1991) is the use of visual organizers. Clarke defines visual organizers as "graphic frames that have been used most prominently to organize student processing of text, in both reading and writing" (1991, p. 526). The use of visual organizers is also supported by Marzano (2001), who refers to these graphic organizers as a combination of linguistic modes and non-linguistic representations. Both visual organizers and graphic organizers use words and phrases along with symbols and arrows to represent relationships. Barton and Heidema (2002) also supported the use of graphic organizers, noting, "teachers can help students grasp embedded concepts as well as how other concepts are related by demonstrating these relationships with graphic organizers" (p. 20). Barton and Heidema (2002) and Braselton and Decker (1994) discuss implications for the use of specific graphic organizers to help students with problem solving. These will be discussed later in the review.

Once students have tools necessary to read the language of mathematics they can begin to apply strategies to aid in the problem solving process. Adams (2003) stated, "a knower of mathematics is a doer of mathematics, and a doer of mathematics is a reader of mathematics" (p. 794). Thus, literacy and mathematics together will help students to be successful.

**Solving Mathematical Word Problems**

Understanding what a mathematical word problem is asking a student to do is often the hardest part of the problem solving process. Marzano (2001) suggested that
using strategies to foster understanding would allow students to construct meaning on their own terms. Using strategies to aid in the understanding of the mathematics language in word problems can further help students conceptualize the problem and choose an strategy, algorithm, or operation(s). Once a student understands the problem they can put their personal opinions towards solving mathematical word problems aside and begin the mathematics necessary to solve the problem.

*Perceptions in problem solving.*

Research has shown that students do not typically enjoy solving problems. Montague (1997) surveyed some of the potential factors for this statement. She suggested that reasons for dislike in problem solving had a broad scope of both academic and social behaviors, as students differed in motivational level, self-perceptions, emotional reactions, and attitudes. Since the results were varied for different students, Montague concluded that strategy maintenance and generalization could be interventions for students with and without learning disabilities.

Despite student like or dislike for problem solving, it is an area of mathematics that is here to stay. According to the New York State Learning Standards for Mathematics (2005), problem solving makes up one-third of the components for mathematics curriculums. When Montague (1997) asked students their thoughts on the importance of mathematical problem solving, their responses suggested that they perceive its usefulness. Research by Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Oliver, and Wearne (1996) proposed that problem solving is what makes mathematics useful. They further suggested that curriculums should be rich in real-life problem solving. However, Hiebert et al. (1996) suggested that problem solving needs to
be presented with some caution, “children need not be asked to think like mathematicians but rather to think like children about problems and ideas that are mathematically fertile” (p.19). In order to achieve this, students need to be provided with opportunities to see the importance of using problem solving throughout the curriculum.

Research-tested strategies for problem solving.

Teachers often use one strategy in particular when teaching problem solving. It is the key word strategy. This is a strategy in which students are taught specific key words to cue them as to what operation to use in solving word problems, as described by Xin et al. (2005). However, Parmar et al. (1996) argued, “the outcome of such training is that the student reacts to the cue word at the surface level of analysis and fails to perform a deep-structure analysis of the interrelationships among the words and the context in which it is embedded” (p. 427). Therefore, they just pick out one key word, for example altogether, and immediately think addition. Other research that supported this fact includes literature by Barton and Heidema (2002) as well as Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts and Ratinckx (1999).

In an attempt to avoid the unfavorable results from the keyword method, the research (De Corte et al., 1985; Davis-Dorsey, Ross, & Morrison, 1991) agreed on a strategy to encourage students to analyze the problem more thoroughly. The strategy encouraged students to read and then reword the problem. Braselton and Decker (1994) noted that if students put the question into their own words it would help them to comprehend what the problem was asking. Davis-Dorsey et al. (1991) added more detail and discussed the role of rewording the problem and then further making a context personalization. Context personalization was accomplished by substituting the names of
the subjects in any given word problem with names of people in a particular student’s life. The study concluded that, these strategies helped students to comprehend the semantic structure of the problem and form correct problem representations (Davis-Dorsey et al., 1991).

Other research made several references to a common problem solving process by George Polya, from his book, *How to Solve It* (1957). His process incorporates four steps (1) Understand the problem, (2) Devise a plan, (3) Carry out the plan and (4) Look back. Shorthand for this method as noted by Van de Walle (2004) is known as the read, plan, solve, and look back method. Much of the literature on problem solving looked at Polya’s four-step process and then adds to, or focuses on, one of the four strategies.

The literature suggested that the use of self-regulated strategies with or without Polya’s method, would aid students in the problem solving process. Case et al. (1992) preformed a study to examine how engaging students to use a plethora of strategies to help themselves, would consequently aid them in solving problems. As part of the self-regulated process, students would have to use the comprehension monitoring strategy. Schurter (2002) completed a study to assess the use of comprehension monitoring throughout the problem solving process. The research emphasized the importance of questioning and understanding of all aspects of the problem-solving process. To achieve this, Schurter (2002), edited Polya’s traditional four-step problem solving process to incorporate a student self-question list as shown in Figure 1. They would then use the list independently, to guide them to think about their thinking process as they work through problem. This research yielded favorable results, and further discussed that emphasizing the use of comprehension monitoring techniques can improve the
Table 2
Modified Form of Polya's Strategy for Solving Word Problems

1. **Understand the problem:**
   - Read Carefully - more than once!
   - Understand technical terms.
   - Make a sketch, diagram, chart, or table to help **visualize** the problem and organize the data.
   - Determine what is *known*, and what is *unknown*.
   - Finally, restate the problem in your own words.

2. **Devise a plan:**
   - Can you use objects to illustrate the problem?
   - Would a **figure, diagram, chart, etc.** help as a visual aid to organize/summarize the data?
   - Is there a relationship or formula that applies?
   - Have you encountered this type of problem before?
   - Is it a "counting" problem?
   - Is there a similar but **simpler** problem you could consider first?

3. **Carry out the plan:**
   - Organize the data and do required computations.
   - Use calculators or computers if required.

4. **Look back:**
   - Is the answer *reasonable* or *expected*? (Can you verify or check it?)
   - Are there other (better) ways you could have used to solve the problem?
   - Are there related or similar problems that can be solved in the same way?
   - Understand and remember the technique used to solve this problem.

