Teaching Vocabulary in Mathematics: the Language of Mathematical Thinking

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Teaching Vocabulary in Mathematics: the Language of Mathematical Thinking

Standardized testing is a widely used practice in all academic areas to assess student achievement. Many of these assessments have moved towards the evaluation of student thinking and communication in addition to the evaluation of content knowledge. The academic area of mathematics is no exception. Students must now provide, not only answers to mathematical problems, but they must show their arithmetic and explain their thinking in words.

The use of language in mathematical thinking and processing has become an area of interest to educators and researchers. Mathematical language is being investigated as a tool for communication of thinking, as well as the delivery tool for instruction and for assessment of learning. Instruction in today's classrooms most often involves the use of language by the teacher to deliver instruction, by students as a way of questioning and processing new information, and by both teachers and students for evaluation of mathematical thinking.

Attention to the lingual aspect of mathematics has not always been a focus in classrooms. Strategies for instruction of mathematical language are currently being researched and implemented to improve student learning. Researchers have also focused on current misconceptions surrounding and involving mathematical language, and the challenges that present themselves in the educational context.

It has been suggested that deliberate instruction and practice of the use of language in mathematics have improved students' ability to communicate their mathematical thinking and access content knowledge to problem solve and process
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mathematical problems. This study investigates the effect of targeted and intense language instruction in mathematics on students’ ability to use language to process, discuss, explain, and describe mathematical thinking.
Literature Review

There has been a recent focus in education on the language of mathematics and the implications of the teaching of this language, or lack there of, for students. The nature of mathematical language and its uses has become a topic of study for many researchers. Common misconceptions, their origins, and other challenges that face teachers of mathematics have also become a current topic for educators and researchers. As a result of recent studies and research, there have been many suggestions in current publications for the teaching of mathematical language for understanding and communication of mathematical concepts and ideas. This paper will review current available literature on these topics.

Uses of Language

Language is the medium of teaching and is the major means of communication between educators and learners (Thompson & Rubenstein, 2000). Therefore, language and its use in the classroom has become a focus for continued improvement of teaching skills and strategies. Vygotsky (1962) suggested that words are the most basic unit of meaning. He also stated that language dramatically affects a child’s cognitive development. Similarly, Thompson and Rubenstein believed that by processing ideas through language, students are able to build understanding. By listening to their oral communications and reading their writings, teachers are able to diagnose and assess students’ understandings. This allows teachers to “first understand children’s difficulties in making sense of mathematical language,” (Thompson & Rubenstein, 2000). Language is clearly an important part of teaching and learning in today’s schools.
More specifically, the vocabulary a student uses provides access to concepts (Monroe & Panchyshyn, 1995). The vocabulary a student acquires is based on the context in which it appears, and these experiences continually develop the overall language of the student. These experiences are opportunities to explore, investigate, describe and explain ideas, which lead to reorganization of concepts and new or deeper understandings. “Generalizing ideas through communication is vital when building mathematical language,” (Steele, 1999).

The Language of Mathematics

It was determined that a child’s ability to learn and perform mathematics is dependent on that child’s ability to read and process language (Knight & Harris, 1997; Skypek, 1981). As a result, the National Council for Teachers of Mathematics has stated that all K-12 students should be able to use communication to organize and consolidate their mathematical thinking and analyze and evaluate the mathematical thinking and strategies of others (2000). The crucial role of mathematical language is the building of sound concepts and subsequent development of mathematical thinking (Raiker, 2002; Barton, Heidema, & Jordan, 2002; Lee & Herner-Patnode, 2007).

The specific language of mathematics has been described in many different ways. It is largely accepted that the language of mathematics includes symbols, words and notations not often encountered in everyday life (Shields, Findlan, & Portman, 2005; Braselton & Decker, 1994). These terms, phrases, signs, graphics and symbols are essential in communicating mathematical ideas (Rubenstein & Thompson, 2002; Adams, 2003; Barton, Heidema, & Jordan, 2002). The terms, or vocabulary, of mathematical
language have been found to be a key component in the understanding of mathematics (Lee & Herner-Patnode, 2007; Miller, 1993; Shields, Findlan, & Portman, 2005; Thompson & Rubenstein, 2000).

Monroe and Panchyshyn (1995) described mathematical vocabulary using four different categories. The first category is Technical Vocabulary. This term refers to the specific terminology of mathematics (e.g. integer, quadrilateral) that can only be defined using mathematical language and concepts. Subtechnical vocabulary describes the set of words with more than one meaning, which is dependent upon the subject in which the word is being used, for instance, the word volume can mean a level of noise or the space inside a three-dimensional object. General vocabulary includes everyday language with meanings that are universal. The final category of vocabulary, as defined by Monroe and Panchyshyn, is symbolic. This group includes all non-alphabet symbols used in mathematics, including signs, numerals, and abbreviations.

In conjunction with the varied categories of vocabulary used in mathematics, there also exists an order of the language that must be learned to fully understanding mathematical language. “Mathematics is a language of order, and reading mathematics requires that one pay attention to several principles that guide how the reading must take place if accurate interpretation, comprehension, and communication are to result,” (Adams, 2003). Adams suggests three principles of order:

- Principle I: Mathematical operations are performed between only two numbers at a time.
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Principle 2: The order in which operations are written is not necessarily the order in which they are performed.

- Principle 3: Formats and presentations of numbers can change and vary with trends of society (e.g. using decimals instead of hyphens for phone numbers).

The language of mathematics is complex and different from other everyday language uses, making it an essential focus of instruction in the classroom.

Misconceptions, Challenges, and Pitfalls

As with all subjects of learning, there exist many areas in the language of mathematics that can create opportunities for misconceptions and challenges or pitfalls for both teachers and learners. This section will explore some of these misconceptions, challenges, pitfalls.

Concepts. Mathematical concepts are largely connected in a hierarchical way. Misconceptions of concepts learned previously can affect the understanding of new concepts. Sound concepts and the development of mathematical thinking are dependent on the precise establishment of meanings of mathematical words (Raiker, 2002). Also, problems can arise from and are often compounded by the spoke language involved, and teachers’ lack of awareness about the power of their use of mathematical language (Raiker, 2002; Tracy, 1994). Another difficult aspect of mathematical concepts is that they are often abstract and require experiences with concrete examples to develop understandings (Tracy).

