Using Three-dimensional Cubes In the Development of Number Sense and Spatial Reasoning of First Grade Students

Deborah R. Parrish-Hill

St. John Fisher College

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of First Grade Students

Deborah R. Parrish-Hill
Math, Science and Technology Education
St. John Fisher College
Rochester, New York
December 18, 2000
Using Three-dimensional Cubes
in the Development of Number Sense and Spatial Reasoning
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OVERVIEW

In this study of six first grade students, my initial goal was to observe and describe the varying developmental levels of spatial reasoning and number sense through which these students would progress as they solved increasingly complex tasks and activities involving one-inch, three-dimensional, primary-colored cubes. These are the types of cubes used in studies by Battista (1998) and that are in most primary classrooms.

Based upon those observations, my ultimate goal was to demonstrate that when young students have meaningful early experiences with one-inch, three-dimensional cubes, they will be able to reach New York State's benchmarked standards (see Appendix A) in 3-D geometry as they are related to the Rochester City School District's standards. To reach that goal and to help classroom teachers more effectively match instructional materials and activities with students' developmental needs, more research was needed on the effects of early experiences with cubes on children's ability to manipulate (physically and mentally) spatial structures.

The development of spatial reasoning and number sense in primary education is important because it should provide young students with the opportunity to build increasingly elaborate schemas of spatial structuring as they progress into middle school and high school where their geometric studies will become more sophisticated.

In classroom textbooks (for example, Silver, Burdett and Ginn Mathematics, 1992), as well as in the research literature (for example, Irons, 1996), there is a remarkable variation of perspectives regarding first grader's understanding of spatial concepts, as well as the type of spatial, number, and problem-solving strategies that teachers should provide. This wide-ranging issue has created a need for a much better understanding of number sense and spatial reasoning processes in first grade students so
that classroom teachers can meet the needs of elementary students and implement state and national standards in their classrooms.

Background for this study was based partially upon my experiences of teaching first graders over a number of years, as well as informal interviews with some of the teachers in my building who use teacher-created activities; that is, activities the teachers themselves have devised, including two pre-school teachers in my building who use one-inch 3-D cubes in their classrooms to foster eye-hand coordination, pattern recognition, stacking/balance understanding, and color identification. One kindergarten teacher uses cubes for counting activities and color identification. Two first grade teachers use the cubes as a math manipulative for counting. A third/fourth grade teacher uses cubes with her students for spatial-related activities.

In addition, several school supply companies offer sets of pattern task cards that teachers can use with one-inch cubes at the K-3 grade levels. For example, I reviewed materials published by the widely-used Silver, Burdett & Ginn Mathematics K-4 textbooks in order to evaluate their approach to promoting students' learning using one-inch cubes. I found that the first and second grade programs offered very few opportunities for students to work with cubes. Most of the lesson concepts consisted of identification of basic shapes, classification activities, and identification of the basic properties of a cube, such as its flat sides, corners, and faces. Lessons offered in the third-grade text provided little information: (a) two pages on observing different perspectives of shapes (top, front and side), (b) one and a half pages on understanding volume, (c) one question in a chapter review test related to using one-inch cubes, and (d) one question in a chapter test on one-inch, 3-D cubes.

The spatial relations activities at the pre-school level can include (see Weikart, Rogers, Adcock, and McClelland, 1971):

**Body awareness and body concept**

**Motoric experience:**

- Body movements
- Activities and projects
Verbal experience

• Name of self and others
• Naming parts of body
• Naming functions of body parts
• Facial expression (e.g., happy, sad)

Position

Motoric experience:

• Body movements
• Activities and projects
• Planning and evaluation

Verbal experience:

• On/off
• On top of/over/under
• In/out
• Top/bottom
• Above/below
• In front of/in back of/behind
• Beside/by/next to
• Between
• First/Next/Last

Direction

Motoric experience:

• Body movements
• Activities and projects

Verbal experience:

• Up/down
• Forward/background
• Around/through
• To/from
• Toward/away from
• Sideways
• Across
Distance

Motoric experience:
- Body movements
- Activities and projects

Verbal experience:
- Near/far
- Close to/far from

A summary view of spatial relations’ activities on classifying shapes by grade level is shown in Table 1:
- Identification of shape
- Classifying of shape
- Counting cubes
- Identifying the basic properties of a cube (flat sides, corners and faces, views)

Table 1. Instructional Activities with Cubes by Grades

<table>
<thead>
<tr>
<th>Grade</th>
<th>Identifying shape</th>
<th>Classifying shape</th>
<th>Counting cubes</th>
<th>Identifying flat sides of a shape</th>
<th>Identifying edges, corners, faces</th>
<th>Seeing views of a cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>X</td>
<td>X</td>
<td>X (3-6, Silver)* (24, Irons)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>X</td>
<td>X</td>
<td>X 18 cubes</td>
<td>X</td>
<td>X cube</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td></td>
<td></td>
<td>X 27 cubes</td>
<td>X</td>
<td>X various shapes</td>
<td></td>
</tr>
</tbody>
</table>