*Figure 1. Self-monitoring problem solving checklist (Schurter, 2002, p. 26).*
mathematical problem-solving performance of developmental mathematics students” (p. 32).

The study by Schruter (2002) related to research completed by Goos and Galbraith (1996) articulating the metacognitive process. Goos and Galbraith (1996) discussed that the “metacognitive process is a way of assessing one’s knowledge, formulating a plan of attack, selecting strategies, and monitoring and evaluating performance by enabling effective decisions to be made regarding the allocation of time, energy, and knowledge resources” (p. 230). The most common strategy used to get students thinking about thinking is think-alouds. Barton and Heidema (2002) discussed how teachers could verbalize their thoughts while reading, processing information, or performing some learning task to model their thinking. Braselton and Decker (1994) supported and further suggested the use of a think-aloud to model the steps for solving a problem. On the contrary, Case et al. (1992) used the think-aloud strategy to have students reveal their understanding regarding the problem solving process. Therefore the think-aloud strategy can be used as during a direct lesson, or used to informally check for student understanding and misconceptions.

In conjunction with metacognition, the use of other self-regulated strategies can promote higher-level thinking. This was supported in research by Fuchs, Fuchs, Prentice, Burch, Hameltt, Owen, and Schurter (2003) who, “experimentally established effects on mathematical problem solving, a domain potentially well suited for self-regulated learning due to the demands for metacognition and perseverance in the face of a challenge” (p. 313). In addition, literature by Fuchs, Fuchs and Prentice (2004) stated that the “combination of explicit teaching with transfer and self-regulated strategies was
an effective intervention to use in whole-class format to enhance student achievement in the area of complex problem solving" (p. 305). Therefore, the literature suggested that a combination of strategies is helpful in solving mathematical problems.

Another research-tested method for solving problems is the schema-based problem solving strategy. This strategy is supported by research completed by Jitendra et al. (1999), Jitendra et al. (2002) and Xin, et al. (2005), which focused on teaching below-average and/or students with mild to moderate learning disabilities. Jitendra et al. (1999) described schema-based instruction as “representational instruction for mathematical problem solving” (p. 50). The literature further described that schema-based strategy instruction emphasized conceptual understanding of the problem structure or schemata (Xin et al., 2005). The strategy required that students find a specific problem pattern or structure. Jitendra et al. (2002) discussed that “a primary characteristic of a schema-based strategy that distinguishes it from other approaches is the use of schemata diagrams to map important information and highlight semantic relations in the problem to facilitate problem translation and solution” (p. 24). In using this strategy, students need to first recognize what the problem type of the question is, and then use the correct strategy to map the important information and establish the relationship. All three studies suggested that students experienced success when using these strategies on mathematical word problems. Jitendra et al. (2002) further noted, “a teacher indicated that participating students were enthused about the strategy and spontaneously applied it when completing word problems on the standardized state test” (p. 36). Schema-based instruction literature does however lack in the area of instruction models. More research or training
would be required to guide students in recognizing the problem type and determining what type of semantic map would be necessary to establish the relationship.

*Useful Problem Solving Strategies*

The research presented a long list of strategies that would help in problem solving. A few of these strategies include, guess and check, reciprocal teaching, and the use of specific graphic organizers. Each of these will be reviewed over the next few pages.

*Guess & check and reciprocal teaching.*

Guess and check is an accepted strategy for problem solving, according to the New York State Scoring Guide from the Guide to the Grades 3-8 Testing Program (2005). However, since the requirements for using this strategy are very specific, some teachers do not encourage its use. Johanning (2007) agreed with this statement and further suggested “guess and check is often dismissed as a unproductive or less sophisticated approach to solving problems” (p. 132). The New York State Scoring Guide stated, “for questions in which students use a trail-and-error (guess and check) process, evidence of three rounds of trial-and-error must be present for the students to receive full credit for the process” (2005, p. 6). Despite the specificities of the process, Johanning (2007) encouraged students to use a systematic guess and check process to broaden the perspective of algebra and algebraic thinking. Johanning described systematic guess and check as “a form of model-based reasoning where the problem solver works with the situational context and applies relational reasoning to solve the problem” (2007, p. 123). During the study conducted by Johanning (2007), students were encouraged to use the strategy as they began to learn and understand algebraic
relationships. The study yielded positive results, “having students articulate their guess and check thinking and share it with others provided opportunities to develop students’ ability to think algebraically while working with an approach that was sensible to them” (Johanning, 2007, p. 132). As long as students are made aware of what steps are necessary for using the trial-and-error strategy, it may be beneficial to their overall understanding of difficult concepts.

Another strategy is the use of reciprocal teaching. Barton and Heidema (2002), Palincsar and Brown (1985) and van Garderen (2004) described reciprocal teaching as an individual or small-group reading strategy in which students learn the skills of summarizing, questioning, clarifying and predicting well enough to perform as an instructor of content. Barton and Heidema (2002) described the importance of reciprocal teaching by discussing fact that “we learn best by teaching others” (p. 118). van Garderen (2004) simplified the strategy and applied it to developing comprehension of mathematical word problems, using the four major components of clarifying, questioning, summarizing and planning. This strategy can also be adapted, as van Garderen (2004) suggested, to accommodate students with special needs. It can also be adapted to start with whole group guided instruction to eventually have students’ lead small groups. Within the small-groups, students need to question each component of the strategy. Prompts can be posted to guide the group or to provide examples of the questions that need to be completed before moving to the next component. According to van Garderen (2004), in solving a word problem using this strategy, the first step would be to discuss the part or parts of the question to be clarified. The group could do this through group verbal communication or further investigations. Next, the group uses
questions to identify the key parts of the problem. Posted prompts may be helpful as students begin to use this strategy. The group then summarizes the purpose of the word problem and finally comes up with a plan to solve the problem. The plan that is selected by the group needs to be specifically stated, and checked by a teacher before they can actually solve the problem. Finally, the problem can be solved individually or cooperatively. Barton and Heidema (2002) suggested going through the reciprocal teaching process several times using whole-group instruction before allowing students to use the entire strategy in small cooperative groups. Palincsar and Brown (1985) and van Garderen (2004), agreed that the use of the reciprocal teaching strategy could provide students with new opportunities to examine mathematics text and to explain to other students how to read and comprehend mathematics material.