A conceptual challenge that can often occur specifically in elementary classrooms is truths that are ultimately untrue. Teachers often create these truths to assist their
students in their current understandings, but in subsequent learning students learn these truths to be false (Tracy, 1994). For example, an elementary classroom teacher may teach students that a bigger number cannot be subtracted from a smaller number, since these students do not yet have the cognitive ability to perform such operations. However, as they progress, these students will learn that it is indeed possible with the introduction of negative numbers.

*Vocabulary.* The vocabulary of mathematics provides several opportunities for misconceptions. One difficulty is terms have meanings in mathematics that are different than their meanings when used in everyday language (Noonan, 1990; Rubenstein & Thompson, 2002; Shields, Findlan, & Portman, 2005; Adams, 2003). Also, word meanings may differ from previous knowledge of the word, even within the same subject context, or they may be used in more than one way (Shields, Findlan, & Portman, 2005; Rubenstein & Thompson, 2002). Terms are often abstract in mathematical language as they usually describe or represent concepts as opposed to objects (Miller, 1993).

Discrepancies between the understandings of word meanings between teachers and students, based on their relative positions on the novice/expert continuum, can also lead to confusion (Raiker, 2002). Teachers can attribute to further confusion by their general lack of awareness of key concepts and lack of planning for the introduction, explanation of meaning, and repetition during teaching of mathematical concepts and vocabulary (Raiker, 2002).

The symbolic category of mathematical vocabulary can also lead to misconceptions as meanings are often represented by multiple symbols, for instance, an
x, parentheses, and a dot can all represent multiplication (Monroe & Panchyshyn, 1995). Furthermore, these symbols are not necessarily universal, which can create misconceptions in an increasingly culturally diverse environment (Furner, Yahya, & Duffy, 2005).

A major challenge for the instruction of mathematical language is opportunities to use mathematical vocabulary are limited to school settings, and the vocabulary is rarely encountered in everyday life. Therefore, students are not likely to have background knowledge for these words (Monroe & Panchyshyn, 1995-1996; Rubenstein & Thompson, 2000 & 2002)

Mathematic Reading Materials. Schell has recognized that texts used in mathematics education can contain more concepts per line, sentence, and paragraph than any other kind of texts (1982). Mathematical texts can also present information in unfamiliar ways to students, including right to left (e.g. number lines), top to bottom (e.g. tables), and diagonally (e.g. graphs) (Barton, Heidema, & Jordan, 2002). Another unfamiliar concept students encounter in mathematic texts, particularly in word problems, is that main ideas of word problems often do not appear until the end of the problem (Barton, Heidema, & Jordan, 2002). Whereas most of the students’ experiences in identifying main ideas in other areas of reading teach them that the main idea is most often found in the beginning sentence or paragraph of a reading selection.

Strategies for Teachers

As awareness of the importance of the development of mathematical language for communication and thinking has increased, so has the attention to research and
implementation of successful teaching strategies to facilitate this type of learning. Some strategies specifically address visual, oral, or kinesthetic practice for students. Other recommendations address more general practices that should be attended during all instruction and practice.

General Recommendations. Although a plethora of specific teaching strategies exist, some general recommendations can also be made in regards to the instruction of language in mathematics. One important observation is that reading comprehension strategies that have been proven effective for general language instruction are also effective for the instruction of mathematical language development (Monroe & Panchyshyn, 1995). Direct teaching approaches must be used for the development of mathematical vocabulary (Monroe & Orme, 2002).

Vocabulary and key terms should be established during planning and teachers must be careful to use correct language when teaching mathematics (Adams, 2003; Furner, Yahya, and Duffy, 2005; Monroe & Panchyshyn, 1995; Raiker, 2002). Formal vocabulary should be carefully spoken, written, spelled, illustrated, and used to ensure students’ accurate understandings (Rubenstein & Thompson, 2002). Differentiation of mathematical definitions from everyday language is also important (Rubenstein & Thompson, 2002). This is particularly important as it is more efficient to teach children correct terms initially than to correct and re-teach misconceptions (Tracy, 1994).

Teachers must also provide vocabulary instruction in a meaningful context for the students as a way to teach and extend knowledge in relation to students’ real-world experiences while paying attention to cultural differences (Adams, 2003; Furner, Yahya,
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and Duffy, 2005; Harmon, Hedrick, & Wood, 2005; Monroe & Orme, 2002; Monroe & Panchyshyn, 1995; Shields, Findlan, & Portman, 2005; Steele, 1999). The activation and link to background knowledge is a fundamental aspect of effective teaching. The sequence of vocabulary instruction should evolve from building concepts first, to expressing understandings informally, and finally to learning formal language (Rubenstein & Thompson, 2000 & 2002; Lee & Herner-Patnode, 2007).

Giving students repeated exposure to experience and practice with vocabulary and concepts is another essential aspect of effective mathematics language instruction (Shields, Findlan, & Portman, 2005; Monroe & Panchyshyn, 1995; Furner, Yahya, & Duffy, 2005; Harmon, Hedrick, & Wood, 2005). Repeated exposure can be facilitated by integrating instruction with other subject areas, particularly in self-contained elementary classrooms (Rubenstein & Thompson, 2002; Furner, Yahya, & Duffy, 2005). Repetition can also be provided in differentiated ways by using a variety of strategies with a multimodal approach that incorporates multiple intelligences (Thompson & Rubenstein, 2000; Furner, Yahya, & Duffy, 2005; Lee & Herner-Patnode, 2007).