* Silver Burdett & Ginn (1992) ** Irons (1996) or "Mimosa"

Taken as a whole, the tasks offered at each grade level did not seem sufficiently well-developed or thought out so as to take advantage of the possibilities of using one-inch cubes to enhance students' understanding of spatial structures. Nor did these tasks comport with the possibilities suggested by recent research (Battista, 1998) on the use of
3-D cubes. Nor did they agree with those from my own teaching practice, in which students have access to the use of cubes on a daily basis and where this material is a favorite manipulative of students.

In this research project, I implemented a series of spatial structuring and number sense tasks that spanned the simple to the more complex. I defined three different classes of strategies involving 3-D cubes that I used to code students' problem-solving activities as they engage these tasks. Analyzing and categorizing these codes enabled me to suggest developmental levels of spatial structuring that will help classroom teachers make timely and appropriate instructional decisions and develop meaningful curriculum and assessment tools.

This paper is organized as follows:

Section I contains theoretical constructs and research findings that have either formed my own theoretical framework or that I suspect might prove useful in the interpretation of my findings.

Section II describes the study and its methodology.

Section III contains the results of the study.

Section IV presents conclusions based upon the study.
SECTION I: RESEARCH BACKGROUND AND THEORETICAL FRAMEWORK

**Background.** The theoretical framework for this study is grounded primarily on the research in spatial reasoning conducted by Michael T. Battista and his colleagues. These researchers have found that in order to understand students' geometrical constructions it is necessary to not only describe students' ways of spatial reasoning but to also determine what mental processes make these constructions possible (Battista, Clements, Arnoff, Battista & Van Auken Borron, 1998). They report that the mental process most critical to understanding the way children deal with quantitative spatial situations is structuring:

We define structuring as the mental operation of constructing an organization or form for an object or set of objects. Spatially structuring an object determines its nature or shape by identifying its spatial components, combining components into spatial composites, and establishing interrelationships between and among components and composites (Battista, Clements, Arnoff, Battista & Van Auken Borron, 1998, p. 503)

Battista and Clements (1996) found, however, that spatial structuring is intimately connected with numerical processes. In describing the children's developing abilities to deal with spatial structuring of 3D rectangular arrays of cubes, Battista and Clements report that children in their earlier stages of development see arrays as unorganized sets of cubes so that when they attempt to organize the cubes their structuring uses "local" (uncoordinated set of faces) rather than a "global" use of layering. That is, they have no overall scheme for organizing all the composites that are present in 3D arrays. It is not until students have a means of iterating rows, columns, or layers of cubes (with layer structuring being the most efficient) that such global coordination and organization are possible. In particular,

[S]tudents who spatially structured an array into columns or layers organized their enumeration by the number of cubes in a column or layer, often skip-counting or multiplying to find the total. Alternatively, students who structured an array as an uncoordinated set of prism faces determined the number of cubes by enumerating cube faces on all or some of the prism faces, often counting cubes along the prism's edges more than once (Battista, Clements, Arnoff, Battista & Van Auken Borron, 1998, p. 504).
In addition to the research cited above on spatial structure and its connection to numeration processes, this study recognizes the role that students' developing problem-solving abilities might play when any attempt is made to assess students' spatial reasoning strategies when engaging mathematical tasks. Following Burns (1999), the following are assumed to be requisite to problem solving:

1) interest in finding solutions to problems
2) confidence to try various stages
3) a willingness to risk being wrong at times
4) ability to accept frustrations that come from not knowing
5) willingness to persevere when solutions are not immediate
6) understanding of the difference between not knowing the answer and not having found it yet

This study recognizes the importance of establishing a classroom culture that promotes these conditions.

Spatial Structuring Strategies. Based on this research, I have identified three classes of Spatial Structuring Strategies and Codes (Appendix B):

- Spatial Structuring Strategies
- Number Strategies
- Problem-solving Strategies

The visual reasoning strategies employed on the Summary of Spatial Codes form has been adapted from the classified strategies that Battista and Clements (1996) used when observing students working with three-dimensional, one-inch cubes in the third through fifth grades. The model applies to students in grades preschool to fourth grade.

The “Summary of Spatial Codes” form is based upon Battista’s four levels of ability, and to meet the ability levels of first grade students, I have made changes that are more appropriate for the first grade student. I used the following three spatial reasoning strategies:

Spatial Reasoning Strategy 1 (VRS1). Students were asked to conceptualize several individual units as a cube unit. In his/her effort to estimate the number
of units in the building, the students were asked to count the faces of the cubes that were visible and not conceptualize the interior cubes.