*Visual or graphic organizers.*

The final strategy is the use of visual or graphic organizers. As mentioned earlier in the review, graphic organizers have helped students to grasp difficult and/or embedded concepts through demonstrating relationships. Two specific graphic organizers to aid students in the problem solving process include a K-N-W-S chart, a modified K-W-L for problem solving by Barton and Heidema (2002), and a diamond shaped organizer with specific section prompts to guide students through Pólya’s (1957) four-step problem solving process.

The K-W-L strategy; *what I know, want to learn, learned*, is used to help students predict and connect new information with prior knowledge. Olge (1986) and Olson and Gee (1991) discussed that the use of this strategy provides students with opportunities to brainstorm, preview the content, and make predictions about what they are going to learn.
Barton and Heidema (2002) presented a strategy based upon the K-W-L, but modified it specifically for mathematical problem solving. This strategy is referred to as K-N-W-S. “Using a word problem, students answer what facts they KNOW, what information is NOT relevant, WHAT the problem asked them to find, and what STRATEGY they can use to solve the problem.” (Barton & Heidema, 2002, p. 112)

With this strategy students can read the problem and decode the information presented. They can then determine what the question is asking and choose a strategy based on all the information they have already gathered. To implement the K-N-W-S strategy, the teacher should first model the proper use of the worksheet, as shown in Figure 2 (Barton & Heidema 2002, p. 113). Students need explicit instructions to be able to complete the chart independently in the future. The use of this chart will enable students to check the information given in a word problem, thus making the mathematical problem solving process more manageable for all students with or without disabilities.

Barton and Heidema (2002) further suggested that this strategy is useful for teachers to quickly evaluate students’ comprehension of the word problem and assist with any misconceptions or misunderstanding that they may have. Another useful graphic organizer is a diamond shaped organizer presented by Braselton and Decker (1994). The shape was chosen because “a diamond shape reinforces the fact that each student begins a word problem with the same information and when successful, arrives at the same conclusion. However, between these two points students should be encouraged to think divergently in order to make full use of the wide variety of problem solving strategies” (Braselton & Decker, 1994, p. 276). The strategy combines Polya’s (1957) four-step
### Figure 2. K-N-W-S Worksheet and Potential Student Example

(Barton & Heidema 2002, p. 113).
problem solving method of read, plan, solve, look back, with Barton and Heidema's (2002) K-N-W-S strategy, where students must decide what they know, what information is unnecessary, what the question is asking, and which strategy they are going to use to solve. This six-step diamond shaped graphic organizer ties together several aforementioned strategies for solving problems.

The first step is to restate the question. Braselton and Decker (1994), Davis-Dorsey et al. (1991) and De Corte et al. (1985) agreed that if students make personal connections by rewording a problem using their own terminology they may then better comprehend what the problem is asking. However, Braselton and Decker (1994) also suggest that for some problems, students can simply restate the question the problem is asking using the terminology presented throughout the question. Like Barton and Heidema's (2002) K-N-W-S worksheet, students in the second step of the diamond have the opportunity to decide what information is needed for solving the problem and disregard any extraneous information.

Once students have determined what information is necessary, in the third step of the graphic organizer, students should choose a strategy and decide how the problem should be solved mathematically. The diamond ensures not only that students decide what calculations to make, but also enables them to decide upon the proper sequencing of their operations. Finally, not until step four do students actually perform all the necessary calculations. "By the time this step is reached, the students have clearly defined the problem, selected the necessary information, and planned a logical sequence of mathematical steps leading to the solution" (Braselton & Decker, 1994, p. 276).
Therefore, the student needs only to focus on the proper execution of the necessary mathematical skills or operations.

In the final two steps, the students are lead back to a more holistic view of the problem solving process (Braselton & Decker, 1994). Here the students ask themselves if the solution they have computed is reasonable. This is decided by going back through the organizer and checking to ensure that there are specific and correct connections between all of the data and the solution. Figure 3 provides an example of a fifth grade word problem and the process through which one student went to come to a solution (Braselton & Decker, 1994, p. 279).


Braselton and Decker (1994) further discussed specific modeling, guided practice and independent practice techniques to assist as they begin to use this graphic organizer. As discussed earlier, problem solving can often be daunting for students. Giving them a word problem and a graphic organizer that they do not understand could be even more traumatic for students. Braselton and Decker (1994) attributed several factors to the success of this graphic organizer. The organizer first required that students slow down and think about each step in the problem solving process. In time, students who were at first uncomfortable with the slow pace did take their time as they were able to improve word problem solving performance (Braselton & Decker, 1994).
Mathematical Problem Solving, 28

**Figure 2. Student's use of the graphic organizer**

Question on test:
Kara baby-sits 3 hours a day for 5 days each week.
How many hours does she baby-sit in a year (52 weeks)?

1. Restate the question:
   How many hours will she sit in a year?

2. Find needed data:
   a. 3 hours a day
   b. 5 days a week
   c. 52 weeks a year
   d. ________________________________

3. Plan what to do:
   a. Multiply 3 × 5
   b. Then Multiply the answer × 52 weeks
   c. ________________________________

4. Find the answer:

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>x 5</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>780</td>
<td></td>
</tr>
</tbody>
</table>

5. Check, is your answer reasonable:
   Yes

6. Answer: 780

---

**Figure 3.** Diamond shaped graphic organizer for problem solving

(Braselton & Decker, 1994, p. 279).
Other positive results yielded from a visual representation of the problem solving process, and cooperative groupings that enabled struggling students to listen to how other students chose their strategies. In using any new strategy, students need to be explicitly told the use and the expectations. When a teacher presents a new strategy or graphic organizer to students, they must scaffold so that the students ease into the new method.

With the large variety of research-tested and non-research tested strategies, teachers should not exclusively rely on the keyword method for teaching students how to problem solve. There is great value in teaching with problems as suggested by Van de Walle (2004). He reasoned, “the learning is the outcome of the problem solving process” (2004, p. 38).

