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The Effect of Using the Conic Graphing Application On Teaching and Learning

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This research studies the use of the Conic Graphing App in three high school pre-calculus classes during the conic sections unit. The Conic Graphing App is an application on TI83/84 Plus calculator that provides a graph of circle, parabola, ellipse or hyperbola in a pre-determined window. The study collects data from four areas: instructional and testing materials, test result, survey completed by the target group, and observation in the classrooms. The results imply that the Conic Graphing App can help improve student achievement, especially among the low-ability students. The study suggests that how graphing calculators are being used in classrooms, and their effect on teaching and learning, depend on the teacher, the student-calculator relationship, and the instruction and assessment materials.

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This research studies the use of the Conic Graphing App in three high school pre-calculus classes during the conic sections unit. The Conic Graphing App is an application on TI-83/84 Plus calculator that provides a graph of circle, parabola, ellipse or hyperbola in a pre-determined window. The study collects data from four areas: instructional and testing materials, test result, survey completed by the target group, and observation in the classrooms. The results imply that the Conic Graphing App can help improve student achievement, especially among the low-ability students. The study suggests that how graphing calculators are being used in classrooms, and their effect on teaching and learning, depend on the teacher, the student-calculator relationship, and the instruction and assessment materials.

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The Effect of Using the Conic Graphing Application on Teaching and Learning

Since the introduction of the first graphing calculator in 1986 and the subsequent development of the handheld calculator technology, along with the widespread usage of calculators in the classrooms, numerous studies on the effect of calculator technology on mathematics instruction, learning and assessment have been conducted. The National Council of Teachers of Mathematics revised the Standards of Mathematics and included the Technology Principle as the sixth principle in addition to the Equity, Curriculum, Teaching, Learning, and Assessment Principles in 2000. The graphing calculator technology continued to evolve and develop rapidly in the last decade of the twentieth century. In-services for teachers on how to use this technology were implemented to enhance mathematics teaching and learning. New generations of textbooks were written to include the graphical solutions. The study of functions was undertaken from three different perspectives: algebraic, graphic and numeric. Changes in mathematics education in response to the inclusion of the calculator technology were real and the effects were far reaching.

However, since the calculator was function-based, the study of the conic sections presented a dilemma because graphing them would require additional work for the students. One graphing calculator maker, Texas Instruments, has incorporated Conics in the Applications file of its recent products to enhance the graphing of the four conic sections (circle, ellipse, hyperbola, parabola). It would graph them in three different modes: function, parametric and polar. Most high school students enrolled in a pre-calculus curriculum would have used graphing calculators for two to three years. They were aware of the capabilities of the graphing calculators as well as the ease of getting a graph of a function. When the pre-calculus program reached the conic sections unit, many students felt they had come to a road block because the calculators were not readily providing the graphs of circle, ellipse or hyperbola. It would require extra work to convert the equations into a form that the calculator would accept in order to provide the graphs of different conic sections.

The purpose of this action research was to examine the effect of using the Conic Graphing App on the graphing calculators in high school pre-calculus classrooms on instruction, student learning and performance in the study of the conic sections. This topic was worth investigating and deemed important in further expanding the graphing power in the hands of students. The method to obtain the graph of a circle was by solving the general form of the equation $x^2 + y^2 + cx + dy + e = 0$ for y and then entered into the calculator as two separate equations, the positive and the negative halves of the circle. In addition to the fact that many students found it difficult to convert from general form to the two-piece form, the equations in terms of y did not reveal information that would have included in the standard form of $(x+h)^2 + (y+k)^2 = r^2$, where information on the center and the radius were shown. Students obtained the graph but had little understanding with regards to these key features. Using the Conic Graphing App to graph would mend this problem because they needed to enter the location of focus, directrix and other features of the conic sections in order to obtain the graph. Knowing what they needed to enter implied that they had a better grasp of the particular conic section.

Graphing calculators have continued to empower both teachers and students at various levels of the study of mathematics since it was introduced in the 1980s. It had rewritten mathematical standards, course content and textbooks. This action research would focus on how enhanced technology in the form of Conic Graphing App on the graphing calculators affected teaching and learning of a particular area of the pre-calculus curriculum. The research would look at the adaptations that were needed in the lesson plans as well as in assessment instruments in order to reflect the new way of effective teaching. Furthermore, it would examine how student performance, content retention and student attitude and belief could have been affected by using the enhanced graphing capability as compared to students using the calculators as a computational aid. This action research would report on the findings from studying the artifacts and data collected from high school pre-calculus classrooms.

Literature Review

The study of the effect of technology on education had its logical beginning in the explorations of the teacher use of technological devices in the classroom. In the early twentieth century, instructional technology was introduced into the classrooms in the forms of film and radio. It held the promise of increased efficiency and productivity for the teaching profession, and the feasibility of presentation of content beyond what was available from a teacher (Cuban,1986). The technology in classroom of film and radio was followed by tape recorder, television, and computer. Nevertheless, further study of technology in education revealed hidden patterns in the teacher-machine relationship. Cuban found that the level of teacher use of technology ranging from five percent of the weekly instruction time in secondary schools to ten percent in elementary. He concluded that the role of classroom technology was determined by the teachers' belief of how the technology could help them solve problems they perceived as important without compromising their authority in the classroom. Teachers would resist or be indifferent to using them if they believed the technology was irrelevant to their practice, increased their burdens without added benefits to student learning, or weakened their control of the classroom.

The framework of this literature review was founded on the graphing calculator technology. While handheld calculator was very different from the earlier classroom technology as in the study conducted by Cuban, there remained two constants. First, the nature of the relationship was that between teachers and machines, and second, the role that technology had in a classroom setting continued to be primarily determined by what teachers thought of it. The difference was, instead of the technology primarily used by the teachers in front of the classrooms, graphing calculator technology was not monopolized by teachers. Students have it in their hands and benefited from it too. It gave students control of their learning to the degree never afforded by any other technology with the only probable exception of the Internet.

Introducing Graphing Calculators in the early 1980s

With the rapid development of calculator technology in the 1980s, teachers, who had been teaching for years and trying to integrate graphing calculator technology into their classrooms, found that effective in-service programs were essential for them to learn new ways of teaching mathematics with the aid of calculators (Bright, 1994). Bright stratified two levels of concerns in the calculator in-service programs. The first one was the low-level concerns that were to make sure that the teachers had: (i) calculator skill, (ii) understanding of mathematics using graphical approaches, and (iii) knowledge of pedagogy related to the incorporation of calculators in instruction . The high-level concerns pointed to the effect of graphing calculator on assessment and curriculum. They were: (i) calculator use and testing, and (ii) teachers' belief about the role of calculators in learning mathematics.

For the low-level concerns such as learning new skills on how to operate a graphing calculator, and augment content and pedagogical knowledge, teachers' major concern was whether they had asked the appropriate questions to help students generate deep understanding of the mathematics. Students had the opportunity to explore and learn independently in spite of the teacher. The teachers' status of authority on knowledge was compromised to being facilitators of learning to help students make connections between the graphical display and the algebraic representation. The high-level concerns focused on assessment issues ranging from what was important in the curriculum to creation of new tests using appropriate questions to evaluate learning with graphing technology. The increase in higher level thinking that students were able to engage in when calculators were available was the subject of study in several other literatures to be examined in the later part of this literature review.

Majority of the literature written on the graphing calculator technology at the end of the 1990s has found that calculators had little negative effect on the learning of mathematics, and some reported that there was no significant differences between the using group and the non-using group (Milou, 1999). Nevertheless, most studies concurred that the use of the graphing calculator in teaching and learning was beneficial in terms of students' attitude towards mathematics. This literature review focused on examining the roles of graphing calculators from three perspectives: first, the relationship with the users; second, what area of mathematics benefited most from using graphing calculators; and third, the constraints of the graphing calculator technology within classroom practice.

Graphing Calculators and Their Users

The Technology Principle, in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), commended that electronic technology, such as calculators and computers, enhanced mathematics learning, supported effective mathematic teaching, and influenced what mathematics was taught. It also stated that the teacher must decide if, when and how technology would be used. With respect to how teachers would use technology in the classroom, many studies in addition to Cuban's (1986) pre-graphing calculator study on technology concluded that teachers' knowledge and beliefs were critical to the success of any new innovation in educational technology (Bright, 1994; Lloyd & Wilson, 1998; Milou, 1999; Doerr & Zangor 2000; Goos, Galbraith, Renshaw, & Geiger, 2001; Wetzel, 2001; Alagic & Palenz, 2006).

Qualitative analysis of the users-calculators relationship was a critical aspect in understanding the interaction between the users and the norm of usage of the tool (Doerr & Zangor, 2000; Goos, Galbraith, Renshaw & Geiger, 2001). Meaning was constructed by the user *for* the tool as it was used, and simultaneously the learner constructed mathematical meaning *with* the calculator (Hiebert, et al, 1997). When students interacted with the teacher, with each other, and with the task using the graphing calculator as a tool in their mathematics class, five patterns of calculator usage could be differentiated (Doerr & Zangor, 2000):

> We found that five patterns and modes of graphing calculator tool use emerged in this practice: computational tool, transformational tool, data collection and analysis tool, visualization tool and checking tool. This suggest that the graphing calculator is a rich, multidimensional tool that the continued study of its use in classroom practice will need to carefully delineate the patterns and modes of use. (p.161)

As a computational tool, graphing calculator was used to evaluate numerical expressions. Accurate entry of parenthesis and symbols, and making sense of the calculator results were self-evident. In Doerr & Zangor's study, students had no tendency to round the number to recognize that real-life measurements could not have six to eight decimal places. They believed that the large number of decimal places to be the more realistic answer. It was as a computation tool that technology could be and had been misused by students that lacked mathematical understanding in context.