**Spatial Reasoning Strategy 2 (VRS2).** Students were asked to conceptualize the cube unit and understand that there are interior cubes, but they had to take the unit apart to see what was in the center.

**Spatial Reasoning Strategy 3 (VRS3).** Students were asked to conceptualize the cube unit in rows and/or layers. This student was asked to systematically compute the contents of the cube unit in an organized manner.

**Number Strategies.** The number strategies employed on the Summary of Spatial Codes form were adapted from the classified strategies that Battista and Clements (1996) observed when working with students in third through fifth grades. The strategies were adjusted to meet the Rochester City School Districts suggested mathematics curriculum for first grade students.

**Number Reasoning Strategies.** Number reasoning strategies employed on the Summary of Spatial Codes form were adapted from those of Battista and Clements (1996) from their work with students in the third through fifth grades. This study looked for student's use of the following strategies:

- **Estimation/Guess and Check (NS1).** An estimation or guess is a tentative calculation. A check is used by a student to examine his/her calculation.
- **Counting (NS2).** Is the act of counting natural numbers in sequential order 1, 2, 3, 4, etc.
- **Counting On (NS3).** A method of addition in which one number is handled mentally and the remaining numbers are combined through counting on natural numbers. Such as $7 + 2 = \_$. The student handles the seven mentally $7 + 1 = 8 + 1 = 9$.
- **Addition (NS4).** Addition is the operation of combining numbers, in which each number represents a separate quantity of measure to produce.
Addition (NS4). Addition is the operation of combining numbers, in which each number represents a separate quantity of measure to produce. A number to represent the whole measure of all parts. \(3 + 9 = 12\).

Doubling (NS5). Doubling is a method of combining a number (\(x \times 2\)) such as 4 and 4, 5 and 5.

Skip Counting (NS6). Skip counting is the mathematical operation of counting sets of quantities, such as counting by 2's, 3's, 4's, 5's, etc.

Multiplication (N7). Multiplication is the systematic combining of several measures of equal sizes to produce a single number \(5 \times 3 = 15\).

Problem-Solving Strategies. Three of the problem-solving strategies used on the Summary of Spatial Codes are a part of the Rochester City School District mathematics curricula to help students think and understand mathematical concepts. These are Look for a Pattern (PS2), Guess and Check (PS3), and Draw a Picture (PS4).

Look for a Pattern (PS2). The students look for patterns in colors, numbers, objects and tasks.

Guess and Check (PS3). The students guess the number of cubes that will fill a box, use cubes to make a building, and check the reasonableness of this guess when completing their task.

Draw a Picture (PS4). The students draw a picture to assist them in carrying out the performance of mathematics and gives them a visual record of their performances. The drawing of a picture by the students presents the teacher with concrete evidence on how they think about the matter at hand.

Act it Out (PS5). The student manipulates cubes to enhance one's ability to construct mathematical knowledge.
SECTION II: THE STUDY

Research Focus. The purpose of this study was to classify and then differentiate first-graders' spatial reasoning-/number-/problem-solving strategies and competencies into developmental levels as they emerge over time as the children engage in solving spatial reasoning tasks of progressively increasing complexity.

Methodology. The major role of assessment is to help teachers to identify the student's present developmental level so that they can support student learning through the appropriate selection of curriculum to promote student learning to the next advanced level. In this study, students were asked to do a series of tasks with three-dimensional cubes (see Appendix C). The students' actions during the block-based spatial reasoning activities were documented by using video taping, recording observations using the evaluation/scoring form (Appendix D), and using student interviews. The evaluation/scoring form (based upon the work of Battista and my teaching experience) was used to help classify student achievement. It can also help teachers classify achievement, select appropriate center activities, determine where the student is at and move the student to the next level of skill at the appropriate time.

There are three levels of students: proficient, practitioner, and explorer, which I defined as follows:

The Proficient (high level) student is functioning on a symbolic level. The student may manipulate the cubes to determine the correct solution to a task but does not need to do so. The student is able to conceptualize the rectangular array in rows or layered building. When estimating the number of cubes in a building or the number of cubes needed to fill a box or rectangular pattern pictures, a prediction is correct 95 to 100%. The number strategy chosen will include multiplication, skip counting, and/or doubling. The student will be able to explain what he/she has learned.
The Practitioner (middle level) will use a partially effective strategy, and can reflect on the problem at hand and self-correct. This student may or may not succeed in finding the correct solution. The student will conceptualize the rectangular array as sub-units of rows, layers, or parts. The student will estimate/guess in close proximity to the correct number of cubes in a building. This student is functioning at the connecting level. The number strategy used will include counting by ones, adding-on or skip counting. This student will be able to identify and describe a basic pattern.