One final general recommendation for teachers, and particularly school administrators, is continuing staff development training in the areas of mathematics language development and vocabulary instruction (Harmon, Hedrick, & Wood, 2005). As with all professional careers, keeping abreast of the latest research and developments in instruction can help teachers ensure that students are receiving effective instruction to maximize their learning opportunities.
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Visual Strategies. Many current effective strategies for vocabulary instruction help connect terms or phrases to background knowledge (Rubenstein & Thompson, 2002; Furner, Yahya, & Duffy, 2005; Harmon, Hedrick, & Wood, 2005). Concept and semantic mapping and webs lend themselves to this type of instruction (Raiker, 2002; Barton, Heidema, & Jordan, 2002; Monroe & Panchyshyn, 1995). Semantic webs and graphic organizers provide visual representations of the connections and associations between words and concepts that are new and those which are parts of background knowledge (Monroe & Orme, 2002; Barton, Heidema, & Jordan, 2002; Shields, Findlan, & Portman, 2005; Braselton & Decker, 1994; Harmon, Hedrick, & Wood, 2005). These visual representations allow students to analyze similar characteristics of mathematical concepts (Barton, Heidema, & Jordan, 2002). Although formal vocabulary is important, first allowing students to invent their own terminology, later to be replaced by formal language, can help activate and link prior knowledge (Rubenstein & Thompson, 2000 & 2002; Adams, 2003). An anticipation guide can also activate prior knowledge for students (Barton, Heidema, & Jordan, 2002; Shields, Findlan, & Portman, 2005). An anticipation guide is developed by teachers for student use prior to instruction to highlight key concepts, activate prior knowledge, and provide a framework for questioning.

Some other suggestions for visual learners include picture definitions (Thompson & Rubenstein, 2000). This strategy pairs written definitions of vocabulary terms with pictures of examples and non-examples, or other visual cues, to help students recall information. Mathematical graffiti is a strategy that can elicit background knowledge,
and is a “visual tool to aid students in thinking about aspects or characteristics of the language,” (Thompson & Rubenstein, 2000). Students create a large poster by including any information they can relate to the topic or word without any organization or sequence. Another familiar strategy used in general vocabulary development is the use of a word walls. Creating a mathematics word wall can help students access terminology and recall different meanings of words specific to the field of mathematics (Rubenstein & Thompson, 2002; Furner, Yahya, & Duffy, 2005).

The study of word origins, including root words, prefixes and suffixes help students decode and analyze words, as well as make associations between words (Rubenstein & Thompson, 2002; Shields, Findlan, & Portman, 2005; Harmon, Hedrick, & Wood, 2005). Words with identical or similar pronunciations, but different meanings, called homophones, can be important tools for vocabulary instruction as well. Attention to these words can help diffuse misconceptions and confusion for students (Adams, 2003).

A less frequently thought of, but equally important, form of exposure for students is the use of numerals in multiple contexts (Adams, 2003). Students need practice reading and using popular formats for numerals. For example, experience with numbers used in Social Security Numbers, phone numbers, prices, and zip codes give students contextual experience with which to relate new information.

Encouraging independent reading provides a wide reading experience that increases exposure to vocabulary and concepts (Harmon, Hedrick, & Wood, 2005). Providing students with experience with literature can also be facilitated by using trade
books with mathematical themes and concepts. Trade books teach concepts in the context of a story, integrate subjects areas, develop mathematical thinking, create a less math-anxious environment, provide for a variety of responses, make historical, cultural and practical application connections, allow for the use of manipulatives as it relates to the story, assesses children’s understandings through reading and questioning, provide a wide range of books, lead to problem solving and active involvement, and provide shared experiences for students and teachers (Rubenstein & Thompson, 2002; Monroe & Panchyshyn, 1995; Furner, Yahya, & Duffy, 2005; Harmon, Hedrick, & Wood, 2005).

Also, nonmathematical material is much easier for a child to read and understand than mathematical material, because nonmathematical material makes use of terms and sentence structures much more familiar to a child (Brennan & Dunlap, 2001).

After initial instruction, it is important to provide multiple opportunities for students to express understandings of terminology in writing using journals, stories, cartoons, bumper stickers, skits, raps, songs, poetry, or writing their own problems (Rubenstein & Thompson, 2002; Monroe & Panchyshyn, 1995; Furner, Yahya, & Duffy, 2005). These written practices can also include nonlinguistic representations [drawings] created by the student to help create meaning, make associations, and make personal connections with vocabulary and content (Barton, Heidema, & Jordan, 2002; Furner, Yahya, & Duffy, 2005).

**Oral Practice.** Kari and Anderson (2003) recommended the use of mental math sessions to help build strong vocabulary to describe thinking. Students share computational strategies, experience repeated discourse, and use multiple problems as a
form a differentiated instruction during mental math sessions. This intensive use of language during group problem solving gives students opportunities to communicate their thinking and practice the use of mathematical language (Thompson & Rubenstein, 2000; Braselton & Decker, 1994; Monroe & Panchyshyn, 1995; Furner, Yahya, & Duffy, 2005). Think Alouds can also be integrated into this strategy to allow the teacher or another experienced other to model their thought processes and use of mathematical language (Barton, Heidema, & Jordan, 2002; Braselton & Decker, 1994; Furner, Yahya, & Duffy, 2005).

More oral practice of mathematical language can be provided with choral responses. This method uses verbal, rhythmic responses by the students to questions or prompts by the teacher as a way for students to orally rehearse knowledge (Thompson & Rubenstein, 2000).

A final suggestion for practice of mathematical language is the presentation of group or individual projects to the class (Thompson & Rubenstein, 2000). This strategy allows for differentiation based on the types of projects completed by the students and gives them opportunities to share their work and critique the work of their peers.

*Kinesthetic Strategies.* A key facet of vocabulary instruction is to link abstract concepts to concrete experiences (Tracy, 1994; Monroe & Panchyshyn, 1995). One way to accomplish this is through the use of manipulatives to demonstrate mathematical concepts and terms (Thompson & Rubenstein, 2000; Furner, Yahya, & Duffy, 2005). For example, the use of algebra tiles to demonstrate concepts of multiplication.
Another hands-on strategy to use with students is writing words on cards for students to organize according to meanings and use in mathematics (Shields, Findlan, & Portman, 2005). The manipulation of the cards helps students with kinesthetic learning needs to interact with terms and concepts.

Other vocabulary games that can provide more kinesthetic practice with mathematical vocabulary include bingo, concentration, Pictionary, charades, and Fake-Out. Fake-Out is a game in which students create false definitions or examples to be added to a list with the real definition or example of a word, and opposing teams must identify the correct definition or example (Shields, Findlan, & Portman, 2005). Card games can also help students learn equivalent verbal, symbolic, story or picture representations of the same values (Rubenstein & Thompson, 2002). Two examples of card games are I have...Who has... and Step Forward and Take a Bow. In the first example, students start by identifying their own card as an example of the descriptors given, and then provide a descriptor of a different card to be identified (i.e. I have ten. Who has a number that is a multiple of 2, but is less than 6?). The second example begins with students standing in a row, each holding a card. The teacher asks for students whose numbers have a certain characteristic to step forward and take a bow. This continues until only one student is left. Participation in such games provides students with opportunities for social interaction centered on the language of mathematics.