Summary

In order to solve mathematical word problems, literature suggests using Polya’s (1957) four-step problem solving method of understanding the problem or read, devise a plan or plan, carry out the plan or solve, and look back. However, in completing these four intuitively simple steps, there exist underlying facets of mathematics and mathematics literacy, which need to be understood before embarking in the problem solving process.

Through mathematics literacy instruction, teachers of mathematics also need to be teachers of literacy. As Adams (2003), Barton and Heidema (2002), Brennan and Dunlap (1985), and Culyer (1988) distinguished that mathematics text presents more concepts per word, per sentence and per paragraph than any other content-area text. Teachers of mathematics literacy also should help students distinguish the overlap between mathematics and everyday English vocabulary, as well as the difference between
mathematical vocabulary, numerals, and symbols, as suggested by Barton and Heidema, (2002).

Once the literacy aspect of problem solving has been addressed, specific problem solving strategies and techniques can be implemented to scaffold students through the problem solving process. Regardless of the strategy or technique chosen by students, depending on their learning profile, studies by Barton and Heidema (2002), Braselton and Decker (1994), Davis-Dorsey (1991), Fuchs et al. (2004), Jitendra et al (1999), Jitendra et al. (2002), Johanning (2007), Schurter (2002), van Garderen (2004) and Xin et al. (2005) agreed that strategy use is essential in the problem solving process.
Methodology

To explore the benefits of using strategies to help students solve mathematical word problems, specifically on state standardized assessments, test scores from two different years were used. The study compared standardized state test scores from students in 2006-2007, with scores from the same assessment booklet taken by the 2007-2008 students.

Participants

For the study, students from a suburban school district outside of Rochester, New York, were assessed. The current demographic makeup of the school district includes just over 800 students. Eighty-eight percent are Caucasian, about 30 percent are eligible for free or reduced-price lunch, and 12 percent are classified in special education. The groups of students for which the study was conducted, included two groups, Group A and Group B.

Group A consisted of 68 sixth graders from the 2006-2007 school year. Of this group, twenty-two percent received Academic Intervention Services or AIS in mathematics for scoring a one or a two out of a possible four using a holistic rubric on the previous years state assessment. This AIS group met every other day for 42 minutes. Students receiving self-contained special education services did not leave the room for additional Academic Intervention Services in mathematics. Students in Group A did not receive any specific strategies other than the mention of Polya’s (1957) four-step problem method.

Group B consisted of 58 sixth graders from the 2007-2008 school year. Within this group 56 percent of the students required state mandated Academic Intervention
Services for scoring a one or a two out of a possible four on the previous years state assessment using a holistic rubric. Due to the large percentage of students below the passing mark, the schedule was changed which allowed all 58 students, including self-contained special education students and students eligible for enrichment to receive two full year math classes. Both classes were taught in 80-minute blocks, on opposite days of each other. The first class was Math 6, a general education mathematics course that followed the New York State Core Curriculum Guide for grade six. There were three sections taught, the class sizes ranged from fifteen to twenty.

The second course was titled Math Lab. The purpose of Math Lab was to provide support to students lacking the skills necessary to be successful in Math 6, or to provide students with enrichment opportunities, preparing them for advanced mathematics courses in the future. There were also three groups of Math Lab, two Math Lab 6A’s and one Math Lab 6E. For these classes the students were grouped according to their New York State Grade 5 Mathematics Assessment score. Students in Math Lab 6E, or enrichment, scored high threes or fours. This course was differentiated since some students were ready for the enrichment-based activities while other students benefited from the extra time allotted to ask questions or participate in skills review. Students in one Math Lab 6A, or AIS, scored high twos or low threes, and the other Math Lab 6A consisted of students scoring low twos or ones. The primary focus of this course was to provide students with skill review to close the missing gaps and prepare students for the grade 6 state mathematics assessments.
It was in all three Math Lab courses that the students received and practiced specific problem solving strategies to aid them in achieving success in Math 6 as well as on future state tests.

**Materials**

Upon the commencement of the study, assessment scores for students in Group A were needed. An analysis was completed to discover if better instruction or strategies needed to be provided in order for Group B to be successful, or if the scores were low based upon poor question structure.

To complete the study for Group B, the analysis from Group A was examined. It was concluded that more exposure to state test-like questions, and the use of specific problem solving strategies might help students in Group B achieve greater success on the state assessments.

A long list of materials was necessary to reach this goal. For Group A, a copy of the specific assessment, as well as, the analysis of the scores was needed. For Group B, lesson plans and materials for specific strategies were needed. The strategies taught included using key actions or phrases for problem solving, and two different graphic organizers. A Smart board and projector were also used to engage the students in whole group instruction.

State test preparation questions were taken from Glencoe, Application and Concepts, Course 2 Mathematics; New York Coach Jumpstart resources; Prentice Hall New York state assessment resources; and additional questions from tests in years past.
**Procedure**

The study to increase test scores on the New York State Grade 6 mathematics assessment was completed over a 4 month time period. Every few weeks in Math Lab, a new strategy was introduced and practiced. Throughout the curriculum, state test preparation questions were also added to increase exposure.

The objective of the first lesson was to encourage students to recognize the key words or actions in word problems and determine which operation should be used. Students were provided with numerous word problems using whole numbers, fractions and decimals. They were expected to read and highlight any important information that would help them to choose the operation.

Throughout the weeks following this first lesson, students were required to use this strategy in solving word problems. Word problems were frequently taken from the Glencoe, Application and Concepts textbook, as well as previous state assessments. The students had to explicitly state which operation they would be using before they found their answer. In addition, students were frequently asked to create their own word problems, given a topic, operation and unit of study. They then needed to use the key words and key actions to create problems for their partners to complete.

In the second lesson, a diamond shaped graphic organizer from Braselton and Decker (1994), was introduced. Students used this graphic organizer in conjunction with the key word and key action strategy. The goal in using this diamond shaped graphic organizer was to help students to slowly work through the four step problem solving process, focusing on all students starting and ending at the same point, but the work they complete in the middle could vary depending on which strategy the students chose.
Students continued to use this graphic organizer to help them when answering multi-step word problems taken from Prentice Hall assessment practice and the Coach Jumpstart resources.