The Explorer (low level) is yet preoccupied with exploring the materials. This student will manipulate the cubes through counting each individual face of each cube or by the process of adding on. There will be little or no evidence of a planned strategy. The student's ability to estimate will lack reasonableness. This student will identify color patterns, but will experience some difficulty identifying relationship in numerical patterns.

The activities (Appendix C) in this study varied in length depending on the complexity of the activity and the ability of the student. I tracked the length of time the students were engaged in each task and also noted the problem-solving strategies, spatial structuring, and number strategies the students used.

In the first week of January 2000, I introduced the students to a unit of preliminary activities on volume, which lasted for a total of three weeks. At the end of the three-week volume unit, I began a one-week series of spatial reasoning and number sense activities to determine levels of spatial reasoning and number sense within the problem-solving tasks of spatial reasoning.

During activities with manipulatives, each student was expected to take an active role in the investigations. Students were also expected to learn that there are usually several different strategies and varying solutions that can be used to solve most problems. The students were told that the goal was not to find the one "right" answer or to be the first student to complete an assignment. The expectation was that individual students were engaged in using their individual initiative to explore, discover, and strive to become an...
independent learner. The students worked in cooperative groupings and pairs to share ideas, model various learning strategies, and motivate each other.

**Activities.** The students completed specific tasks or activities that include (see Appendix C for complete descriptions):

**Activity 1 Tasks:** To develop spatial structuring skills and work with buildings

**Activity 2 Tasks:** To replicate cube buildings

**Activity 3 Tasks:** To learn about one-inch grid paper and to replicate simple structures on the grid paper

**Activity 4 Tasks:** To estimate the number of cubes that will fill a box

**Activity 5 Tasks:** To estimate the number of cubes that will fill “gray bottom” rectangular pattern pictures on grid paper.

**Activity 6 Tasks:** To estimate the number of cubes that will fill rectangular pattern pictures.

**Evaluation/Scoring Form.** The evaluation/scoring form is based upon the literature reviewed in Section I of this paper. The form contains the codes for the following spatial structuring strategies:

1) Spatial reasoning strategies
2) Number strategies
3) Problem-solving strategies
4) Comments

**The Students.** The six students were from a first-grade class at The Roberto Clemente School, a seven-year-old, two-story structure serving pre-kindergarten to fifth grade students in Rochester, New York. The school serves a low-income neighborhood where 97% of the families live at the poverty level. Of these 20 students in the classroom, five were Hispanic and 15 were African-American. One student received Ritalin to control hyperactivity; five students received speech services, where the speech therapist pulled them out of the class twice a week and worked with the entire class once a week for forty-five minutes; and two students received (ESOL) English as a second language
services, where the ESOL teacher taught in the classroom five days a week for thirty minutes, generally working with a group of four to six students. In addition, a paraprofessional worked in the classroom part-time to facilitate student learning.

The six participating students were selected from among the 12 girls and 8 boys in the class and three pairs of students were matched by using their midyear Mathematics Assessment scores through classroom observation of their ability to verbally express their thoughts, their tenacity and problem-solving, and their ability to interact cooperatively. I chose two males who scored in the 95th to 100th percentile on the Rochester City School District mid-year Mathematics Assessment for first graders. One student was hyperactive. He was not receiving medications for his hyperactivity. It was anticipated that his hyperactivity might undermine his performance ability.

I also chose two females students who scored in the 85th to 75th percentile to represent the median group. One student was classified as having a severe speech problem. The two female students represented the lower quartile by scoring in the 60th percentile range on the mathematics assessment. One of these students was classified as speech impaired. Both students are motivated to learn and work hard to be successful.

The classroom was arranged around nine interest centers and a central meeting area. The interest centers supported the curriculum and provided many opportunities to support a spatial structure program. These opportunities ranged from building a castle in the block area, to designing a garden at the science table, to working with Kid Piks on the computer. The nine centers included the following:

- Literacy Corner
- Sand Table
- Science Table
- Computer Center
- Art Area
- Math Lab
- Block Area
- Family Area
- Manipulative Area

* These areas offered many opportunities to support spatial structuring on a daily basis.

The central meeting area was where the total class could come together at tables or on a carpet area for whole class participation. This classroom arrangement, with wall hangings that provided a variety of self-teaching strategies, enabled the students to work as a whole class, in small groups, with parents, and independently. Of special interest for
this study were the manipulatives used in the centers. The manipulatives were sensorial, open-ended, and multi-faceted in the belief that children absorb knowledge from their environment simply by being (Montessori, 1984).