Combination Strategies. As previously stated, the most effective approach to vocabulary instruction is multimodal and one of variety. A large goal of mathematics instruction is to teach students problem solving strategies such as working backward,
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drawing a picture, making a simpler problem, looking for a pattern, trial and error, acting out, using a table, the 4-step process: read, plan, do, check (Furner, Yahya, & Duffy, 2005; Adams, 2003). These problem solving strategies are often used in conjunction with each other and lend themselves to the use of many of the previously discussed strategies. For example, a teacher may integrate writing and problem solving by creating a graphic organizer with two columns: one for computation and one for writing explanations of thinking that was used for problem solving, and later orally presented to the class (Thompson & Rubenstein, 2000).

Collecting examples of math terminology from media, writing about what they learned, and critiquing the presentations of peers not only incorporates many learning modalities, but also links mathematical language to real-word examples (Thompson & Rubenstein, 2000). As media comes in many forms, this strategy can be differentiated for different learning modes and needs.

By using internet fieldtrips and mathematics software, teachers can integrate technology into mathematics language instruction and address multimodal learners (Furner, Yahya, & Duffy, 2005). Furner, Yahya, and Duffy suggest heterogeneous groupings for computer and other cooperative activities to maximize the use of diverse background knowledge and maximize the benefits from balancing strengths and weaknesses of members of the group (2005).

Assessment. The strategies previously discussed also provide teachers with ample opportunities for the assessment of their students' mathematical communication and use of language in thinking processes. Student journals and product-oriented semantic webs
[webs created after a unit of study] provide insight as well as create a record of learning that can be used in cumulative assessments like portfolios (Thompson & Rubenstein, 2000; Shields, Findlan, & Portman, 2005). These written pieces show evidence of student knowledge, display meaning connections, and assist with recall.

Thompson and Rubenstein also suggest the practice of being the silent teacher to allow students to lead discussions and choose language and vocabulary used in discussions. This practice gives tremendous insight into the students' uses of mathematical language and any misconceptions they may have through authentic conversations with their peers without intervention from the teacher (2000). One observation that can be made during any assessment after the use of these strategies is that vocabulary development can improve students' self-esteem about learning math and can encourage them to work on challenging tasks (Lee & Herner-Patnode, 2007).

Summary

It is evident that the use of mathematical language is a key component to the development of mathematical thinking and communication. As with any academic area, instruction of specialized language must be deliberate and thoughtful. The effective use of language must be monitored and assessed in authentic ways, and instruction should be modified to the needs of the students. Through the use of targeted vocabulary instruction, common misconceptions can be avoided or corrected, and students' ability to think mathematically and communicate their thinking in effective ways can be improved.
Methodology

An investigation into the direct instruction of vocabulary during mathematics lessons was conducted to evaluate its effectiveness in increasing students' ability to describe and communicate mathematical concepts, thinking, and processes. Students received daily instruction in mathematics with a structured focus on vocabulary and use of language. Student learning was facilitated and evaluated using a binder system with five components with an evaluation rubric for each section.

Participants

For this research, a fifth grade class of 16 students was used. The student body of the classroom consisted of 8 males and 8 females, ranging in age from 10 to 12 years of age. The students in the class were culturally diverse, consisting of 44% African American/Black students, 19% African American/Black and Hispanic, 19% Hispanic, 12% White, and 6% White and African American/Black.

The class included 8 students who have been classified as individuals with special needs. The disabilities of the students receiving special education services include learning disabilities, Attention Deficit/Hyperactivity Disorder, high lead levels (which presents with similar symptoms as ADHD), and auditory processing deficits. Three other students in the class were identified as at-risk as they were performing significantly below grade level in mathematics and English language arts.

The teaching staff in the classroom consisted of two full time teachers, one general education teacher and one special education teacher. The general education teacher had 8 years of teaching experience with students in grades 5 and 6. The special
education teacher had 6 years of teaching experience in the area of special education in grades kindergarten through 12th grade.

The school where the research was conducted is located in an urban setting. The student body ethnic profile of this urban district is as follows: 65% African American/Black, 21% Hispanic, 12% White, and 2% Asian/Native American/East Indian/Other. Other student body profile available through public access through the school district includes: 88% eligible for free/reduced-price lunch, 17% with special needs, 8% with limited English proficiency, 35 different language groups, 50% of schools at 90% poverty or higher, and the school district has the highest poverty rate among the largest 5 districts in the state.

The elementary school where the research was conducted housed grades kindergarten through sixth grade. The cultural make-up was as follows: American Indian 0.2%, Asian 1.6%, African American/Black 62.8%, Hispanic or Latino 18%, Multiracial 0.2%, and White 17.1%. 91-100% of the population at the school was on public support.

The classroom contained two teacher desks and chairs, 18 student desks and chairs, two kidney bean-shaped tables with six chairs each, four desktop computers on two tables with chairs, one carpeted area, an easel with chart paper, one large white board, six medium sized chalkboards, an overhead projector with viewing screen, and variety of educational materials including grade level textbooks, below-at-above leveled trade books, and mathematical manipulatives.
Instruments and Materials

Teacher instruments and materials included Investigations fifth grade instructional series, chart paper and easel, overhead projector and screen, model of student binder, and a word wall separated into alphabetical sections (two letters per page).

Student instruments and materials included student binders. The binders consisted of a 1 or 1 ½ inch 3-ring binder with five tabbed dividers. The sections of the binder were as follows: 1-Do Now, 2-Class Notes and Work, 3-Homework, 4-Word Wall, and 5-Learning Reflection Log. Each section consisted of a one page list of expectations and loose-leaf paper, except for the word wall section which contained lined pages with two letters, in alphabetical order, per page. Students used pencils as a main writing utensil for work; however, colored pencils and crayons were sometimes used as well.

Data Collection

Data was collected by assessing students’ work in each section of the student binders. Work was assessed on several dimensions, including correct mathematical answers to problems, effective communication of processes and thinking occurring during problem solving, and the inclusion of written explanations, numerical representations, and pictures/drawings as representations.