The third lesson introduced a K-N-W-S chart from Barton and Heidema (2002). Using this chart, students needed to critically read the question using the strategies from the first lesson to record what they know from the information given in the problem. In the second step students had to decide what information from the problem was not needed. Third, students need to use the key actions to determine what the question is asking them to find. Finally, in the last step, students needed to choose a strategy to use in solving the problem.

After all three lessons were taught, students continued to work on solving word problems. They had the option of using either of the two graphic organizers to guide them throughout each problem.

When the four months were complete the students in Group B had received several strategies for solving problems. In addition they were exposed to the wording of problems similar to that of the problems on the New York State math assessments.

To assess the success of using these strategies and exposure to different word problems, the students in Group B were then given the exact assessment previously completed by Group A.

The students in Group B completed the assessment under similar conditions as the students in Group A. They completed the questions from part one on the first day, and then completed the questions from part two on the second day. The results were then examined and compared to the analysis completed for the students in Group A.
Results

Four components contributed to the results of the study of using strategies to aid in mathematical problem solving. The first component was an analysis prepared after Group A completed the assessment. Second, the results from the strategy lessons were discussed. Third, an additional analysis was compiled after Group B completed the assessment, and finally, the two evaluations were compared.

Group A Analysis

Upon completion and regional scoring of the assessment given to Group A, a gap analysis was completed. The analysis listed all the questions on the exam, their specific New York State standard performance indicator and whether the question was a multiple-choice question or a constructed response question. With this information, the analysis listed the percent correct for general education, special education, district total and then comparisons to scores from the districts BOCES affiliate, as well as scores from districts across Western New York.

Using this data, a Teacher Data Analysis Log was completed to find out both the strengths and weaknesses of Group A, as well as to focus instruction throughout the year. Furthermore, a reflection was completed to determine why these gaps might have existed. Table 1 shows the Teacher Data Analysis Log with the strengths of Group A. For the purpose of this analysis, an 85 percent correct or higher was considered a success. It is also noted that standards and questions with 75 percent correct and higher were also included. Some standards had multiple questions on the test.

In Table 1, topics considered to be strengths of Group A included, finding volume and capacity, simple ratio, fraction, and probability concepts, as well as solving and
Table 1

From District Results to Classroom Instruction: Teacher Data Analysis Log based upon Gap Analysis for State Assessment – Group A: Strengths

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Percent Correct</th>
<th>Skills/Knowledge assessed by this question</th>
<th>85% or higher: Why did students achieve mastery?</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 CR 7 MC</td>
<td>94 75</td>
<td>Finding the volume of a rectangular prism</td>
<td>Structured lesson, specific state test practice</td>
</tr>
<tr>
<td>6 MC</td>
<td>93</td>
<td>Record experiment results using fractions/ratios</td>
<td>Specific state test practice</td>
</tr>
<tr>
<td>3 MC</td>
<td>91</td>
<td>List possible outcomes</td>
<td>Assumed they did probability in 5th grade</td>
</tr>
<tr>
<td>5 MC</td>
<td>88</td>
<td>Solve proportions using equivalent fractions</td>
<td>Specific state test practice</td>
</tr>
<tr>
<td>27 CR</td>
<td>88</td>
<td>Solve and explain one step equations</td>
<td>Structured algebra unit</td>
</tr>
<tr>
<td>1 MC</td>
<td>85</td>
<td>Express equivalent ratios as a proportion</td>
<td>Specific state test practice</td>
</tr>
<tr>
<td>16 MC</td>
<td>85</td>
<td>Translate verbal expressions into algebraic expressions</td>
<td>Structured algebra unit</td>
</tr>
<tr>
<td>17 MC</td>
<td>80</td>
<td>Order rational numbers</td>
<td>*Students did poorly on the constructed response of this standard</td>
</tr>
<tr>
<td>13 MC 2 MC</td>
<td>80 79</td>
<td>Identify capacity of customary units</td>
<td>Structured lesson, specific state test practice</td>
</tr>
<tr>
<td>4 MC</td>
<td>75</td>
<td>Translate two-step verbal expressions into algebraic expressions</td>
<td>Structured algebra unit</td>
</tr>
<tr>
<td>23 MC</td>
<td>75</td>
<td>Identify capacity of metric units</td>
<td>Structured lesson, specific state tests practice.</td>
</tr>
</tbody>
</table>

MC = Multiple Choice  CR = Constructed Response
translating one-step algebraic equations and translating two-step algebraic equations. The fourth column discussed some potential reasons for students to be successful on these standards. However, for some of these standards, corresponding constructed response questions did not yield the same successful percentages.

The second part of the Teacher Data Analysis Log evaluated the weaknesses of students in Group A on this assessment. For the purpose of this analysis, less than 75 percent was considered a weakness. Table 2 shows weakness starting at the lowest percent and then increasing. Some questions that fell between 75 and 58 percent were not considered in this overall evaluation for the Teacher Data Analysis Log. Topics considered to be weaknesses of Group A included, number sense: percents, operations with fractions, and multiple representations of rational numbers and geometry: plotting points, plotting points to find shapes and finding perimeter on the coordinate plane.

In the fourth column of Table 2, connections were made to possible poorly written test questions. This conclusion was drawn from below average percentages for students in all of Western New York.

**Strategy Lessons**

Students received three different strategies to help them with the problem solving process. Throughout the lessons, a form of lesson study protocol was followed. In using lesson study, colleagues together planned a lesson, and then one teacher taught while another teacher observed the lesson. After the lesson is completed the colleagues again met to discuss the pros and the cons of the lesson, discussing how changes should be made in order for students to better understand the content of the lesson.
Table 2

_From District Results to Classroom Instruction: Teacher Data Analysis Log based upon Gap Analysis for State Assessment – Group A: Weaknesses_