As a transformational tool, graphing calculator changed tedious computational task into interpretative task in the classroom. The teacher focused students' attention on the interpretation of the result, rather than on the actual computation. Students attended to making sense of the result and validating it in the context of the task. The class continued to find solution using paper-and-pencil method. Nevertheless, it was the numerical estimate of the calculator as opposed to exact answer that changed the nature of classroom learning from a computational focus to an interpretative one. The shift fostered classroom discourse and social interaction that was inductive to connections among ideas and reorganization of knowledge (NCTM, 2000).

As a visualization and data analysis tool, students used the graphing calculator in four ways: (i) finding the equations that fit the data set, (ii) finding appropriate window to view the graph and determine the nature of the function, (iii) connecting the visual representation to the context of the task, and (iv) solving equations.

As a checking tool, the graphing calculator could be used to check conjectures proposed by students as they engaged in problem investigations. After students posing a conjecture about a possible function to fit a set of data, they used the calculator to check how well it fit graphically. Since students generally chose an appropriate regression model, they rarely questioned the *fit* of the equation to the data. Teacher's role was to lead the mathematical discussion to justify the graph by algebraic reasoning. Difficulties in interpreting a graph might be due to the limitations of the calculator's screen that the units on the coordinate axes were not labeled.

Another qualitative analysis of the users-calculators relationship became valid when the graphing calculator was used in conjunction with a peripheral technology such as the projection device. The calculator's screen output was viewed by the whole class and therefore provided an opportunity for collaborative inquiry. With the discourse going on about the projected graphing calculator screen, teachers could focus on student thinking (NCTM, 2000).

A study based on a three year study of senior secondary school classrooms theorized the four metaphors for the use of technology in relation to teaching and learning interactions (Goos, Galbraith, Renshaw & Geiger, 2001):

> The relationship between technology usage and teaching/learning environment is not one of simple cause and effect. The four metaphors of master, servant, partner, and extension of self are intended to capture some of the different ways in which technology enters into the mathematical practices of secondary classrooms. (p. 10)

The hierarchy was based on the levels of sophistication with which teachers and students worked with the graphing calculators. They were not related to the level of mathematics taught. It showed how technology aligned the user and the tool, depending on how knowledge was formed and applied. The first case of technology as master occurred when teachers and students had limited technical competence and the usage was restricted to a limited variety of operations. The users were subservient to the technology. Students depended on the technology for solutions. They might even lacked the ability to enter correct symbols and evaluate the accuracy of the output of the calculator.

The second case of technology as servant occurred when teachers and students used it to produce answers for routine exercises, replacing mental or pen-and-pencil calculations, or to demonstrate calculator operations to the class through the overhead projection viewscreen. Teachers used it in conventional instruction and did not change the nature of the activity. Technology remained as a medium that was not used in any creative ways.

The third case of technology as partner became apparent when teachers and students interacted with it as a partner in learning that maintained a two-way communication and responded to their commands. Users verbalized their thinking in response to the output of the graphing calculator, and held peer discussion when they compared their screens. Through the use of the overhead projector viewscreen, teachers could present alternative mathematical conjectures and draw students into whole-class discourse or collaborative group investigations. When teachers used technology as partner, they provided students with learning opportunities by selecting mathematical tasks that took advantage of what technology could do effectively and well, and that was graphing, visualizing, and computing (NCTM, 2000, p.26). For example, comparing the graph of $y = 8 \times 2^x$ with the graph of $y = 2^{x+3}$, teachers had the opportunity to peer into students thinking by posing similar graphs quickly (Doerr & Zangor, 2000).

The fourth case of technology as extension of self developed when users integrated technological expertise as an central part of their mathematical endeavor. Teacher would write course material that was technology-rich, and students would use a range of technological resources into the construction of a mathematical conjecture. In this mode of usage of technology, teachers were capable to make instructional decisions that facilitated students learning by encouraging them to make sense of mathematics, generate conjectures and justifications, and through collaborative inquiry develop deeper understanding.

Another study that also focused on the user-machine relationship examined the learning and the thought process of one student using Bloom's taxonomy (1956). It correlated the three stages of mathematical thinking processes with the six cognitive levels of Bloom's taxonomy. The three stages were intuitive, operative and applicative (Choi-Koh, 2003). The intuitive stage corresponded to Bloom's knowledge level, operative stage to comprehension and application, and applicative to analysis/synthesis/ evaluation. In the study, when the student observed the properties of the sine function as shown on the graphing calculator, he was at the intuitive (knowledge) stage. When he found a pattern between the numerical and visual data, he understood the properties and was able to explain the roles of coefficients. The student has transited from intuitive (knowledge) to operative (comprehension/application) stage. Soon after operating from this cognitive level, he began to abstract, generalize, and systematize by formulating conjectures, and reflecting by switching between the algebraic and the graphical models. The study concluded that, with the graphing calculator, the student reached the applicative (analysis/synthesis/evaluation) stage sooner, and less dependent on the teacher in the process of learning. It was also observed that because of the window (scaling) of the calculator screen the student switched to radians measure from degrees measure when he found it to be helpful to discover answers to equations with a pattern of π, although he had resisted using radians measure in pen-and-pencil calculation (Choi-Koh, 2003).

The review of the literature on the user-machine relationship suggested that in order to utilize technology to its fullest potential and to optimize its effect on the teaching and learning of mathematics, the users must recognize the mode in which they used the technology. The ability of the users to maximize the benefit of using technology was a function of their knowledge of the technology. When teachers posed appropriate and effective questions, that was made possible by the graphing calculator's fast and accurate graphing and multiple representations, students could visualize the task better and be able to explore mathematical ideas as supported by technology. The graphing calculator should not be used as a servant just to get a quick answer. When teachers asked why and how in response to the output generated by the graphing calculator, they stimulated meaningful mathematical thinking among students. When students used graphing calculators to learn at the applicative stage, technology was used as a partner and as a transformational tool that changed disseminating knowledge to inquiry.

Graphing Calculators and Mathematics Learning

NCTM's *Principles and Standards for School Mathematics* (2000) stated that technology influenced what mathematics was taught, and when a topic appear in the curriculum. It continued to comment:

> As some skills that were once considered essential are rendered less necessary by technological tools, students can be asked to work at higher levels of generalization or abstraction.... Because of technology, many topics in discrete mathematics take on new importance in the contemporary mathematics classroom. (NCTM, 2000, p. 26)

Furthermore, it emphasized that while technology should support effective mathematics teaching and learning, it should never be used in such a way as to supplant basic understanding and intuitions among students (NCTM, 2000).

The areas of mathematics in which the graphing calculator had the most impact were the topics that made up the major part of the algebra and pre-calculus curricula (Barrett & Goebel,1990). The topics included solving equations, analyzing functions and data analysis. While some literature was general in scope, such as discussing the shift in instruction emphases when graphing calculator was used as part of instruction and learning (Demana & Waits, 1990; Milou, 1999), others contributed to specific findings of the effect of graphing calculator on student understanding and assessment in relation to targeted areas of mathematics, such as mathematical reasoning, pre-calculus, calculus, quadratic equations, algebraic variables, polynomial division, and graphing activity. The second part of the literature review would differentiate the vast collection of literature on the effect of calculator technology into three areas: (a) on elementary, middle, and secondary mathematics, (b) on instruction and testing, and (c) on specific mathematics topics.

Effect of calculators on pre-college mathematics

In a study on the effects of hand-held calculators in pre-college mathematics education, it was found that students of all grades except fourth grade that were of average ability improved in both paper-and-pencil and problem solving skills (Hambree and Dessaart, 1986). It was concluded that calculators might not be appropriate for all mathematical topics or levels. Allowing elementary students to use calculator as a computational and checking tool was not desirable before they mastered certain basic mathematical skills. But using calculator did have a positive effect on students' attitudes towards mathematics for reasons as simple as novelty and more hands-on.

It should note that the calculators being used at elementary and middle school levels were scientific calculators that eased computational burden but did not enhance visualization and interpretation. The performance of eighth grade students in the computation and problem solving portion of a test in Iowa was enhanced by the use calculators, but not on the concept portion (Lewis & Hoover, 1981). A test was designed and conducted to determine the usefulness of a calculator to attain a correct answer. The result was that significant calculator effect was found for questions that required complex computations. But for questions in which computations could relatively easy be done by hand there was a non-significant trend in favor of the calculator group (Lloyd,1991). The effect of calculator as a computational tool on the lower grades was not specifically meaningful to this action research but it provided an overall review of the graphing technology.