Several of the manipulatives used in the center activities were self-correcting and offered a variety of curriculum concepts that students in beginning explorations conceive as play. From (Piaget, 1972) we know that play has a strong cognitive tone and learning can be supported in a play domain.

The problem-solving strategy "Stop, Think and Plan" was borrowed from Carol Captuo-Schwartz, a Speech Therapist who worked within the classroom once a week for forty-five minutes. The Stop-Think-and-Plan strategy was designed for use in therapy with brain-damaged patients.

During regular classroom sessions, students (working with partners or in small groups) were presented with a problem-solving project. Through their “stop-think-and-plan” sessions, they were expected to learn to

- first, stop and silently think of their individual plan
- second, listen to their partner or teammates’ plan, and
- third, develop a verbal plan.

By the end of a typical year, students discuss their plans verbally and are able to communicate their ideas in writing. This provides a written record for the student and offers the teacher evidence on how the student is thinking.

In this study, I tried to determine if students use this strategy when they complete the study tasks (see the "Evaluation/Scoring" form), and I developed forms to assist in the process of evaluation and to provide a systematic means of collecting, recording, and communicating data. These are included in the appendices.

The "Summary of Student Performance" form was used to document evidence of students performance dealing with spatial structure, number reasoning strategies, problem-solving strategies and a timeline in which these strategies took place within the students engagement in activities. The "Summary of Spatial Structuring Codes" form provides the name for the strategies characterized in this study (spatial structuring, number strategies, and problem solving), their respective codes, and a description of the
features of each strategy. The "Three-dimensional, One-inch Cube" rubric (see Appendix E) contains the elements of the developmental progression in spatial structuring number strategies and problem-solving strategies that the students were expected to meet at the proficient, practitioner, and explorer levels.
SECTION III: RESULTS

As an experienced teacher, I knew that students enjoy working with colored, one-inch, three-dimensional cubes, which meant that I would not need to spend extensive time motivating the students to engage in the planned activities. After starting the video camera, I gave directions to the students, and the students immediately began manipulating the cubes. The process of mental problem solving and spatial reasoning was also evident immediately.

My purpose for this research was to gather evidence to classify and differentiate the first-grader’s spatial reasoning/number sense/and problem solving strategies into developmental levels. To do so, I used specific tasks/activities (see Appendix C) and evaluation/scoring forms (see Appendix D) to help track the skills the students used to brainstorm reasonable estimates and manipulate the cubes to make specific shapes ("buildings"). When scoring the students’ performances, I used the scoring forms and in addition, I constantly referred to the videotapes because a great deal of the students’ explanations included behaviors such as non-verbal gestures, facial expressions, pointing, and other hand movements.

The videotapes were essential because, even though I encouraged the students to share ideas and strategies verbally to promote thinking skills, students still used the non-verbal techniques, as had been anticipated. In fact, I found that sharing of information and communication between partners was hampered by the young student’s lack of mathematical vocabulary distinctive to the tasks.

To promote a caring and an academically supportive classroom environment, the students were encouraged to work with a partner to support each other’s learning. Each pair demonstrated a different interaction style; however, when one partner clearly displayed a greater understanding of the task, the less competent student would begin to

1 Although I had to substitute one student at the last minute because the student who had been chosen originally had to leave town for a week, I carefully chose partners who had worked together successfully on previous projects and who had established a relationship that recognized their partners for a strength that would support a friendly partnership.
compete with the stronger student, and I then had to remind the pair that they were working as a team.

If these activities had taken place in a regular classroom situation as learning activities, such competitive situations would not have occurred. To prevent competition between unequal partners in a regular classroom environment, a successful teacher works with each student on activities that are most appropriate for that individual student's level of understanding. If the teacher observes a student experiencing difficulty at a particular level, the teacher can return to reinforce the concepts of the proceeding level and move to the next higher level as the student achieves a satisfactory level of understanding.

The major role of assessment in a classroom is to help teachers identify the student's present developmental level and support student learning through the appropriate selection of curriculum that will promote student learning to a more advanced level. Related to the tasks in this study, this performance-based assessment process consists of

1) observing the students' choice in mathematical problem solving strategies
2) observing the students' spatial reasoning ability
3) observing the process and end products of individual and collaborative efforts.

A program with a sequenced set of tasks offers teachers the advantage of working with each student on their individual level. Implementing such a program involves training the teaching staff so that they understand the theory behind the design and the practices they will follow and adapt for use in their classroom.