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Percent Correct</th>
<th>Skills / Knowledge assessed by this question</th>
<th>Less than 75%: What common errors do you see? What else contributed to incorrect answers?</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 MC</td>
<td>13</td>
<td>Absolute Value</td>
<td>This standard was overlooked</td>
</tr>
<tr>
<td>34 CR</td>
<td>25</td>
<td>Finding percent of a whole number</td>
<td>The students had a difficult time understanding percents. WNY - 40%</td>
</tr>
<tr>
<td>10 MC</td>
<td>32</td>
<td>Plot points to form basic geometric shape</td>
<td>Did not give appropriate practice in reviewing 5th grade standards</td>
</tr>
<tr>
<td>18 MC</td>
<td>36</td>
<td>Calculate perimeter of basic geometric shapes on a coordinate plane</td>
<td>Not enough coordinate plane review</td>
</tr>
<tr>
<td>11 MC</td>
<td>36</td>
<td>Add/Subtract fractions with unlike denominators</td>
<td>More practice and maybe new approach is necessary. We spent quite a bit of time on this topic. WNY - 49%</td>
</tr>
<tr>
<td>15 MC</td>
<td>37</td>
<td>Find multiple representations of rational numbers</td>
<td>Students struggled with converting fractions into decimals. WNY - 57%</td>
</tr>
<tr>
<td>31 CR</td>
<td>43</td>
<td>Finding area and circumference of a circle</td>
<td>The question on the test poorly written. It was hard for the students to visually pick out the radii of the two circles. WNY - 34%</td>
</tr>
<tr>
<td>8 MC</td>
<td>42</td>
<td>Commutative and Associative property</td>
<td>More practice is necessary throughout the curriculum</td>
</tr>
<tr>
<td>14 MC</td>
<td>50</td>
<td>Finding the range</td>
<td>Students did not remember which measure was range. WNY - 58%</td>
</tr>
<tr>
<td>33 CR</td>
<td>57</td>
<td>Plotting points in quadrant 1</td>
<td>Did not give appropriate practice in reviewing 5th grade standards</td>
</tr>
<tr>
<td>30 CR</td>
<td>58</td>
<td>Create a sample space and determine the probability of a single event</td>
<td>Did not give appropriate practice in reviewing 5th grade standards</td>
</tr>
</tbody>
</table>

MC = Multiple Choice  CR = Constructed Response
In this study, three strategies were planned and taught. Different colleagues observed the lessons over the four-month period. After the observations, conversations and reflections, the colleagues assisted in further enhancing the teaching of the strategy in future lessons.

The first strategy implemented was for students to critically read the question and highlight or underline important information and key words necessary to understand and solve the problem. Students were successful in deciding what operation and strategy to use when specific key words were present in the language of the problem. However, when the problems lacked the key words, students were often unable to choose the correct operation and strategy to solve. Once change made to help students in these instances, was to familiarize them with the key actions associated with each operation. Table 3 makes the connection from operation to key action. For multiplication and division, two actions were given. The first action's intended use describes multiplying or dividing whole numbers, while the second action described multiplying or dividing fractions. Using these actions students had a greater success in choosing an operation and accurately completing the problems.

For the next strategy, students had to incorporate their knowledge of finding and using key actions with a graphic organizer. They used Braselton and Decker's (1994) diamond-shaped graphic organizer. In using this organizer, students were forced to complete a series of steps before beginning to solve the problem. Some students struggled with this slow pace, specifically, if they read and immediately understood the question and then knew what operations and strategies to use. However, the slow pace
Table 3

*Connection between mathematical operation and action verb associated with the operation.*

<table>
<thead>
<tr>
<th>Operation</th>
<th>Key Action Verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>Put together or Combine</td>
</tr>
<tr>
<td>Subtraction</td>
<td>Find out how much more or less</td>
</tr>
<tr>
<td>Multiplication</td>
<td>Put together in groups of equal parts</td>
</tr>
<tr>
<td></td>
<td>Finding parts of parts</td>
</tr>
<tr>
<td>Division</td>
<td>Separate into equal groups</td>
</tr>
<tr>
<td></td>
<td>Find out how many pieces in the group</td>
</tr>
</tbody>
</table>
and different prompting questions did help several students to guide themselves through the problem. This diamond-shaped graphic organizer forced the students to critically read the question to pull out the important information before choosing how they will solve the problem. Furthermore, this graphic organizer allowed space for multi-step operations. Lastly, at the very bottom of the organizer the final question asked students to decide if their answer was reasonable. The students very often just wrote the word yes on the line, not really knowing if their answer was or was not reasonable. This was a skill that the students continued to struggle with as they worked through different word problems.

The final strategy used was the KNWS chart as explained by Barton and Heidema (2002). Students used this strategy to answer the questions from old state assessments. Figure 4 is a multi-step question the students completed, taken from the New York State, Grade 6 Math Assessment, Sample Test 2005. In completing the KNWS chart for this question, students were instructed to read the entire question including parts A and B. Next they had to categorize the information into the four columns. The first column was labeled K – what facts do I know from the information in the problem. Students successfully wrote the facts that were explicitly stated in the question. They did find difficulty in pulling out the facts that were implied through the language of the wording. Many students had difficulty choosing the part and the whole, or stating that 120 out of 200 cars were in the parking lot on Friday. After completing the first column, students easily completed the second column N – what information do I not need? Students could accurately decide what information was extraneous. In the third column W – what does the problem ask me to find, students were able to copy the question part of the problem
On Friday and Saturday, there were a total of 200 cars in the parking lot of a movie theater. On Friday, 120 cars were in the parking lot.

**Part A**
What percent of the total number of cars were in the parking lot on Friday?

*Show your work.*

**Answer** __________ %

**Part B**
What percent of the total number of cars were in the parking lot on Saturday?

*Show your work.*

**Answer** __________ %

*Figure 4: Example of a multi-step percent question from the New York State, Grade 6 Math Assessment, Sample Test 2005.*

(New York State Department of Education, 2005, p. 3).
down, but often times had trouble figuring out what that portion of the question was asking. When asked to write the question in their own words, students struggled greatly. Lastly, column S – what strategy/operation/tools will I use to solve the problem, was the most difficult. Here students often wrote down several different strategies so that they would have multiple choices. However, they were unsuccessful in choosing the correct operation and strategies chronologically. Students needed more guidance once they chose a strategy to help them correctly work through the appropriate steps to solve the problem. After filling in the KNWS chart completely, students had to be reminded to go back to their strategy column and make sure they were actually using that strategy. For multiple part questions, students consistently forgot to go back to their chart and revise the columns and use the strategies chosen.

After all three strategies had been completed; students had the option to choose the different organizers as they completed sample test questions in preparation for the assessment of this research as well as the upcoming State Assessment for their grade level.