Data was collected daily by informal observations of student work and class discussions. Data was collected weekly by collecting student binders to formally assess student work. Work was assessed by both teachers in the room to assure consistency.
Procedures

This study was conducted using a five-week unit plan entitled Mathematical Thinking. The daily lesson plans can be found in Appendix F: Daily Lesson Plans for Mathematical Thinking Unit. A general lesson plan format was used for each lesson in the unit to establish a predictable pattern for the students. For each lesson, key vocabulary was briefly introduced or reviewed at the beginning of the lesson. The identified vocabulary was correctly pronounced by the teacher and repeated twice by the students. Each word was correctly written on either the white board or a piece of chart paper so it was visible to the students throughout the lesson, and defined using a grade level appropriate definition.

Each word was used at least 5 times by the teachers throughout the lesson. When students correctly used an identified vocabulary word, they were praised. When students failed to use terminology to explain their thinking or used terminology incorrectly, the teachers offered redirection and encouragement to the correct use of vocabulary. The teachers provided at least two examples of each vocabulary term during each lesson, including visual representations when appropriate.

As part of the closing of each lesson, students recorded any new vocabulary terms into their personal word walls of their binders. The teachers encouraged students to use their own words or kid language when writing definitions in personal word walls. Students also were required to include numeral representations and/or pictures when appropriate in definitions. As part of the word wall procedure, students also modified existing definitions as understandings deepened, or new examples were discovered.
Students were also encouraged to discuss with their understandings with partners to compare their definitions and check for accuracy. During these procedures, teachers circulated to assist students and ensure accuracy in definitions and examples being added to the students’ word walls.

Another closure procedure used each day was 5 to 10 minutes of journal writing. Students used journals to give a written description of key ideas or mathematical processes learned or practiced during the day’s lesson. One expectation of the journal writing was that students were to include at least one of the identified key vocabulary terms from the lesson correctly in their written description. They were also encouraged to include any vocabulary learned in previous lessons as appropriate. Either during their writing or immediately after, students highlighted vocabulary terms used in their journal entry by circling, underlining or highlighting the terms.

Encouragement during these procedures included verbal discussions, both as a class and on an individual basis, to recall information from the lesson. Students were also encouraged to refer to class notes and work, visual representations in the classroom, and word wall entries when completing tasks.
Results

Assessment of student learning was mainly based on the contents of the students’ math binders. These binders included five sections, including: Do Now, Word Wall, Class Work, Learning Reflection Log, and Homework. For the five-week unit of study, binders were collected on a weekly basis and reviewed for completion of work, accuracy of computations, and effective language use for insightful mathematical thinking. Rubrics used for the evaluation of student work can be found in Appendices A-E.

Do Now

The expectation for this section of the binder was that students practice math problems involving skills previously learned. Students were required to show all computational work, use pictures, numbers and words to explain problems as appropriate, and to identify important words and symbols in word problems.

The following is an example question from this section during the first week of instruction: Draw the number rectangles (arrays) that can be made using the number 48. The assessment of this section after the first week of instruction resulted in two students consistently using complete sentences to explain the processes used in problem solving. An example of a consistent and appropriate response is the student drew 10 rectangles and labeled the dimensions of the rectangles, which are the factors of the number. The student also included a written statement to describe his/her thinking that stated: “First I started with the number one and made a rectangle that was 1 x 48 and 48 x 1. Then I tried to see if I could make a rectangle that was 2 wide and I could. It was 24 long. And I knew I could do 24 x 2. Then I tried 3, 4, 5, and 6 and I made rectangles for all those...
numbers except 5.” Two students, 12.5% were able to consistently provide this level of response in the Do Now section of their notebooks.

The next level of response included correct use of rectangles for the question, but lacked the written description of thinking or provided a limited description. For example, “I made rectangles with the numbers for 48.” Four students, 25%, responded to questioning with at this level.

Eight students, 50%, were able to complete the question with some, but not all, rectangles that answered the problem and omitted any written response. And, finally, an additional two students, 12.5% did not provide any accurate responses to the question.

After five weeks of instruction in this unit, responses in the Do Now section of the binder improved. Although the content of the questions changed throughout the unit, the numerical, pictorial, and written criteria for responses remained constant. At the end of the fourth week eight students, 50%, were at a consistently accurate level of response. Five students, 31.25%, were at a somewhat consistently accurate level; two students, 12.5% were at an occasionally consistently accurate level; and one student, 6.25%, was not able to produce any consistently accurate responses.

Word Wall

The expectation for this section of the binder was that students would enter mathematic vocabulary into this section as new words were introduced during lessons and as needed when they encountered words they thought were appropriate to add. Students were expected to use pictures, numbers and words as applicable to define and exemplify the concept.
An example of a consistent accurate response for this section would be the word *rectangle* written in the *r* section of the word wall with a definition "a polygon with 4 sides, 2 pairs of parallel sides and 4 right angles." Students also included a picture of a rectangle as an example, and another shape as a non-example.

After the first week of instruction, all students were dependent on teacher-provided definitions. However, all students were able to provide their own examples and non-examples for entries during the first week. Through informal observations, it was also noted that, when prompted, students referred to entries in their word walls during lessons and work time as a support.

During the fifth week assessment of this section of the binders, it was observed that students continued to use teacher-provided definitions; however, they were also including synonyms to help them understand definitions. An example of this would be "a rectangle is a polygon (shape) with 4 sides, 2 pairs (sets) of parallel sides (like train tracks), and 4 right angles (like the corner of a paper)." Students continued to be able to provide pictures or numerical representations as examples and non-examples for their word walls.

*Class Work*

This section of the notebook included students’ notes from class and any examples or problems worked on during lessons. The expectations were that students would include all written information presented during the lesson completely and accurately, including underlining any key terms and making any side-notes that helped them with comprehension. All students were able to consistently meet these criteria
Teaching Vocabulary 30

throughout the unit, although some students with special needs required extra time to copy some information that was presented visually. In addition, all students included teacher-provided example pictures and arithmetic examples accurately.

*Learning Reflection Log*

This section of the binder required student to reflect on their learning using written descriptions. Students were encouraged to use vocabulary discussed during lessons and included in their word walls to describe their learning. Students also used this section to pose questions or seek clarification.