The Study. In this section, I use the transcript data to summarize the observations that I made as students completed their 3D block tasks/activities. I sequentially present

- Transcript Data on Pairs of Students
- Transcript Data on How Pairs of Students Approached Tasks
- Transcript Data on Individual Student's Unfolding Developmental Skills
- Results for Individual Students (Transcripts of Activities)
- A summary of Findings For Students Arranged by Skills
Transcript Data on Pairs of Students. I am summarizing the data on pairs of students first, because as the students worked in pairs, their interactions helped reveal the students' developmental stages. There were three pairs of students:

- Students A and B
- Students C and D
- Students E and F

Students A and B. The following transcript for Student A and Student B on Activity 2A, shows that the students perform a series of sequential tasks to solve the problem. They visually determine which building has the most cubes, use guessing to estimate the number of cubes in each building, construct the building with cubes, and seek their answer.

1. Student B: Mine is 16 [her answer to the number of cubes in building X]. Should I do the other one?
2. Student A: Mine is 14
3. Teacher: Yes, you may start working on building Y. [Student A has constructed a building with six layers with four cubes in each layer and continues to change her formation.]
4. Student B: Your not going to trick me!!!! [laughing]
5. Student A: I changed my building because I looked at the paper. [Indicating that the building did not look like what was on the paper.]
6. Student B: It's the same as the first one. [Stating that building Y is the same as building X.] If I stand this up [Pointing to building Y] it will be the same [As building X]. It's the same height. [Student B used her hands to suggest the movement necessary to turn the building up right. Student B then physically manipulated the entire building upright to prove her point.]
7. Student A: I came up with 18 [by counting the number of cubes in the building] but she came up with 16.
8. Student B: Should I tell her?
9. Teacher: No, I want Student A to try a different way to check her answer.
10. **Student A:** 2. [Student A removes two cubes] 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. [Counted the right corner cubes twice.]

11. **Teacher:** Try checking your answer a different way.

12. **Student A:** 2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18. 1, 2, 3, 4 [Taking the building apart one cube at a time and moving each cube to the side.] 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

**Task Summary.** From the activities of these two students, we can see that two students, both of whom scored in the middle range on their Mid-year Math Assessments, do not perform equally on a spatial ability task.

**Students C and D.** As the following transcript shows, Student C and Student D, who scored high on their Mid-year Math Assessments, again performed differentially on a spatial ability task. The following scripts from tasks 6B and 6C show how the student began to offer several mathematical solutions to one problem and tried to help Student D find the "pattern" that would lead to the correct answer.

13. **Teacher:** OK Next one, 6b

14. **Note:** Student D begins cutting.

15. **Student C:** I know the answer. Can I put my answer down?

16. **Teacher:** (to Student D) Did you make your estimate?

17. **Student D:** No

18. **Teacher:** You need to make an estimate before you move ahead.

19. **Student C:** My answer is 12, because there are 6 in the middle. 1,2,3,4,5,6, 6 in the middle.

20. [Student D folds the pattern sides up to help him guess his answer.]

21. **Student C:** Hold on, I haven’t told her my answer, there are 6 in the middle.

22. [Teacher stops activity to settle students returning from art class.]

**Student C:** That’s your answer? 6?

23. **Student D:** Yes.
24. **Student C:** My answer is 12 because there are 6 in the middle and there are 2 rows on the side, and you have to put another 6 and 6x2 is 12 and my answer is 12.

25. [Student C stacks 12 cubes to be placed in the box that Student D is putting together. Before Student D finishes putting on the last two strips of tape he looks inside the box and indicates that the answer is 12.]

26. **Student C:** You probably don't know the pattern. Do you know the pattern?

27. **Student D:** No.

28. **Student C:** You have to figure out the pattern. I know the pattern.

29. [Student D begins taking the cubes from Student C’s stack of cubes.] [He appears uncomfortable not knowing the pattern.]

30. **Student C:** Are you counting the cubes while you're putting them in? If they all fit in that means I’m right. [Very confident and proves that he knows the “pattern”.]

31. **Student D:** They all do.

32. **Student C:** The answer is 12.

33. **Student D:** 12. 6x2

34. **Teacher:** OK, this is the last activity, are you ready?

35. **Student D:** Let’s do it together. You passed a row.

36. **Student C:** No. I didn’t. 1,2,3,4,5,6,7,8,9,10. See this is how I get my answer. I think it’s 30. I mean 40.

37. **Teacher:** Can you explain it?

38. **Student C:** Because it’s ten right here (pointing to the middle, then pointing to one row on a side to indicate ten is going to take up this one), and ten is going to take up another row and that’s 20, that’s 30, and that’s 40.

39. [Teacher observes Student D comparing 6a and 6b and asks what he is doing.]

40. **Student D:** Erasing because I didn’t want this answer. He begins erasing, and I don’t want 40 either.

41. [Student C begins cutting.]

42. [Student D: I know what I can do. (Begins stacking cubes)]

43. **Teacher:** What did you write down as your guess?