In a survey of classroom usage of graphing calculator, Milou (1999) reiterated the result of a wide range of research that teachers' enthusiasm and perceptions of graphing calculator was paramount for the successful integration of the technology into the classroom. Many middle school teachers and algebra I teachers did not perceive this technology to be appropriate in their classrooms. One reason could be due to there was little pressure on teachers coercing them to use graphing calculator. Another reason might be due to many teachers' belief that dependence on graphing calculator for Algebra I students would prevent them from mastering algebraic manipulation skills crucial for the future study of mathematics (Schmidt & Callahan, 1992).

While there were many reasons for students having difficulty progressing from arithmetic to algebra during the middle school years, their failure to understand the concept of variable could be the most detrimental (Graham & Thomas, 2000). In a study that allowed students using the STORE feature on the graphing calculator to experience variation and symbolization, the result indicated that the graphing calculators had helped build a versatile understanding of algebraic variables.

A meta-analysis of 54 studies that was completed between 1983 and 2002 had suggested the following (Ellington, 2006):

> When calculators were part of instruction but not used in testing, the skills needed to solve problems on mathematics achievement tests improved. On the other hand, paper-and-pencil skills and the skills necessary to understand mathematical concepts were maintained but did not get better as a result of using calculators. When calculators were included in both testing and instruction, students experienced improvement in overall mathematics achievement (p.17).

But it was concluded that educators needed to resolve many pedagogical issues before students could have the benefit to the fullest extent from calculator use in the study of mathematics. Furthermore, with respect to student understanding of the concept of function, the results were found to be more favorable for the calculator but still inconclusive because traditional skill-based testing was used for assessment in these studies.

In a study that lasted three semesters involving 710 pre-calculus students in a college, researchers examined whether there existed a difference on the final exam performance between students taught using a graphing calculator and students taught in the traditional way using a scientific calculator. The study showed that students using graphing calculators attained a mean of 14.21 higher than students using scientific calculators. It concluded with three reasons to explain the phenomenon: the more interactive presentation of topics in the classrooms, the immediate feedback and the ability to check the answers, and the development of visualization skills might have caused the improvement in scores when graphing calculator was used (Quesada and Maxwell,1994).

Texas Instruments commissioned a review of 43 studies on the use of handheld graphing technology in high school mathematics (Burrill, Allison, Breaux, Kastberg, Leatham, & Sanchez, 2002). The findings were first, students using calculators had better understanding of mathematical concepts including functions, variables, applications of algebra, and the interpretation of graphs; second, for lower ability students, the improvements in achievement were more noticeable; and third, students using graphing technology spent more time in mathematical investigations and problem solving than students not using it. And finally:

> Students were likely to use graphing calculator when they believed that a graph would help the problem solving process, but when they felt the situation did not require looking at a graph they were less likely to incorporate other features of the graphing calculator in the working of the problem (p.32).

This meta-analysis concluded that graphing calculators had become an integral part of the study of mathematics and helped student understanding of mathematical concepts. It was concluded that (Ellington, 2006):

> There were no circumstances under which that students taught *without* calculators performed better than the students *with* access to calculators. However, students receive the most benefit from graphing calculators when they have access to them during both aspects of the learning process (instruction and testing). (p.24)

Effect of calculators on teaching and testing mathematics

Teaching mathematics through an interactive technological approach implied a two-folded challenge, changes in emphases of content and testing. Demana and Waits (1990) wrote that with the capability of graphing calculator, students could graph numerous functions quickly, establish common properties of classes of functions, explore and discover mathematical concepts, and adopt graphical solution to solve realistic application problems. On the part of teachers, asking appropriate questions and providing supporting activities to help students understand concepts visualized by technology became pivotal. On the part of students, technology changed the types of understanding that they needed to have. They were expected to understand the effects of scaling on graphs on the calculator screen, solve realistic applications, control the error in solution, and operate within and between multiple representations of the same problem setting. These four skills were fundamental in effectively applying technology to enhance mathematics learning (Demana and Waits, 1990).

Data analysis involved making scatter plot and finding equation of regression line, with which prediction could be made and anomaly identified. Graphing calculators made learning data analysis effective and more meaningful. As Barrett and Goebel (1990) stated:

> This topic is virtually nonexistent in the curriculum of most secondary schools, despite the recommendations of experts in NCTM's *Agenda for Action and Curriculum and Evaluation Standards* (p.206).

A study on test scores between a group of secondary students using graphing calculators on a regular basis versus a non-calculator group as conducted by Ruthven (1990) had the following result. On symbolization items, questions asking for an algebraic expression of some graphs, regular calculator users exhibited superior performance over students not using graphing calculators. However, similar performance was not repeated on interpretative items, such as questions on interpreting contextualized graph. Ruthven explained that graphing calculator had no direct use in solving interpretative questions. That depended on students' skills to synthesize verbal, contextual, and graphical information. He continued to state that calculator use could not help students develop such skill. Graphing technology improved the quality of information available to students, facilitated checking, reduced uncertainty and anxiety on the part of students were all factors contributing to the better performance attained by the calculator group (Ruthven, 1990).

The effects of calculator use on SAT test scores was studied by Bridgemen, Harvey, & Braswell (1995) that the use of calculators resulted in a modest score increase on a test included mathematical reasoning. They explained:

> If access to a calculator changed a difficult conceptual problem into a routine calculation problem, low-scoring students would be benefited. But if the calculator eliminated routine computational error within a difficult conceptual

> problem, high-scoring students would be benefited, which would result in a widening of the gap between the high- and low-scoring groups (p. 339).

It therefore implied that students with prior experience in using calculators, in spite of their ability level, would likely be benefited in testing situations. Therefore, students who wanted to maximize their performance on a test" that allowed calculator use should take courses in which calculators were used in instruction and practice. The research also analyzed individual items in the test to construct validity when calculators were permitted and had the following conclusion (Bridgeman et al.,1995):

> Questions that measure estimation skills or that require some mathematical insight in a no-calculator group might measure trivial computational skills when calculator is permitted. Other items might become purer measures of mathematical reasoning when calculators are used to reduce computational errors that are secondary to the main focus of the items (p.339).

The study concluded that test speededness was about the same between the group using graphing calculators and the group not using calculators in a test, and the calculator effect could be either present or absent in difficult and easy items. Therefore, generalizations about which group would be hurt or helped relative to the other group on tests when calculator use was permitted could not be made because different set of questions could produce very different results (Bridgeman et al.,1995).

The recommendation in the *Curriculum and Evaluation Standards* (NCTM, 1989) for an increased emphasis on using calculators in assessment led to the need to examine how the use technology might impact on each item in a test (Senk et al.,1997). Teachers needed to consider the appropriate balance between paper-and-pencil solution and graphical solution with technology. There were little inservice on how to balance the two approaches in assessment. Three levels of technology dependence: active, neutral, or inactive, were devised for a survey to help determine the potential impact of technology with respect to graphing or drawing features and align assessment using systematic coding of the calculator effect. The study suggested research to be conducted on test materials development to ensure assessment reflect the mathematics that students should know and be able to do as impacted by graphing technology. Waits and Demana (2000) also commented that standardized test must change to reflect the advances in technology so that teachers and textbook publishers would be willing to make the needed changes to tackle the calculator issue. Nevertheless, Hornsby (2002) lamented that a new generation of textbooks rushed to include graphical solutions and neglected some time-honored methods of calculation and approximation.

Effect of calculators on specific topics in mathematics

The current calculator technology has branched out to do more than what it could do a decade ago. Numerous studies had been conducted by educators to investigate the effect of these advanced calculator features on targeted areas of mathematics. The TI-92 Plus calculator (replaced by Voyage 200) was used as a demonstration devices in geometry instructions (Din et al., 2001). Programs were written so that a TI-83 Plus calculator could perform polynomial or synthetic division (Perera, 2002). When Calculator-Based Laboratory was used in conjunction with graphing calculator, students could collect real-time data and visualize the different physical events of position-time, velocity-time, and acceleration-time graph (Lapp, 2001; Wetzel, 2001).

The new generation of graphing calculators with flash ROM technology that enabled calculators to download programs or to be upgraded electronically had revolutionized the applicability of calculators in the twenty-first century (Waits & Demana, 2000). Calculators that came with a bundle of application files had rendered the capability of calculators became boundless. Application files including Cabri Jr ®., transforms graphing, conic graphing, polynomial root finder and simultaneous equation solver, probability simulations and others made the study of the effect of calculators on mathematics learning open-ended. The action research that followed aimed to study the effect of the conic graphing application on teaching and the achievement of students in high school pre-calculus classes.

Constraints of the graphing technology

Students found that answers were easy to obtain for mathematical problems when they mastered the use of graphing calculators. However, they often tended to develop misconceptions due to the limitations of the calculators or inappropriate use. The main areas of difficulty for students were interpreting the outputs of the calculators, working among the multiple representations of contextual problems, and knowing when the use of calculators was appropriate (Forster & Mueller, 2001).

Doerr and Zangor (2000) found the constraint of the calculator when it was used as a personal device and explained as follow:

> While we did observe… that students frequently used their calculators while the teacher or other students were talking in lecture or whole class discussion, we also observed that this personal use of the technology serve to breakdown group communications (p. 160).