An example of a consistently accurate entry in the reflection log was “Today we talked about the kinds of factors a number has. When we listed factors sometimes numbers had a lot of factors and sometimes they only had two. We learned that there’s a name for when number has only two factors, it’s called prime. And all the other numbers are called composite. An example of a prime number is 3. A composite number is 6.”

After the first week of instruction, two students, 12.5%, created consistently accurate and insightful entries in their reflection logs. Twelve students, 75%, included reflections that lacked either completeness or accuracy. The remaining 2, 12.5%, students were unable to independently produce a written reflection on learning.

Following five weeks of instruction, five students, 31.25%, were able to reflect on learning with consistent accuracy and insightfulness. Five students, 31.25%, were able to reflect on learning with somewhat accurate and insightful reflections. Five students, 31.5%, were able to occasionally produce accurate and/or insightful reflections. One
student, 6.25%, remained unable to produce accurate and insightful reflections on learning.

Homework

The homework section of the students’ binders was included as a central location to record responses to all homework assignments. As these assignments varied from computational practice to the solving of word problems with written statements to describe mathematical thinking, this section was generally evaluated for completeness, accuracy, and overall use of language.

The evaluation of homework after the first week of instruction showed five students, 31.25%, able to consistently produce accurate and complete homework. Seven students, 43.75%, were able to produce somewhat accurate and complete work. The remaining four students, 25%, were able to occasionally produce accurate and complete work. It was observed that 72% of inaccuracies were computational, with the remaining 28% as a result of incomplete work or inaccurate descriptions of mathematical concepts and application.

The evaluation of homework following five weeks of instructions showed minimal changes in the students’ work, except for the improvement of two students quality of work from somewhat to consistently accurate and complete. However, the evaluation of inaccuracies revealed a shift from 72% that of computational errors to 80%, and from 28% to 20% of errors involving incomplete work or inaccurate descriptions.
Discussion

The hypothesis of this investigation posed that the direct instruction of mathematical vocabulary can increase students’ ability to describe and communicate mathematical concepts, thinking and processes. In general, the results of this study support this hypothesis. Students showed improvements in all areas evaluated using the sections of the student binders. This is most evident in the data shift observed in the inaccuracies in the homework section with more complete and accurate use of language.

As discussed in the review of current literature, specific strategies to focus instruction on the use of vocabulary and language in mathematics increased student learning. The specific strategies recommended in the literature that were included in this study include the use of a word wall for vocabulary, the use of discussion both guided by teacher and students, practice using written and verbal explanations of mathematical thinking, and repeated exposure. Another strategy discussed in the literature that was a main focus for this study was for teachers to focus on the use of vocabulary during instruction. Mathematical vocabulary was introduced in meaningful ways, multiple meanings of words were discussed with emphasis on the mathematical use, correct grammar usage and spelling was consistently modeled, and formal language use was also modeled and encouraged. Students were given opportunities to explain thinking in their own terms, followed by teacher support to restate thoughts using formal language when applicable.
This study produced positive results with use of strategies recommended in the literature reviewed. The establishment of vocabulary and key terms during planning, as recommended by several previous studies allowed teachers to focus on the careful and correct use of specific language during mathematical instruction (Adams, 2003; Furner, Yahya, and Duffy, 2005; Monroe & Panchyshyn, 1995; Raiker, 2002). This practice was evident in Appendix F with inclusion of focused vocabulary for the unit.

Rubenstein and Thompson stated that formal vocabulary should be carefully spoken, written, spelled, illustrated, and used to ensure students' accurate understanding (2002). This strategy was followed with the use of modeling for specific sections of the student binder including class work and word wall. These sections also allowed teachers to differentiate between mathematical definitions and everyday use of language as well.

The general lesson plan format used in this study allowed students to build concept knowledge, then express understandings informally, followed by learning formal language and connecting formal language to concepts learned. This practice was recommended by repeatedly by Rubenstein and Thompson in 2000 and 2002, as well as by Lee and Herner-Patnode in 2007.

Repeated exposure to formal mathematics language was achieved through direct instruction, review of previous concepts, visual aids in the classroom, and independent practice for the students using a variety of visual, oral, and kinesthetic practice strategies. Repeated exposure was frequently referenced in the review of literature as a successful strategy for the development of mathematical language (Shields, Findlan, & Portman,
Some possible applications of this study and the accompanying review of literature include the inclusion of language development as part of mathematical curriculum in schools. Furthermore, the evaluation of learning in mathematics should include the use of language. Although mathematics is often simply thought of as the use of numbers to represent concepts, operations, and processes, educators must shift this thinking to include the natural use of language as part of the human processing of information, including mathematical thinking. By giving students the tools to participate in dialogue, both internal and with others, teachers help deepen students' understanding of concepts and processes.

Children acquire language through emersion and exposure to language, including intentional introduction, exposure, and practice. These practices apply to the acquisition of mathematical language as well. Therefore, educators must also consider the prior knowledge and associations students may have with mathematical language, particularly misconceptions students may hold, and plan instruction based on student needs.

**Challenges**

Some variables of this study could not be controlled and therefore could have affected results. One of these variables is the use of multiple concepts used throughout the unit of study. Evaluation of learning differed based on content and task.

Another variable that may have unintentionally influenced results is the differences of the students. The sample for this study was not large and consisted of a
high ratio of students with special needs, including speech and language difficulties. Also, students participating in this study had not previously received intense language instruction in mathematics and may have lacked prerequisite language and content knowledge to process and master current grade level concepts and processes.
Conclusion

The use of language in mathematics is clearly a large part of learning and developing deep understandings. Implications in education focus on instruction and assessment of learning. Although many recommendations have been made in current literature, further research is needed to support the use of suggested strategies in mathematics instruction. Many recommended strategies for mathematics language instruction are extensions of current strategies used in general reading and writing language development, but do not have empirical evidence to support their use in mathematics.