44. **Student D:** 80
45. **Teacher:** Student D, I want you to focus on the task that we are doing now.
46. **Student D:** I am.
47. [(Teacher assumes that he's stacking 80 cubes as Student C had stacked 12 cubes in task 6b.)]
48. **Student D:** Look there are steps.
49. **Teacher:** Student D, what should you be doing right now?
50. **Student C:** You should be trying to find the pattern.
51. **Student D:** Why do I need a pattern when I can use my fingers?
52. [Student C asks for the tape after realizing that Student D has dropped out. Student D then focused back to task and passed Student C the tape. Student D became a little aggressive after counting to 30.]
53. **Student D:** You're right.
54. **Teacher:** How did you get all three tasks correct?
55. **Student C:** Because I know the pattern.
56. **Teacher:** What is the pattern?
57. **Student C:** Six, right here, will take up one block (row) 6x2=12.
58. **Teacher:** How did you figure out the answer to 6?
59. **Student C:** Did you count them? 10 at the bottom. 10 would take up another one that's 20, then 10 would take up another one, that would be 30, and another one is 40. You can skip count by 10's, 10, 20, 30, 40, or you could do 10x4=40 or 10+30=40.

**Task Summary.** Student C was able to successfully use his spatial structuring skills to come up with the right answer, whereas Student D was not able to do so because of a lack of spatial structuring ability. In addition, Student C continually demonstrated improvement in spatial and mathematical skills with the completion of each task.

**Students E & F.** Students E and F both scored low on their Mid-year Math Assessments. The following scripts for them on Activities 5A and 5B provide insights into their spatial reasoning, number sense, and problem-solving experiences. For example, throughout each task, Student E consistently wore a smile of confidence, quite often making the comment, "I'm good at this!" Student E frequently experienced
difficulty if the pattern picture had more than one row; however, the student was very involved in the tasks, carefully watching a partner completing the tasks. After watching, Student E would change an answer after noticing from the height of the boxes that an additional level of cubes could be added.

60. Teacher: Look at the design carefully and then guess how many cubes you will need to fill the box.
61. Student F: 20
62. Teacher: Write it down. Why do you think you'll need 20 cubes?
63. Student F: Because I counted the squares.
64. Teacher: Does this paper look a little different from the paper we used yesterday?
65. Student E: It doesn’t have the black coloring spots on it
66. Student F: You’re right, it doesn’t have the gray area to show you where the bottom is. You will have to find the bottom part own your own. Teacher: Student E, what are you doing?
67. Student F: She’s changing her answer to 8.
68. Teacher: Why are you changing your answer?
69. Student E: When you put it like a box, and put these ends up, it will be 8.
70. Student F: But it doesn’t have any black marks?
71. Teacher: What happened to 20? I thought that 20 was your answer.
72. Student E: Yeah, but it’s not 20 when you put it like a box. It will be 8 (8 cubes).
73. Student F: Yes, it is [counting all the squares in the pattern] 1-20. Now watch it’s going to be 20.
74. Student F: 8
75. Teacher: Let’s not argue. Student A will make the box and then we can check our answers.
76. [As Student A places the cubes in the box. Student E counts by ones.]
77. Student F: 1,2,3,4,5,6,7,8. I’m right, I’m good at this.
78. Teacher: You’re doing a great job.
79. Teacher: Here is your next task. Think about what your answer might be and write it down.
80. **Student E:** Ooh, I know, I'm very good at this. It's 6.

81. **Student F:** I think it's 6. [Doesn't seem to know where to begin without the gray area as a visual cue, substituting pencils and crayons for the gray area.]

82. [Student E tapes the box together.]

83. **Student E:** Now I know, it's 12. [She has now observed that the box has two levels and mentally doubled or skip counted to 12.]

84. [Placing the cubes in the box, Student E counted by ones, 1-12.]

85. **Teacher:** Do you know what you missed on your first guess?

86. **Student E:** Yes, but I only counted to 12 (smiling). [The smiling seemed to indicate, yes, my first estimate was incorrect; however, I adjusted my answer correctly before I placed all the cubes in the box and therefore I'm right and I'm good at this.]

**Task Summary.** This transcript documents the interaction of Students E and F as they estimate the number of cubes that will fill a rectangular pattern picture (Task 6C). From the transcript, we can see how Student E continued to scaffold her answers by testing her predictions as she worked with the cubes.

**Transcript Data on How Pairs of Students Approached Tasks.** The following scripts demonstrate how two sets of partners approached the same problem, sometimes in very different ways. For example, when Students A and B worked as a team, they correctly completed this activity in one minute, whereas Students C and D when working as a team, took 17 minutes to complete the same building correctly – but using different approaches.