This was because when students began to work individually on a problem, it was hard to get them back to function as group. However, the overhead projection viewscreen provided the opportunity for alternative interpretation by the teacher, or different conjectures and group presentations by other students to make the classroom more learner-centered.

The most critical problem that educators contended with using the graphing technology was to find a balance between teaching paper-and-pencil techniques and teaching with technology (Waits & Demana, 2000). Teachers needed to communicate to students that basic traditional arithmetic and algebra skills were still very important, and mental mathematics as a skill would increase in value in an ever increasing technologically driven society. Students should do the old-fashioned, pen-and-pencil way and then support using the graphing calculators. They needed to understand to some degree why procedures worked the way they were, and when they were applicable. Waits and Demana (2000) continued to write:

> However, it does mean that our objectives for mastery and understanding shift from speedy paper-and-pencil computation in division and factoring problems to making sense of the operation and their proper use (p. 221).

In an attempt to achieve the balance between paper-and-pencil techniques and technology, knowing what items to be assessed with technology and which items without it was an urgent and important problem. New pedagogical approaches needed to be tested and piloted. Heugl, Klinger, & Lechner (1996) proposed a strategy called *black-box-white-box* and the scaffolding principles, and used long division of polynomials as an example to elucidate. Waits (2000) explained it as follow:

> In the white-box phase no calculators would be used except perhaps to check results. Paper-and-pencil procedures would be developed that illustrate the division algorithm and why it works. Later in the year, when division is needed in a problem, students would be allowed to use a calculator for the computation (black –box phase) (p.11).

All the stakeholders in mathematics education would like to achieve a balance between technological and paper-and-pencil math skills. The balance would not be static, depending on the background of the users and the level of the mathematics. Use of technological tools in classroom would continue to change with every advance in calculator technology. Teaching and learning would continue to evolve, and curriculum and assessment would continue to make progress in response to technology. As Waits & Demana (2000) looked into the future of technology in mathematics classroom, they concluded that mathematics curriculum and practice would continue to change with every advance of technology. As some aspect of technology became obsolete and so was some content of mathematics. The use of technology must truly be integrated into the fabric of classroom practice, from curriculum design, to instructional strategies, and forms of assessment (p.14).

Methodology

A high school pre-calculus curriculum provided a dynamic environment to study the effect of technology on teaching and learning of mathematics. The focus of the action research was on the Conic Graphing App that either came with, or could be downloaded, in several models of the Texas Instruments' graphing calculators. The application enhanced the calculator's capability to graph or trace circles, ellipses, hyperbolas, and parabolas (Appendix A). Since this was an attempt to gain an understanding of the effect of this application on teaching and learning, adaptations in lesson plan and instruction, student performance in the assessment instruments of the unit, comparison of student performance with a controlled group, teacher's own observation, and a student survey were included as part of the methodology employed by this action research.

Participants

The two groups of students involved in the action research were from a suburban senior high school that had approximately 1,450 tenth to twelfth graders. The target group was from the three sections of the pre-calculus course taught by the writer of this action research. The sample size was 58. Among them, 48 were twelfth graders and ten were eleventh graders; 30 were female and 28 were male. The researcher was in her third year of teaching pre-calculus in the school district.

The second group of students was from the three sections of the pre-calculus course taught by the collaborating teacher at the same senior high school. The collaborating teacher was in his seventh year of teaching and first time teaching pre-calculus. The sample size was 53, with 29 female and 24 male. The assessments in the

The target group of students was taught with the aid of the Conic Graphing App on the TI-83 Plus or TI-84 Plus graphing calculators. The students in the control group taught by the collaborating teacher did not have the Conic Graphing App on their graphing calculators. They used them in a conventional way as a computational aid and mainly for graphing parabolas.

The teachers in the sample designed their lessons for the conic sections unit based on a textbook chosen by a textbook selection committee at the senior high school in early spring of 2006. The textbook was *A Graphical Approach to Pre-Calculus with Limits* by Hornsby, Lial and Rockswold. It had a copyright of 2007 presuming that it would provide the most current graphical approach in teaching and learning pre-calculus.

Data sources

On the effect of using the Conic Graphing App on lesson planning and teaching, data was based on the writer's own observation and analysis of artifacts. The lesson plan that used the Conic Graphing App approach was based on a lesson plan prepared for the control group. Construction of the database on the effect of the Conic Graphing App on student learning involved three sources: the assessment instrument completed in the target and control groups, teacher's own observations, and a student survey (Appendix B). The survey asked the students in the target group to respond to six questions related to their experience while working with the Conic Graphing App, and was conducted in class the day after the test at the end of the conic sections unit.

The majority of the students in the target group was under eighteen. A letter (Appendix C) notifying the parents of the action research at the beginning of the conics unit was sent home together with the consent form for the minor (Appendix D) to obtain parental permission letting their daughter or son participate in the survey at the school at the end of the unit.

Data analysis

Initial analysis was conducted with the instructional materials. The essential questions at this stage included what students could learn from using the Conic Graphing App on their calculators, what changes in the lesson plan, and in the assessment, reflected the enhanced graphing capability in teaching. Subsequent analyses of the effect on students began by comparing the quiz and test results from the target and the controlled groups, followed by compiling and interpreting data from the student survey, and concluded by the teacher's own observation of the process, classroom environment and student attitude towards learning, and her own experience from the action research.

Action research was not meant to guarantee a predetermined notion of success for the teachers. However, for the researcher, success meant motivating her to continue to engage in learning how to use effective teaching and assessment strategies, develop inquiry toward one's practice, and strengthen one's knowledge of mathematics and technology, and further understanding of student mathematical thinking. And for the students in the target group, success was revealed in their thinking process and their feeling towards learning during the conic sections unit, and measured by their assessment result as compared to the controlled group.

Results

The three pre-calculus classes in the study completed the unit on conic sections in seven blocks as scheduled. Day one of the unit was on circles and an introduction to the Conic Graphing Application on the graphing calculators. From day two to day four the classes had instruction on parabolas, ellipses and part of the material on hyperbolas. On day five the classes had a quiz on circles, parabolas and ellipses and finished the hyperbolas section. Day six was for reviewing the entire unit and day seven was the unit test. The survey was conducted on the day after the unit test when the test was returned to the students. The three control classes taught by another teacher also completed the conic sections unit along a similar timeline.

The research resulted in a collection of data from four sources: 1) the instructional and testing materials, 2) the test result, 3) the survey completed by the target group, and 4) the researcher's observation in the classroom.

The instructional and testing materials

The teachers of the target and control groups planned together and made class notes for each conic section. The class notes were copied and distributed to the students each class. They provided the basic outline for instruction and blanks and graphing grids for students to take notes and draw graphs. The researcher then added calculator notes with specific reference to how to use the Conic Graphing App (Appendix C). The homework assignments were the same for both groups. It was a selection of problems from the exercise sections of the textbook. Since the textbook did not make any reference to the Conic Graphing App, none of the practice problems in homework required students to use the application on the graphing calculators.

The test was made by the teacher of the control group and another veteran teacher at the high school with reference to the tests provided by the textbook publisher (Appendix D). An analysis of the test indicated that five of the eight questions were not calculator-sensitive, meaning that students could not simply get the data from the question without any further processing and enter into the calculator to obtain an answer. The other three questions that constituted the rest of the test were highly calculator-sensitive, meaning the equations in the questions were in standard form and students could get the data from the questions without any additional work and enter into the calculator to obtain an answer

The test result

The average test score of the target group was 84 and that of the control group was 82 (Table 1). The target group had a better performance of 2 points over that of the control group. The control group had a range from 48 to 99. The range was less for the target group, between 64 and 98. The outlier of 48 in the control group was 16 points lower than the target group's minimum of 64. If this outlier was removed from the data, then the average improved by 0.7 points to 82.7 for the control group. This was insignificant for further consideration and disregarded. The median of 86 and 83, and the maximum of 98 and 99, respectively for the target and control groups, did not constitute any gross differences between the two groups on the study of the effect of using the Conic Graphing App on the graphing calculator. The standard deviation, the degree of dispersion of the values around its mean, was less for the target group.

Table 1. A summary of the unit test result

A statistical summary of the unit test result of the two groups in the study:

	Target Group	Control Group
Sample Size	58	48
Average	84	82
Minimum	64	48
Median	86.0	83
Maximum	98.0	99
Standard Deviation	8.8	10.5

The survey

There were six items in the survey. The survey used a five-level scale that ranged from level one representing the opinion of "strongly disagree", to level three being "neutral", and finally to level five indicating the belief of "strongly agree". To reveal the calculator experience of the target group, survey item 1 asked the participant about the number of years he or she had used graphing calculator. The result showed that 70% had used graphing calculator for four years, 22 percent three years, and 8 percent five years.

Survey item 2 sought the opinion of the participant whether he or she perceived that the teacher was knowledgeable about teaching the conic sections using graphing calculator technology and the Conic Graphing App. While 87 percent selected "strongly agree" at levels 4 or 5, 11 percent remained neutral, and two percent strongly disagreed.