This study has focused on the use of language in mathematics; however, further investigation is warranted into the effect of mathematical language development on a student’s overall ability to communicate effectively in all areas. Two main areas of learning for mathematics are reasoning and logic. By increasing a student’s ability to communicate thinking in these areas, educators may influence the student’s ability to communicate in other areas involving these critical thinking skills.
References


### Appendix A

**Do Now Rubric**

<table>
<thead>
<tr>
<th>Do Now</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tbody>
<tr>
<td>Accuracy of computations and/or modeling</td>
<td>Computations and models are inaccurate.</td>
<td>Some computations and models are accurate.</td>
<td>Computations and models are mostly accurate with one or two errors.</td>
<td>All computations and models are accurate.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Explanations are unclear or absent.</td>
<td>Explanations lack clarity and may contains some inaccuracies.</td>
<td>Explanations are mostly clear and accurate.</td>
<td>Explanations are clear and accurate.</td>
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</table>
## Teaching Vocabulary
### Appendix B
**Word Wall Rubric**

<table>
<thead>
<tr>
<th>Word Wall</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td><strong>Accuracy of definition</strong></td>
<td>Definition is incomplete or inaccurate.</td>
<td>Definition is missing one key aspect.</td>
<td>Definition is complete and accurate with some grammatical errors.</td>
<td>Definition is complete, accurate, and grammatically correct.</td>
</tr>
<tr>
<td><strong>Examples and non-examples</strong></td>
<td>Examples do not exemplify the concept, or are not included.</td>
<td>Either examples or non-examples are provided to model the concept.</td>
<td>Examples and non-examples exemplify the concept.</td>
<td>Examples and non-examples clearly exemplify the concept.</td>
</tr>
<tr>
<td><strong>Use of clarifying language</strong></td>
<td>Everyday language is used incorrectly to clarify formal language.</td>
<td>Everyday language is included to clarify some formal language when prompted.</td>
<td>Everyday language is included to clarify formal language when prompted.</td>
<td>Everyday language is independently included to clarify formal language.</td>
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### Appendix C
#### Class Work Rubric

<table>
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<tr>
<th>Class Work</th>
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<tr>
<td>Accuracy of computations and/or modeling</td>
<td>Computations and models are inaccurate.</td>
<td>Some computations and models are accurate.</td>
<td>Computations and models are mostly accurate with one or two errors.</td>
<td>All computations and models are accurate.</td>
</tr>
<tr>
<td>Complete work</td>
<td>Information from class discussions and/or instruction is documented with several errors or omissions.</td>
<td>Some information from class discussions and/or instruction is documented.</td>
<td>Most information from class discussions and/or instruction is documented.</td>
<td>All information from class discussions and/or instruction is clearly documented.</td>
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## Appendix D

### Learning Reflection Log Rubric

<table>
<thead>
<tr>
<th>Learning Reflection Log</th>
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<th>4</th>
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<tr>
<td><strong>Content</strong></td>
<td>Key concepts and/or skills from instruction are included with errors or omissions.</td>
<td>Some key concepts and/or skills from instruction are included.</td>
<td>Most key concepts and/or skills from instruction are included.</td>
<td>All key concepts and/or skills from instruction are included.</td>
</tr>
<tr>
<td><strong>Language Use</strong></td>
<td>Formal language is used to describe few concepts and skills, without clarifying everyday language.</td>
<td>Formal language is used to describe some concepts and skills, with some clarifying everyday language as necessary.</td>
<td>Formal language is used to describe most concepts and skills, with clarifying everyday language as necessary.</td>
<td>Formal language is used to describe concepts and skills, with clarifying everyday language as necessary.</td>
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## Appendix E

### Homework Rubric

<table>
<thead>
<tr>
<th>Home Work</th>
<th>1</th>
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<td>Accuracy of computations and/or modeling</td>
<td>Computations and models are inaccurate.</td>
<td>Some computations and models are accurate.</td>
<td>Computations and models are mostly accurate with one or two errors.</td>
<td>All computations and models are accurate.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Explanations are unclear or absent.</td>
<td>Explanations lack clarity and may contain some inaccuracies.</td>
<td>Explanations are mostly clear and accurate.</td>
<td>Explanations are clear and accurate.</td>
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# Daily Math Plans for Mathematical Thinking Unit

**Grade Level:** Fifth  
**Unit:** 1  
**Mathematical Thinking at Grade 5**

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
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**Whole Group Lessons**

<table>
<thead>
<tr>
<th>Investigation 1, Session 1</th>
<th>Investigation 1, Session 2</th>
<th>Investigation 2, Session 3</th>
<th>Investigation 1, Session 4</th>
<th>Investigation 1, Session 5</th>
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<tbody>
<tr>
<td>• Introducing the Mathematical Environment, p. 4</td>
<td>• Continue with Building Number Rectangles</td>
<td>• Number Shapes, p.9</td>
<td>• Review Yesterday’s homework—Student Sheet 2</td>
<td>• Teacher Checkpoint: Solving Number Puzzles, p.20</td>
</tr>
<tr>
<td>• Building Number Rectangles, p.5</td>
<td>• Special Math Words, p.7</td>
<td>• Discussion: Names for Rectangles, p.9</td>
<td>• Using Puzzle Clues, p.18 (300 chart)</td>
<td></td>
</tr>
</tbody>
</table>

**Homework**

| Send home family letter, p. 103 | Student Sheet 2, Factor Pairs from 1 to 25, p.105 (graph paper) | Student Sheet 3, Skip Counting, p.106 | Student Sheet 6, What’s the Number, p.109 |

**Teacher Support**

<table>
<thead>
<tr>
<th>Teacher Note: Read Caring for and Storing Materials, p.12</th>
<th>Teacher Note: Read Using Mathematical Vocabulary, p.14</th>
<th>Dialogue Box: Read Solving a Number Puzzle, p.24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dialogue Box: Read Talking About Calculators, p. 15</td>
<td>Notes: Make sure you have 300 charts available. You may want to discuss the use of 300 chart for solving the puzzles.</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary**