*Group B Analysis*

Once students in Group B had completed all necessary mathematical content lessons and been exposed to and practiced the different problem solving strategies, they were given the same Grade 6 Math Assessments as the students in Group A. The assessment was given in a similar environment with the same time allotments. Some students with special needs did not receive their test modifications and therefore their grades may have suffered.
After completion of the assessment, another gap analysis was completed. This analysis listed the topic, standard, question number and designation, either multiple choice or constructed response, and the percentage of the total students who got the question correct. From that gap analysis, an additional Teacher Data Analysis Log was created to determine strengths and weaknesses of the Group B students. For the purpose of this analysis, an eighty-five percent were considered strengths, however, scores ranging from 98 to 78 percent were included in this strengths chart. Topics and standards considered to be the strengths of Group B include volume of rectangular prisms, exponents, one-step algebraic equations and expressions, and using fractions to evaluate ratios and proportions.

In determining the weaknesses of Group B another chart was compiled. For the purpose of this chart, less than seventy-five percent was considered a weakness. This chart however only shows scores ranging from 45 to 71 percent. Scores between 72 and 77 percent were not considered in this data. Topics considered being weaknesses of Group B included, rational numbers on a number line, operations with fractions, capacity conversions, and problems involving percent. Overall, the common reflections from the fourth column include more integration of topics throughout the curriculum and more specific state test practice is necessary for student achievement.

Comparison Analysis: Group A versus Group B

In comparing the data from Group A and Group B, overall Group B performed better on the assessment. The average percentage of correct responses for Group A was 63.37%, when Group B was 74.73%. For Group A, there were 13 standards and questions on the strengths list and 11 weaknesses compared to Group B with 16 standards
Table 4

From District Results to Classroom Instruction: Teacher Data Analysis Log based upon Gap Analysis for State Assessment – Group B: Strengths

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Percent Correct</th>
<th>Skills/Knowledge assessed by this question</th>
<th>85% or higher: Why did students achieve mastery?</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 CR 7 MC</td>
<td>98</td>
<td>Finding the volume of a rectangular prism</td>
<td></td>
</tr>
<tr>
<td>1 MC</td>
<td>95</td>
<td>Express equivalent ratios as a proportion</td>
<td></td>
</tr>
<tr>
<td>5 MC 28 CR</td>
<td>91 78</td>
<td>Solve proportions using equivalent fractions</td>
<td></td>
</tr>
<tr>
<td>27 CR</td>
<td>89</td>
<td>Solve and explain one step equations</td>
<td></td>
</tr>
<tr>
<td>35 CR</td>
<td>88</td>
<td>Solve simple one-step equations</td>
<td></td>
</tr>
<tr>
<td>3 MC</td>
<td>87</td>
<td>List possible outcomes</td>
<td></td>
</tr>
<tr>
<td>22 MC 32 CR</td>
<td>87 87</td>
<td>Order of operations/exponents</td>
<td></td>
</tr>
<tr>
<td>33 CR</td>
<td>86</td>
<td>Plot point in the first quadrant</td>
<td></td>
</tr>
<tr>
<td>21 MC</td>
<td>85</td>
<td>Identify radius, diameter, and chords</td>
<td></td>
</tr>
<tr>
<td>6 MC</td>
<td>84</td>
<td>Record experiment results using fractions/ratios</td>
<td></td>
</tr>
<tr>
<td>17 MC</td>
<td>80</td>
<td>Order rational numbers</td>
<td></td>
</tr>
<tr>
<td>2 MC</td>
<td>78</td>
<td>Identify capacity of customary units</td>
<td></td>
</tr>
<tr>
<td>31 CR</td>
<td>78</td>
<td>Finding the circumference of a circle</td>
<td></td>
</tr>
</tbody>
</table>

MC = Multiple Choice  CR = Constructed Response
### Table 5

**From District Results to Classroom Instruction: Teacher Data Analysis Log based upon Gap Analysis for State Assessment – Group B: Weaknesses**

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Percent Correct</th>
<th>Skills/Knowledge assessed by this question</th>
<th>Less than 75%: What common errors do you see? What else contributed to incorrect answers?</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 MC</td>
<td>45</td>
<td>Add/Subtract fractions with unlike denominators</td>
<td>Confusing wording for students. Should have been reminded of their action verbs.</td>
</tr>
<tr>
<td>20 MC</td>
<td>47</td>
<td>Absolute Value</td>
<td>A more integrated approach is needed. Absolute value needs to be incorporated into different topics.</td>
</tr>
<tr>
<td>10 MC</td>
<td>47</td>
<td>Plot points to form basic geometric shape</td>
<td>More integration of vocabulary. Students failed to read the directions carefully.</td>
</tr>
<tr>
<td>8 MC</td>
<td>56</td>
<td>Commutative and Associative property</td>
<td>More integration of property vocabulary throughout curriculum</td>
</tr>
<tr>
<td>23 MC</td>
<td>58</td>
<td>Capacity conversions using metric units</td>
<td>Students struggle greatly when multiplying or dividing by powers of 10. A district wide math audit is being performed to detect gaps.</td>
</tr>
<tr>
<td>15 MC</td>
<td>62</td>
<td>Find multiple representations of rational numbers</td>
<td>Students struggled with converting fractions into decimals. Although a 25% increase over Group A.</td>
</tr>
<tr>
<td>4 MC</td>
<td>64</td>
<td>Translating written expressions into algebraic expressions</td>
<td>Students did not critically read as practiced</td>
</tr>
<tr>
<td>24 MC</td>
<td>65</td>
<td>Order of operations with exponents</td>
<td>Students got confused when the parentheses included an exponent</td>
</tr>
<tr>
<td>34 CR</td>
<td>70</td>
<td>Finding percent of a whole number</td>
<td>Students are still struggling with determining the part and the whole.</td>
</tr>
<tr>
<td>13 MC</td>
<td>71</td>
<td>Capacity conversions using customary units</td>
<td>Students struggle visualizing which units are small or large and making the connections when completing the conversions.</td>
</tr>
<tr>
<td>9 MC</td>
<td>71</td>
<td>Solve percent problems involving percent, rate, and base</td>
<td>Students struggle determining the part and the whole.</td>
</tr>
</tbody>
</table>

**MC = Multiple Choice  CR = Constructed Response**
and questions classified as strengths and still 11 tagged as weaknesses. However, with this same data, the average increase of correct percentages on the weakness data list was 20.5%. An interesting observation occurred with questions from the capacity standards.