Survey items 3 and 4 inquired about the usefulness of the Conic Graphing App in helping student in the test. Item 3 asked if it was possible to use the graphing calculator to obtain part of or all of the solutions for the questions in the unit test; but the questions could reasonably be answered without using it. 78 percent responded "strongly agree" at level 4 or 5, 13 percent neutral, and nine percent answered "strongly agree" at level 1 or 2. Item 4 queried whether the use of the graphing calculator with the Conic Graphing App was necessary to obtain a solution or greatly simplified the work needed to get a solution in the test. The result to this item was that 40 percent chose "strongly agree" at level 4 or 5, 36 percent was neutral, and 24 percent "strongly disagree" at level 1 or 2.

Survey items 5 and 6 tried to find a general outlook on the Conic Graphing App. Item 5 asked if the application helped the participant visualize ideas, understand, and retain the concepts taught in the conic sections unit. Nearly half of the respondents answered positively at level 4 or 5, 33 percent “neutral”, and 18 percent “strongly disagree” at level 1 or 2. Item 6 inquired if learning to use the application on the graphing calculator was easy. 62 percent responded “strongly agree” at level 4 or 5, almost a quarter of all participants answered “neutral”, and 13 percent “strongly disagree” at level 1 or 2.

Observation in the classroom

When the teacher demonstrated the Conic Graphing App on day one of the unit using the overhead viewscreen, the students were attentive. She emphasized the dual-capability of the application. The first one was to provide users with the graphs of the conic sections based on the standard forms of the equations, with the only exception for circle where the general form of the equation $Ax^2 + Ay^2 + Bx + Cy + D = 0$ was also accepted. The second feature that the application was capable of doing was to trace along the conic curve after it was displayed on the pre-determined viewing window. By tracing, the coordinates of the vertex or vertices could be obtained as shown on the bottom of the screen. She also pointed out that the students must use paper-and-pencil way to show the analytical work and to attain the graph and then check their work for computational error using the graphing calculators. The next time the teacher showed the class the Conic Graphing App again was on the day of review. She demonstrated how to use the rest of the application with regards to the parabola, ellipse and hyperbola in order to obtain a graph and be able to trace the curve.

The response from the students on wanting to try the Conic Graphing App was polarized. After the teacher loaded the Conic Graphing App onto students’ calculators, students that owned their calculators were excited to check and try the new feature. It was

Discussion and Conclusion

The discussion that followed focused on four areas. They were the role of the teacher, the relationship between learner and the calculator on conceptual understanding, the inadequacy of the instructional and assessment resources in teaching with technology, and finally, the limitations of this study and future considerations. All the discourse was supported by the data collected from the study including the teaching and testing artifacts, the test scores, the survey, and the classroom observation artifacts. It was interesting to see that the findings of the literature written on the subject were coming to life during the discussion and confirming many of the ideas that the literature had already uncovered.

The role of the teacher

Many studies (Cuban, 1986; Schmidt & Callahan, 1992; Bright, 1994; Milou, 1999; Doerr & Zangor, 2000; Alagic & Palenz, 2006) concurred that it was the teacher, not the technology used, that held the key to the success of the mathematical learning environment. Integrating calculator technology and attaining the full potential of the technology in the mathematics curriculum depended on many factors such as the teachers' calculator skill and understanding of mathematics using graphical approaches, and other issues such as teachers' belief about the role of calculators in learning mathematics. The survey completed by the target group indicated that while 87 percent of the students agreed that the teacher possessed the essential calculator skill, but less than half of them, only 40 percent, perceived the graphing calculator as necessary or greatly simplified the work needed to get a solution. The under-utilization of the feature on the graphing calculator could be an indication that students did not value the added benefit of the enhanced graphing capability or simply did not know when and how to use it. This might have also been because of their teacher telling them that they needed to learn and show the math analytically, and used the graphing calculator to help them with graphing and computation. The survey result supported the base concept that the teacher's belief of the role of technology in the learning process ultimately determined the quality and quantity of student learning with regards to using graphing calculator technology in the classroom.

On the issue of approximately how much class time was spent teaching students how to use the Conic Graphing App, it was recounted that about ten minutes were used. And the amount of time using the calculator as a transformational and visualization tool throughout the whole unit, it was about thirty minutes out of the approximately 480 minutes of the six blocks assigned to teaching the conics. The total of forty minutes accounted for 8.3 percent of the instructional time for the unit, equivalent to no more than five percent of the weekly instruction time. This aligned surprisingly with Cuban (1986)'s study that said the level of teacher use of classroom technology in secondary schools was about five percent of the weekly instruction time. Cuban was referring to the older version of classroom technology such as television and tape recorder. This perennial phenomenon affirmed the widely held precept that teachers' belief of the importance of technology in learning would prevail as long as they believed that using the graphing calculator added benefits to student learning without compromising their authority on the subject in their classrooms. In this case, believing the Conic Graphing Application was relevant to classroom practice, the teacher showed the class her calculator knowledge, bolstering her authority, and instructed the class her expectation of how the technology should be utilized.

The mean test score of 84 for the target group and 82 for the control group show a difference of two points. It did not seem to be significantly better for the target group, especially when other factors such as disparity in grading between the two teachers was taken into consideration. However, the net result indeed reflected the findings by some studies that the use of graphing calculator in teaching and learning was beneficial in terms of students' level of understanding and achievement in algebra and pre-calculus (Milou, 1999). It also agreed with the conclusion of other studies that there were no circumstances under which the students taught *without* calculators performed better than the students *with* access to calculators (Ellington, 2006). The range of 34 points in the test scores for the target group, compared to that of 51 points for the control group, was in line with reports that poor-performing students benefited more from using the enhanced graphing feature on the calculator in learning and testing than those in the non-using group. The poor performing students in the target group might have benefited from the Conic Graphing App in the test and, consequently, the low score was higher for comparable students in the control group.

The result of the action research also supported the position taken by many studies and mathematical organizations, including the NCTM, that "initially, the teacher must decide if, when, and how technology will be used" (NCTM, 2000, p. 26), and "students learn mathematics through the experiences that teachers provide" (NCTM, 2000, p. 16). In the study of the conics, the pre-calculus students in the target group had a different learning experience with the graphing calculator, as compared with the control group. However, the teacher did not change her instructional approach when using the graphing calculator to teach the conics, except to provide more visual examples. She also did not use it to facilitate discussion, encourage students to conjecture or to provide ideas. She did not shift her role when using the Conic Graphing App might be explained by the fact that she lacked the training or was not prepared to transform instruction to exploring with technology based on her instructional materials. She failed to capture the opportunity provided by technology to make the classroom more learner-centered, and less equation-based to enhance students' mathematical thinking. This was in part due to the teacher's belief, instructional materials and pedagogical strategies that ultimately determined the learner-calculator relationship in the classroom. The potential of using the graphing calculator to promote collaborative learning by the teacher was thwarted in the classroom when the nature of the interaction with technology was not an extension of self, but a servant to users.

The learner- graphing calculator relationship

As students used the calculator, they learned to use it better and more effectively to help them understand in depth. Since one of the goals of this action research was to examine how the graphing calculator technology affected learning in a mathematics classroom, and the relationship between the learner and the tool was fluid and dynamic, the relationship between them must be examined. The following discussion was based on the ideas presented by Goos, et. al.(2001) and Doerr et. al. (2000) that theorized or illustrated this relationship.

Goos' four roles of calculator were a master, servant, partner, or extension of self. More than 75% of the students in the target group have had four or more years of experience working with the graphing calculator. To them, the calculator was definitely not their master, in the sense that their usage was limited to basic operations. Most of the time they used the calculator as a servant for a fast and accurate replacement of mental or pen-and-paper calculations,. In this mode, they did not used the calculator in a creative way that changed the nature of the task. Some of them were able to interact with the technology and respond to its error messages, treating the calculator as their learning partner. When using the Conic Graphing App, students might encounter error message such as "*Allowed parameter values:* $0<B<A$" if they entered values that were not feasible for the standard equations of circles, parabola, ellipse or hyperbola. For example, a negative *B* would not be possible for a circle and the message "*Parameters create a non-real answer*" would appear. For several students this provided a stimulus for peer discussion. For other, they sought teacher for immediate support. During the learning of the conic sections, neither the teacher nor the students of the target group had used the calculator as an extension of self, in which they constructed or hypothesized mathematical arguments, included other technological resources, and conducted inquiry for deeper understanding of the content.

Doerr categorized five patterns of use of technology within a classroom. They were computational, transformational, data collection and analysis, visualization, and checking. A calculator as a checking tool was when it was used to check conjectures made by learners as they engaged with problem investigation, accepting, or rejecting and re-conjecturing to further their learning process. It was comparable to Goos' extension of self. The students in the action research were not facilitated to use the calculator as a checking tool. Furthermore, if the students were able to use the calculator to develop visual parameter matching strategies to find equation, find appropriate views of the graph, link the visual representation to the analytical representation, and solve equation, they were using technology as a visualization tool according to Doerr's interpretation. The time allotted for calculator use in the classroom for the target group was not sufficient to foster the development of understanding through visualization using calculator. A tool has to be used often enough to for learners to experience and value it as an instrument to expand understanding and create new meaning in the same context. The survey result that less than half of the respondents, 49 percent, agreed that the Conic Graphing App helped them visualize ideas, understand, and retain the concepts, indicated a majority of the students did not appreciate the calculator as a learning partner or a tool for visualization. Another data that only five percent of the class time were spent using technology in the conics unit showed the lack of time for students to develop the transition in their relationship with the calculator from transformation to visualization tool.