| rectangles | squares, factors, odd even | prime, composite | |

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Appendix F
<table>
<thead>
<tr>
<th>Day 6</th>
<th>Day 7</th>
<th>Day 8</th>
<th>Day 9</th>
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<td><strong>Lesson</strong></td>
<td><strong>Lesson</strong></td>
<td><strong>Lesson</strong></td>
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| Investigation 1, Session 6  
- Make Your Own Puzzle, p.22 | Investigation 2, Session 1  
- Strategies for Remembering Factor Pairs, p.31 | Investigation 2, Session 2  
- Teacher Checkpoint: Factor Pairs of 100, p.35 | Investigation 2, Session 3  
- Factor Pairs of Multiples of 100, p.37 | Investigation 2, Session 4  
- Building Rectangles with 1000, p.40  
*Have students focus on efficient strategies* |
<p>| <strong>Homework</strong>  | <strong>Homework</strong>  | <strong>Homework</strong>  | <strong>Homework</strong>  | <strong>Homework</strong>  |
| Student Sheet 8, Find the Counting Numbers, p.119 | Student Sheet 9, More Factor Pairs, p.120 | Student Sheet 10 Counting Backwards, p.121 | Student Sheet 7, Make an Impossible Puzzle, p.110 (Review) |
| <strong>Teacher Support</strong> | <strong>Teacher Support</strong> | <strong>Teacher Support</strong> | <strong>Teacher Support</strong> | <strong>Teacher Support</strong> |
| Teacher note: Read Two Important Ways of Building Numbers, p.33 | Dialogue Box: Read Counting Around the Class, p.34 | Notes: If students are having difficulty finding factor pairs of 100, use graph paper to make all the rectangles that have exactly 100 squares. | Notes: It is important to have the discussion about the strategies that students used to find the factor pairs of 200 and 300. Have students explain the strategies that they used. | Notes: Remind students that it is important for them to write down how they know that their rectangles have 1000 squares. Does the student have a particular strategy besides counting all the squares to know they have 1000? |
| <strong>Vocabulary</strong> | <strong>Vocabulary</strong> | <strong>Vocabulary</strong> | <strong>Vocabulary</strong> | <strong>Vocabulary</strong> |
| multiples | | | | |</p>
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<tr>
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**Whole Group Lessons**

<table>
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<tr>
<th>Investigation 2, Session 4 con’t</th>
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<tr>
<td>*What Rectangles Did We Make?, p.40</td>
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<table>
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<tr>
<td>*Numbering Squares in Our Rectangles, p.44</td>
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<tr>
<td>*Displaying 10,000 squares, p.46</td>
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<table>
<thead>
<tr>
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<tr>
<td>*Counting to 100, 1000, and 10,000</td>
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<table>
<thead>
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<tr>
<td>*Solving Multiplication Clusters, p.55</td>
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**Teacher Checkpoint:**

<table>
<thead>
<tr>
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**CATCH UP DAY**

**Homework**

<table>
<thead>
<tr>
<th>Practice Page A, p.123</th>
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<tbody>
<tr>
<td>Student Sheet 11, What Could You Count By?, p.123</td>
</tr>
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</table>

**Teacher Support**

**Dialogue Box:**

| Read Relationships Among Factor Pairs of 1000, p.43 |
| Notes: Students should be able to locate numbers on the chart without counting by 1’s. |

**Notes:** Have students have focus on efficient strategies. Each pair will need a 20 x 50 or 25x40 rectangle. IF pairs did not make these rectangles, if pairs did not make these rectangles give them time to, for the next session.

**Teacher Note:**

| Read Powers of 10: How Much Larger, and How Many Times Large?, p.54 |

**Notes:** You may want to do some more modeling of solving problems before students work on Student Sheet 12. It is important that students understand they do not have to use all the problems to solve the cluster.

**Teacher Note:**

| Read About Cluster Problems, p.62 |

**Vocabulary**

<table>
<thead>
<tr>
<th>strategy, cluster</th>
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</thead>
</table>

**Vocabulary**

<p>| multiplication, addition |</p>
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<th>Unit: 1 Mathematical Thinking at Grade 5</th>
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<td>5.N.13, 5.N.14</td>
<td>5.N.13, 5.N.14</td>
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<td><strong>Whole Group Lessons</strong></td>
<td><strong>Whole Group Lessons</strong></td>
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<tr>
<td>Investigation 3, Session 2</td>
<td>Investigation 3, Session 3</td>
</tr>
<tr>
<td>- Finish Student Sheet 12 if needed</td>
<td>- Making Our Own Problem Clusters, p.58</td>
</tr>
<tr>
<td>- Sharing Our Cluster Strategies, p.57</td>
<td><strong>Whole Group Lessons</strong></td>
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<tr>
<td><strong>Investigation 3, Session 4</strong></td>
<td><strong>Investigation 4, Session 1</strong></td>
</tr>
<tr>
<td>- Solving Division Clusters, p.58</td>
<td>- In pairs, have students play <strong>Close to 1000</strong> for 30 minutes and then have then play <strong>Close to 0</strong> for the remainder of the time.</td>
</tr>
<tr>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
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<tr>
<td>Student Sheet 13, More Multiplication Clusters, p.126</td>
<td>Student Sheet 18, Writing About Multiplication Clusters, p.131</td>
</tr>
<tr>
<td><strong>Teacher Support</strong></td>
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<tr>
<td><strong>Teacher Note:</strong></td>
<td><strong>Teacher Note:</strong></td>
</tr>
<tr>
<td>Read <em>The Relationship Between Division and Multiplication</em>, p. 63</td>
<td>Read <em>What About Notation?</em>, P.64</td>
</tr>
<tr>
<td><strong>Dialogue Box:</strong></td>
<td><strong>Notes:</strong></td>
</tr>
<tr>
<td>Read Ways to Solve 46x25, p.62</td>
<td>In your discussion encourage students to think how using multiplication to solve the problems. Record strategies on chart paper as students are sharing</td>
</tr>
<tr>
<td><strong>Vocabulary</strong></td>
<td><strong>Vocabulary</strong></td>
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<tr>
<td>division</td>
<td>division</td>
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<tr>
<td>Lesson</td>
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</table>
| **Investigation 4, Session 3**  
- Review Choice Time expectations | **Investigation 4, Session 4**  
- Review Choice Time expectations | CATCH UP DAY | **Investigation 4, Session 5**  
- Hat Can You Say About 10,000?, p.89 | **Investigation 4, Session 5**  
- Assessment: Add a Clue, p.91  
- Factor Pairs of 10,000 |
| **Homework** | **Homework** | **Homework** | **Homework** | **Homework** |
| Student Sheet 28, Problems, From Close to 0, p.144 | Practice Page G, p.168 |  |
| **Teacher Support** | **Teacher Support** | **Teacher Support** | **Teacher Support** | **Teacher Support** |
|  |
| Teacher Notes: Read Assessment: Add A Clue, p.94-95 |

Vocabulary  
Vocabulary  
Vocabulary  
Vocabulary  
Vocabulary