Group A achieved success for both questions on the assessment, while Group B only had one of the questions on the strength chart, while the other appeared to be a weakness.

Another interesting analysis was that geometry standards that were weaknesses for Group A appeared to be strengths for Group B.
Discussion and Conclusion

The goal of getting students to achieve a greater success on the state assessment was definitely attained throughout the use of specific strategies. The greatest aspect that attributed to students achieving greater success on the assessment was the exposure to many different types of word problems and different approaches to finding a solution.

Throughout the process of preparing students to be better problem solvers some observations were made through both teacher reflections and lesson study conversations. To inevitably raise test scores across the board, students need to learn and understand the mathematical content. More tenured teachers or teachers with more experience in teaching these content areas have a greater knowledge of what types of instructional strategies work best. The way in with the mathematical content strands are taught or presented may change the way a student learns or understands the material. In addition to learning mathematical content, students need exposure to problem solving questions where they have the opportunity to use the things they have learned to solve a real life problem. As Van de Walle (2004) stated, “the process of problem solving is now completely interwoven with the learning; children are learning mathematics by doing mathematics” (p. 37). Many state assessments are designed to incorporate problem solving, by asking students questions that may involve real world applications. That is why incorporating questions of this type throughout the curriculum are beneficial to students. In addition, when these real-world applications may pique student interest, therefore motivating them to learn (Martin, 2007).

In using the different strategies, there were many similar findings. When students were using the keyword method, where they were just looking for words such as,
altogether or total, and then adding. Barton and Heidema (2002) and Verschaffel, DeCorte, Lasure, Van Vaerenbergh, Bogaerts and Ratinckx (1999) suggested that this method was not beneficial to students because they fail to critically read to find out what the question is actually asking of them. In changing this strategy, for the purpose of this research, to have students read the question and decide what key action the question is asking them to perform, they were able to achieve greater success in answering word problems. Students were more likely to correctly solve questions with operations using whole numbers. For future use of this strategy, it may be helpful to have the conversation with students about the difference between a key word and a key action. More practice and perhaps more research is needed to help students achieve success for problems containing fractions or multiple steps.

In the second strategy, students used a Braselton and Decker’s (1994) diamond shaped graphic organizer for problem solving. The first step in using this organizer was for students to re-state the question. At first, students would copy the question right out of the problem. When asked to use their own words, they struggled greatly. Some students complained that the question was already stated clearly, and there was no need to state it differently. While many student struggled with the task of re-wording the question. Perhaps using the strategy of Davis-Dorsey (1991) where students re-word the problem and make a context personalization by substituting names of subjects with names of people in a particular student’s life. The goal would be for students to identify with what the question is asking. For the next three steps, finding the needed data, planning what to do, and finding the answer, students did fairly well on. It wasn’t until the end of the organizer that the students really struggled. In the fifth step, students were
asked to check their answer to make sure it was reasonable. Most students just wrote yes on the line, unable to describe or explain why their answer was reasonable. A possible reason for this was that the students felt that since they had done so much work that their answer must be correct. Often, students with incorrect answers did indeed believe that their answers were reasonable. Based on collegial conversations, this continues to be a topic students struggle with as they go through upper grade levels in school. Justifying the reasonableness of answers would be a topic of further research.

In using the KNWS chart, students had similar successes and struggles as the diamond shaped organizer. Students did well pulling important and non-important information out of the problem. They again struggled writing the question in their own words, especially for multiple step problems. Finally, in the last column, when they were asked to list what strategy/operation/tools should be used to solve, they listed several different strategies, but then struggled when choosing which one would be most useful. More guided practice would have been helpful, or more brainstorming on how to fill out the fourth column might have helped students in achieve greater success with the chart.

Overall, students seemed to prefer the KNWS chart to the diamond organizer. When students had the option to choose one or the other, more students tended to choose the KNWS chart. In general, students seemed to feel that the diamond shaped organizer slowed them down. Even though, as research by Braselton and Decker (1994) suggested, the slower pace allows more time for student to comprehend what the problem is asking.

In order for students in Group B to practice the literacy strategies they needed to practice with a variety of word problems. There were several occasions where Group B had more opportunity to practice completing these problem solving type word problems
than Group A. It was the exposure to problems similar to the problems on the assessment that gave students more confidence in answering the questions. Students in Group B also practiced highlighting and/or underlining the important information from the problems to assist them in understanding what the question was asking. This was considered the simpler version of using the graphic organizers, since the state assessments to have a time limit and the students are not allowed to use scrap paper to create the graphic organizers.

Along with the literacy strategies practiced by Group B, there were also some additional factors that could have attributed to the higher test scores. First, students in Group B received eighty minutes of mathematics instruction or practice every day, while students in Group A received only 42 minutes of instruction daily. Also, students in Group B received several different instructional strategies for learning the content. This coupled with more time for math instruction could have been a factor in greater success on the assessment.

The current study did not provide information that would allow a conclusion to be drawn as to what specific factors aided in increasing assessment scores. Forty-four percent of the Group B students did not enter sixth grade with a strong mathematical content base. Factors that could of provided students with skills to help them succeed include different instructional strategies for learning the content, different literacy strategies for solving word problems, specific state assessment practice, along with more time for learning mathematics content.

In conclusion, for future success on assessments, three aspects of learning need to be in place and used as part of the daily mathematics routine. First, multiple instructional strategies need to be used to present the content to students in a way which fosters
understanding. Second, literacy strategies, such as word walls and graphic organizers, should be incorporated to help students understand the vocabulary and apply it to understanding word problems for problem solving purposes. Finally, students need continuous exposure to questions similar to those that will be seen on their state assessments. This last step is crucial to decrease test anxiety and stress going into the assessment. In the end, if students are comfortable with the content being taught and have a bank of strategies for solving word problems in their repertoire, the assessment should just be a small hiccup in their daily mathematics routine.
References