The relationship between learners and the calculator exhibited a cause-and-effect cycle, with one enforcing another, as dictated by the learning environment that was influenced by the teacher's belief, attitude and knowledge on calculator use and graphical approach of instruction, and flexible pedagogical strategies. The teacher of the target group had not allotted adequate time to promote the use of technology and to enhance the learner-calculator relationship with the goal of allowing students to reach the optimal level of technology use in a mathematical learning environment.

Instructional resources and assessment

The target group used the same instructional material as the control group. The only difference was the additional information on the Conic Graphing App (Appendix E). This implied that the instruction by the teachers of both groups would follow the same pattern in a didactic and teacher-centered environment. Without modifying the instructional materials that put technology in the forefront, instruction in the target group would probably be similar to that in the control group. Teaching using the graphical approach and changing the nature of learning to inquiry-based with the help of technology started with planning that deliberately incorporating technology as the starting point. The researcher found it to be a daunting task for individual teachers. Besides drafting new instructional materials based on graphical approaches before class and changing the pedagogical strategies while in class, assessment was the third facet that required considerable recalibration to complete the integration of technology in teaching to garner the full benefit in mathematical learning.

Both groups also used the same test (Appendix F) for assessment and were allowed to use calculators on the test. An inspection of the questions on the test showed that only three out of the total of eight questions had significant calculator effect (Bridgeman, et. al. 1995), meaning that a solution could easily be obtained using calculator with the Conic Graphing App with little or no additional work using pen and paper, provided the students knew when or how to it. Questions 2, 3, 5, 7 and 8 on the test had moderate to low calculator effect, meaning that the Conic Graphing App was not directly helpful in obtaining the answer. In other words, few questions on the test actually required the Conic Graphing App to solve. Students used calculator on those questions simply as a tool to avoid computational mistakes, not to acquire the solution in context. The survey result showed that more than half of the respondents, 60 percent, either strongly disagreed or remained neutral when the item asked whether the use of graphing calculator with the Conic Graphing App was necessary to obtain a solution or greatly simplified the work needed to get a solution. In addition, 78 percent of those surveyed strongly agreed that it was possible to use the graphing calculator to obtain part of the or all of the solutions, but the questions could reasonably be answered without using it. The survey result painted a picture that the assessment used did not require students to demonstrate higher level of understanding such as the multiple representations of the topic as best facilitated by graphing calculator. Instruction and assessment were not synchronized to maximize the benefit of using the Conic Graphing App in a mathematics classroom.

The Conic Graphing App was designed with the sole intention of providing the graphs of the four conic sections. To obtain other solutions such as the location of the directrix and the focus of a parabola, students would draw on their understanding of the analytical representation of the parabola to match their visual interpretation of the graph of the parabola. It was a skill that teachers could easily help students develop with some changes in instructional strategies and the help of the calculator technology, provided the calculator was totally integrated in the fabric of instruction. When the teacher of the target group reflected on the amount of time spent using the Conic Graphing App, she lamented the minute quantity and poor quality of its use in teaching the unit. If the Conic Graphic App was adequately used in class with changes in instructional strategies, aiming for deeper understanding of the topic by linking the multiple representations of the conics, the teacher would have provided the opportunity to help students recognize that mathematics could be understood, and the analytical task that was considered essential should concede to less importance when students could work at higher levels of generalization or abstraction.

Limitations of the study and future considerations

Integrating calculator technology to make learning a learner-centered process in multiple representations setting, provide for rich, on-going assessment, and facilitate learning for understanding and retention were the ultimate goals for using the technology in any learning community. This research reaffirmed many findings of previous studies on the effect of the calculator technology on teaching and learning of mathematics. Calculators were found beneficial for students at all ability levels by improving their disposition toward mathematics, generating positive changes in classroom dynamics, and improving mathematical achievement especially among low ability students. The sample size might be small relative to other studies, the benefit however was particularly noticeable, in terms of relative achievement for the weakest students in this research, with 64 as the minimum score of the target group versus 48 of the control group.

The Conic Graphing App provided one micro area of calculator technology for study. It might be unreasonable to ask students to respond to a survey with regards to a subject that they had used only for a small amount of time and after six blocks of class time. Nevertheless, students' experience in using calculator made the survey result relevant.

The researcher found that the lack of resources and support in terms of instructional and assessment materials that included calculator as a tool for meaningful learning to be the major and immediate problem for teaching based on graphical approach. The textbook used in the research was written thirty years after the introduction of the first graphing calculator. The graphical approaches in the book were mostly transformational, meaning that they merely changed the representation from the analytical form to graphic form. Most of the problems in the exercise sections and tests did not require students to use the calculator as an extension of self, meaning the problems were not designed to ask students to use the calculator to construct mathematical argument, engage in investigation, accept or reject the argument, and perhaps make another conjecture and repeat the process again. In the presence of technology, the ability of students to be able to operate within and between different representations of the same concept or problem setting was fundamental in effectively applying technology to enhance mathematical learning (Demana & Waits, 1990).

The lack of mentoring and coaching by teacher with calculator technology expertise and intermittent professional development opportunities were among the long-range concerns for the researcher. Time was needed for personal growth to become proficient and dexterous in teaching based on graphical approach and explorations, and to use the calculators more than just for verifying student work. More research on determining the percentage of calculator sensitive items in tests and other assessments would be beneficial as graphing calculators became increasingly available to more students and frequently used in daily instruction. The researcher also wondered whether the low ability students might become too dependent on the calculator in learning mathematics and was concerned about how some of them used the calculator as a black box without understanding or meaningful interpretation of the result provided by the calculator. Further study on the retention of mathematical content and skills among the high ability students that were flexible enough to work between multiple representations would shed light on the extent of the effectiveness of learning using graphing calculator technology. Following differentiated groups of students on how they learned with

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Appendix A

Conic Graphing Application

TI Apps Demonstration:

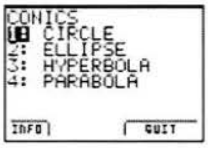
Conic Graphing App

For the TI-83 Plus and TI-83 Plus Silver Edition

The Conic Graphing Application provides enhanced conics functions to the already powerful TI-83 Plus. Graph or trace circles, ellipses, hyperbolas, and parabolas and solve for the conic's characteristics. Present equations in function, parametric, or polar form.

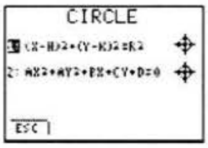
1

Select the App by pressing the APPS key and selecting *Conics*. If the App does not behave like this script, then press and ensure the window setting is "AUTO".



2

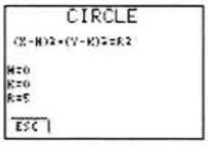
At the main menu, select from the four conic types. The main menu allows you to use the ENTER key, number keys, or soft keys (for INFO, or and for QUIT). Press the INFO soft key and the splash screen will appear for a few seconds.



3

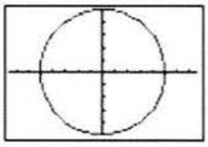
Circle In Function (X,Y) Form

Press or to select circle. Here are the two equations for circle in the XY form. Select Equation 1 by pressing .



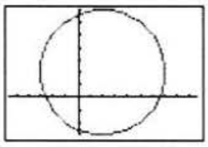
4

Enter *H=0, K=0, and R=5.* Press GRAPH.



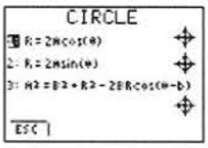
5

The circle is displayed. Press the key to go back. Press TRACE to show the points along the curve.



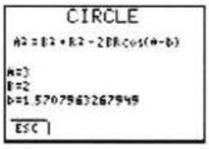
6

Change the H value to 2 and the K value to 2. Press the GRAPH key. Note that the center of the circle is now at 2,2. Press the key, then É SOLVE (above) to show the new center.



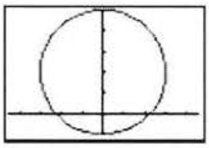
7

Circle In Polar Form. Press 3 and change the *CONIC SETTINGS TYPE* to *"POL"* Press the ESC soft key. If the equation screen of the circle is visible, the polar equations will now be displayed.



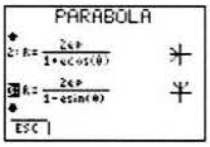
8

Note that the B and b are the polar form of the offsets. Select equation 3 and enter A=3, B=2, and b=p/2. Note that p/2 is evaluated. Different functions are evaluated in this screen. For example, enter *B=2*sin(p/2)*. This results in the value 2.



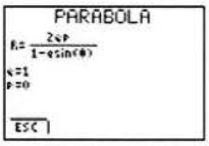
9

Press GRAPH. Then press TRACE to show the points along the curve and note the different coordinate system used. To continue with the circle, press CLEAR and change values. Using the ESC soft key, go back to a different equation or change the mode to parametric.



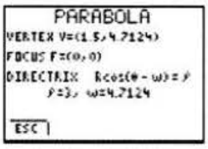
10

Parabola In Polar Form. From the circle, press the ESC soft key to return to the main menu. Press and the parabola equations appear. Since the handheld is in polar mode, there are 4 different polar equations for the parabola. Use the C and D keys to scroll to choose one. Select equation 3 (Press or highlight ∏).



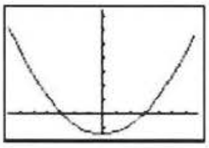
11

Fix the eccentricity of the parabola to 1. Change the P value to be 1.5 and press the É SOLVE key.



12

Here, the solutions to parabola specific terms are shown in polar form and also reflect radian mode. Exit the App and change the mode setting to degrees, re-enter the App and show the difference. The App does retain the last value for P on exit.Press the s key.



13

The graph is displayed. Press the TRACE key and use the arrows to move along the curve.

Appendix B

Survey Form

Your response to the following questions with regards to using the Conic Graphing App would be appreciated. Except for question 1, please circle the number that best represents your opinion with a "**1**" meaning you "**Strongly disagree**", and a "**5**" meaning you "**Strongly agree**."

1. How long have you been using a graphing calculator and do you own one? **Yes / No**

1 2 3 4 5

2. The teacher was knowledgeable about teaching the conic sections using graphing calculator technology and the Conic Graphing App.

Strongly disagree		Neutral		Strongly Agree
1	2	3	4	5

3. For the questions in the quiz and test of the conic sections, it was possible to use the graphing calculator to obtain part of or all of the solutions; but the questions could reasonably be answered without using it.

Strongly disagree		Neutral		Strongly Agree
1	2	3	4	5

4. For the questions in the quiz and test of the conic sections, use of the graphing calculator with the Conic Graphing App was necessary to obtain a solution or it greatly simplifies the work needed to get a solution.

Strongly disagree		Neutral		Strongly Agree
1	2	3	4	5

5. The graphing calculator with the Conic Graphing App helped me visualize ideas, understand, and retain the concepts taught in the conic sections unit.

Strongly disagree		Neutral		Strongly Agree
1	2	3	4	5

6. Learning to use the Conic Graphing App on the graphing calculator was easy for me.

Strongly disagree		Neutral		Strongly Agree
1	2	3	4	5

Thank you for completing the survey!

Appendix C

Letter to parents

To: Parents/Guardians
From: Mrs. Wong
Date: January 2 , 2007
Re: Class survey

I am your daughter or son's math teacher and would like to inform you that we started the analytical geometry unit today. In this unit the students will learn about the conic sections including circles, parabolas, ellipses and hyperbolas.

At present I am a graduate student at St. John Fisher College. To fulfill the graduation requirement, I am writing a master thesis based on an action research conducted in the class that your daughter or son is attending. This research is about studying the effect of using the Conic Graphing App, a program for graphing the conic sections on TI-83 /84 Plus graphing calculator, on teaching and learning mathematics in a high school pre-calculus curriculum. The class will be conducted as it normally would with the added feature of using the enhanced graphing capability to support effective graphing, and better understanding of the content. At the end of the unit, in about three weeks, your child will be asked to complete a survey that I prepared. The six survey questions are all related to the learning of the unit using the Conic Graphing App. He or she will complete the survey anonymously. When the survey result is gathered and analyzed, all survey forms and material will be destroyed by shredding. Please be assured that there is no risk involved and note that your child will not be compensated for completing the survey because this is for my academic research purpose only.

I thank you in advance for your cooperation and permission of letting your daughter or son participate in the survey. Enclosed are three documents for your reference:

1) information on the Conic Graphing App provided by Texas Instruments,
2) the survey that you daughter or son will be completing in class at the end of the unit, and
3) the consent form.

Please return the consent form in the envelop provided after you sign and approve your child's participation in the survey. If you have any questions or concerns please contact me via email at lwong@rhnet.org or call the R-H voicemail system at 359-5189, mailbox #2681.

Sincerely,

Laiman Wong

Appendix D

Informed Consent Form (for use with minors)

St. John Fisher College
INFORMED CONSENT FORM
(for use with minors)

Title of study: The effect of using the conic graphing application on teaching and learning

Name(s) of researcher(s): Laiman Wong

Faculty Supervisor: Dr. Diane Barrett Phone for further information: 585-385-8366

Purpose of study: The study focuses on exploring the effect of using the conic graphing feature of the graphing calculator on teaching and learning the conic sections in three high school pre-calculus classes that the researcher teaches. The purpose is to collect data for the action research in order to write a Master's Thesis. Types of data include quiz and test results, a student survey, and the author's own observation.

Approval of study: This study has been reviewed and approved by the St. John Fisher College Institutional Review Board (IRB).

Place of study: Rush-Henrietta Senior High School
Length of participation: 5 minutes
Risks and benefits: The expected risks and benefits of participation in this study are explained below:

There is no risk for the students involved in the survey because it is anonymous. The survey will be conducted in class. It should take no more than ten minutes. The survey is attached for reference.

Method of compensation, if any Students are not compensated.

Method for protecting confidentiality/privacy:

Your rights: As the parent/guardian of a research participant, you have the right to:

1. Have the purpose of the study, and the expected risks and benefits fully explained to you before you choose to allow your minor child to participate.
2. Withdraw from participation at any time without penalty.
3. Refuse to answer a particular question without penalty.
4. Be informed of appropriate alternative procedures or courses of treatment, if any, that might be advantageous to you or your minor child.
5. Be informed of the results of the study.

I , the parent or guardian of ______________________________, a minor ________ years of age, consent to his/her participation in the above-named study. I have received a copy of this form.

Print name (Parent/Guardian) Signature Date

Laiman Wong

Print name (Investigator) Signature Date

If you have any further questions regarding this study, please contact the researcher listed above. If you or your child experiences emotional or physical discomfort due to participation in this study, contact the Office of Academic Affairs at 385-8034 or the Wellness Center at 385-8280 for appropriate referrals.

Appendix E

Informed Consent Form

St. John Fisher College
INFORMED CONSENT FORM

Title of study: The effect of using the conic graphing application on teaching and learning

Name(s) of researcher(s): Laiman Wong

Faculty Supervisor: Dr. Diane Barrett Phone for further information: 585-385-8366

Purpose of study: The study focuses on exploring the effect of using the conic graphing feature of the graphing calculator on teaching and learning the conic sections in high school pre-calculus classes that the researcher teaches. The purpose is to collect data for the action research in order to write a Master's Thesis.

Approval of study: This study has been reviewed and approved by the St. John Fisher College Institutional Review Board (IRB).

Place of study: Rush-Henrietta Senior High School Length of participation: 5-10 minutes

Risks and benefits: The expected risks and benefits of participation in this study are explained below:

There is no risk for students involved in the survey because it is anonymous. The survey will be conducted in class. It will take no more than five minutes. The survey is attached for reference.

Method for protecting confidentiality/privacy: All data collected will be destroyed at the end of the thesis writing process by shredding.

Your rights: As a research participant, you have the right to:

1. Have the purpose of the study, and the expected risks and benefits fully explained to you before you choose to participate.
2. Withdraw from participation at any time without penalty.
3. Refuse to answer a particular question without penalty.
4. Be informed of appropriate alternative procedures or courses of treatment, if any, that might be advantageous to you.
5. Be informed of the results of the study.

I have read the above, received a copy of this form, and I agree to participate in the above-named study.

____________________ ____________________ Date: _______________

Print name (Participant) Signature

Laiman Wong_______ ____________________ Date: __________________

Print name (Investigator) Signature

If you have any further questions regarding this study, please contact the researcher listed above. If you experience emotional or physical discomfort due to participation in this study, please contact the Office of Academic Affairs at 385-8034 or the Wellness Center at 385-8280 for appropriate referrals.

Appendix F

Class notes on circles and part of parabolas

Name____________________________ Chapter 6
Pre-Calculus Notes

6.1 Circles and Parabolas (Day 1)

Circle –

Center-Radius Form of the Equation of a Circle –

Equation of a Circle with Center at the Origin –

EXAMPLE 1
Find the center-radius form of the equation of a circle with radius 6 and center (-3, 4). Graph the circle by hand. Give the domain and range of the relation.

EXAMPLE 2
Find the equation of a circle at the origin and radius 3. Graph the relation, and state the domain and range.

EXAMPLE 3
Using a graphing calculator to graphing each circle in a square viewing window.

a) $x^2 + y^2 = 9$

b) $(x+3)^2 + (y-4)^2 = 36$

General form of the equation of a circle:

Notice that the signs of x
By completing the square, we can get the equation of the form:

If $m > 0$, then

If $m = 0$, then

If $m > 0$, then

EXAMPLE 4 *Finding the Center and Radius by Completing theSquare*

Decide whether each equation has a circle as its graph.

a) $x^2 - 6x + y^2 + 10y + 25 = 0$

Isolate the x terms and y terms in parentheses on left side of the equation. Keep the constant term on the other side.

Complete the square for x and y separately.

Factor and **add**.

Center: ___________

Radius: ___________

b) $x^2 + 10x + y^2 - 4y + 33 = 0$

Center: ___________

Radius: ___________

c) $x^2 + 10x + y^2 - 4y + 33 = 0$

*To complete the square, make the coefficients of x^2 and $y^2 = 1$

Center: ___________

Radius: ___________

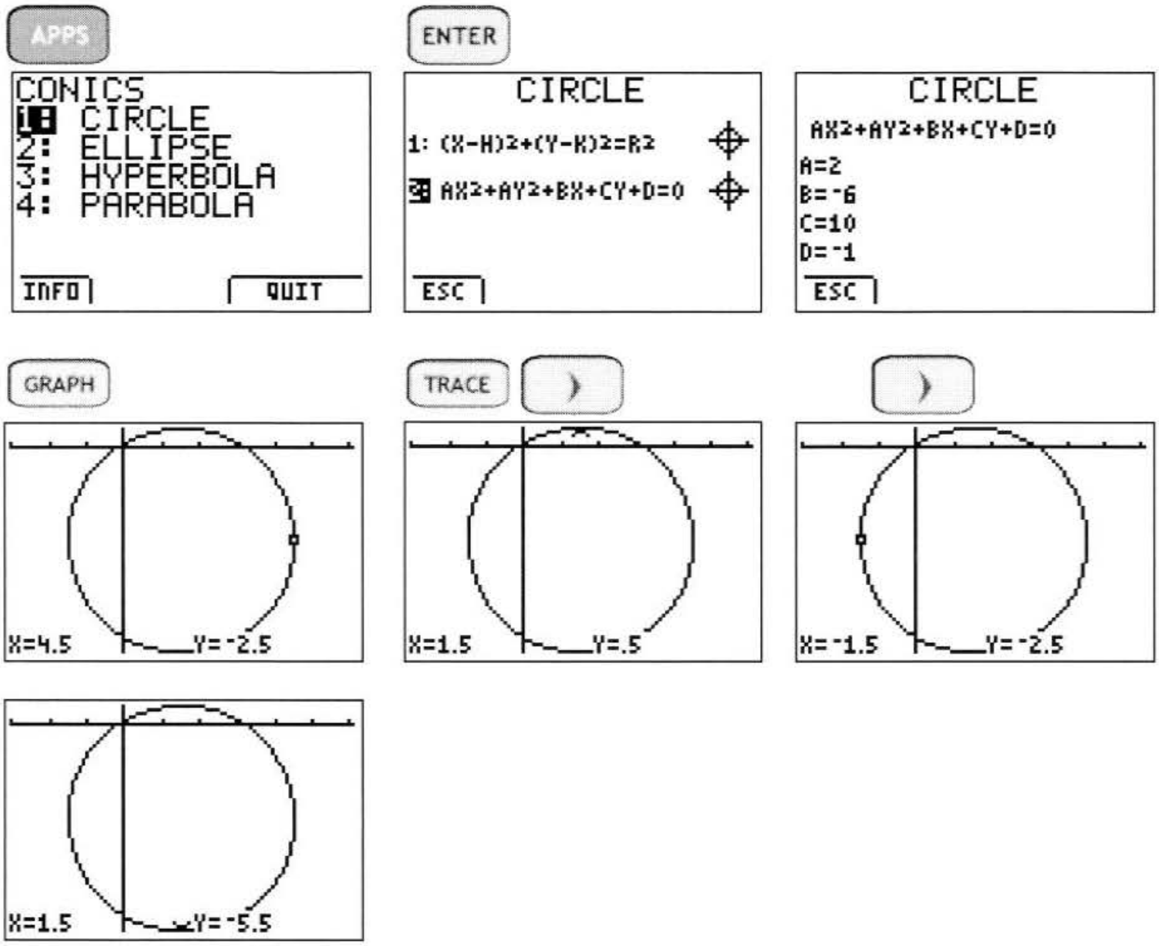
Try this:
$x^2 + y^2 - 6x - 2y = 26$

Center: ___________

Radius: ___________

OTL: Sect. 6.1 (Day 1) p. 428, # 1, 2, 11-19 (odd), 31, 36, 41, 43, 45, 48, 50, & 52.

Using the Conics App to check your work(completing the square): Circle



Center: ___________

Radius: ___________

SHOW ALL WORK ON COMPLETING THE SQUARE.

Appendix G

Assessment

Name__________________________Block____ Chapter 6
Pre-Calculus Test **A**

TEST

1. Identify the type of conic section represented by the equations below.

a) $5x^2 + 6x = 2y^2 + 3y + 18$ a) ____________________

b) $\frac{(x-3)^2}{16} + \frac{(y+1)^2}{23} = 1$ b) ____________________

c) $-3x^2 - 2x - 3y^2 + 9y + 4 = 0$ c) ____________________

d) $2(x-3)^2 + y = \frac{7}{2}$ d) ____________________

2. Put the equation of the parabola in standard form.

$x^2 - 4y - 10x + 1 = 0$ ______________________

3. Put the equation of the conic section in standard form.

$8x^2 - 7y^2 - 16x + 42y - 111 = 0$ ______________________

4. Graph the ellipse. Find the coordinates of the foci, center, co-vertices, and vertices. Give the domain and range and eccentricity.

$$\frac{(x-3)^2}{9}+\frac{(y+1)^2}{36}=1$$

Center____________________

Vertices__________________

Co-Vertices_______________

Foci______________________

Eccentricity______________

Domain____________________

Range_____________________

5. Graph the hyperbola. Find the coordinates of the foci, center, and vertices, and the equations of the asymptotes. Give the domain and range and eccentricity.

   $4x^2 - 4y^2 = 100$

Center___________________

Vertices_________________

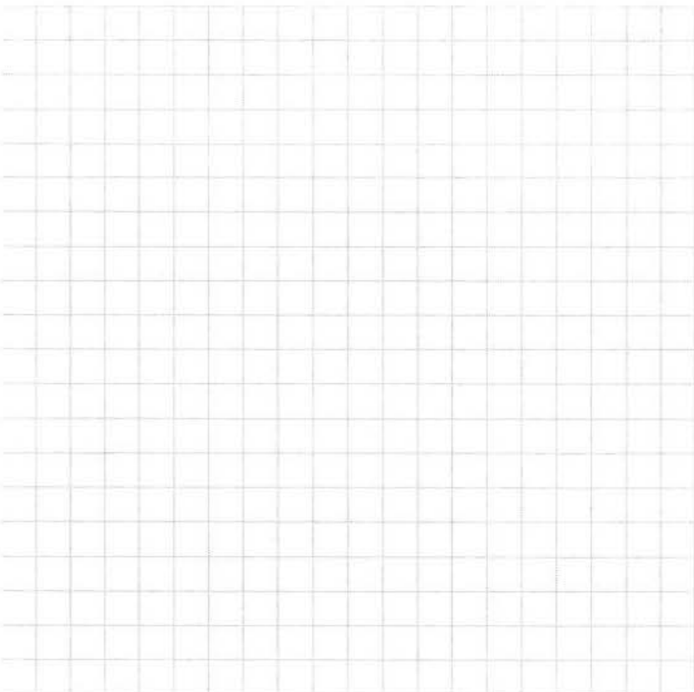
Foci____________________

Asymptotes______________

Eccentricity_____________

Domain_________________

Range__________________



6. Graph the parabola. Find the vertex, focus, and directrix. Give the domain and range.

$$(y+2)^2 = -12(x-1)$$

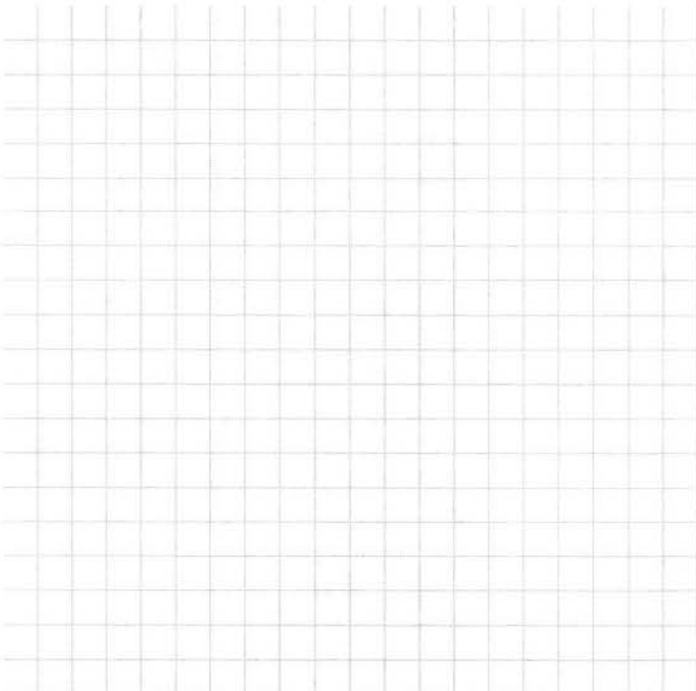
Vertex__________________

Focus__________________

Directrix________________

Domain_________________

Range__________________



7. Determine the equation of an ellipse in standard form whose minor endpoints are (4, 11) and (-6, 11) and where the distance between the two foci is 18. (The use of the graph is optional.)

____

8. Find the equation of the conic section in standard form, given the following information. (The use of the graph is optional.)

   Centered at (-4, 3); Focus at (2, 3); Eccentricity = 3

____