In Task 3A, it appeared that students needed to construct three buildings; however, it was actually one building facing three different directions. It was not possible for the students to make a single building. This hands-on, concrete experience was a good activity; however, an activity having the students fill in one row of four, or three rows of two, would have added more value and complexity to the final activity.

As the transcripts show, it is interesting to observe how each student’s problem-solving skill and style led to specific estimates and final products.
Students A and B.

87. Student B: Look at this.
88. Teacher: What would you like me to look at?
89. Student B: This is going to fall down. There’s no block under this.
90. Teacher: Did you try to make it?
91. Student B: Why? It’s just going to fall down.
92. [Student B smiles as she clearly expresses her confidence and at the same time dares me to present her with a more challenging task.]

Students C and D.

93. Teacher: I have our last task for you to try. Place your name on your paper please.
94. Student C: (reading) Make each building with the cubes.
95. Student D: My building is going to look better than yours.
96. Teacher: Remember that you are teammates and that you should work together.
97. Student D: How do you do that?
98. Student C: I don’t know.
99. Student D: That’s hard. Can you help me, when you finish?
100. Student C: This is tricky.
101. Student D: I don’t know how to make that.
102. Student C: Well, that’s not how to do it.
103. Student D: If I figure it out, I will help you.
104. Student C: Does anybody have any super glue?
105. Student D: Is it just 1? You just have to hold it up.
106. Student C: Can we use glue?
107. Teacher: Can you make this building with cubes only, no glue?
108. Student D: Nope, I need help.
109. Teacher: What do you need help with?
110. Student D: This last one, this is too hard.
111. Teacher: Why is it hard? What makes it hard? What about X and Y, you did those very quickly?
112. Student D: That one’s harder because you’ve got to stand it up.
113. **Student C:** I did it, I did it.

114. **Teacher:** Does it look like the building on your paper?

115. **Student D:** No.

116. **Student C:** Yes.

117. **Student D:** This is not supposed to be on the ground (table).

118. **Teacher:** What can you do?

119. **Student D:** Hold it up? [laughing]

120. **Teacher:** I don't see any hands on the paper holding it up. Student C, what do you need to make it work?

121. **Student D:** Lay it down flat?

122. **Teacher:** That's interesting. [An Ariel view].

123. **Student C:** It's something like this.

**Task 3A Summary.** I found it interesting that Student B looked at this task for 30 seconds and confidently said that it could not be done, whereas Students C and D worked at the activity for 17 minutes. When I agreed with Student D's eventual decision that "you can't do it," they seemed more disappointed than Student B in the fact that it could not be built. I think that they were expecting a magical mathematical explanation of how to make the building.

**Transcript Data on Individual Student's Unfolding Developmental Skills.** In the following section, I focus on Students C, D, and E, and observations on how their developmental skills unfold. I am focusing on Student C because this student had exceeded my expectations, Student D because my expectations based upon math scores were not met, and Student E because of a unique ability to scaffold throughout the project, as well as an ability to come through the project successfully while using lower-level mathematical computation skills.

Table 2 provides a view of the spatial reasoning, number sense, and problem-solving strategies used by Students C, D, and E to complete Tasks/activities 1-6.
Table 2. Spatial Reasoning, Number Sense, and Problem-solving Skill for Activity/Task1, Developing Spatial Structuring Skills and Working with Buildings.

<table>
<thead>
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<th>Spatial for 1A</th>
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Table 3. Spatial Reasoning, Number Sense, and Problem-solving Skills for Activity/Task 2, Replicating Cube Buildings

<table>
<thead>
<tr>
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<th>Spatial for 2a</th>
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<td>SRS3</td>
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Table 4. Spatial Reasoning, Number Sense, and Problem-solving Skills for Activity/Task 3A and 3B, Introducing One-inch Grid Paper/Replicating Simple Structures on Grid Paper

<table>
<thead>
<tr>
<th>Student</th>
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Table 5. Spatial Reasoning, Number Sense, and Problem-solving Skills for Activity/Task 4A, 4B, and 4C, Estimating Number of Cubes to Fill “Gray Bottom” Rectangular Patterns on Grid Paper

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Table 6. Spatial Reasoning, Number Sense, and Problem-solving Skills for Activity/Task 5A, 5B, and 5C, Estimating Number of Cubes to Fill “Gray Bottom” of Rectangular Patterns on Grid Paper

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Table 7. Spatial Reasoning, Number Sense, and Problem-solving Skills for Activity/Task 6A, 6B, and 6C, Estimating Number of Cubes to Fill Rectangular Pattern

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From these tables (Tables 2-7), we can see that for spatial reasoning there was a clear progression (improved ability) for Students C and D. However, Student E continued to experience difficulty with spatial structuring or reasoning throughout the project.

Related to number sense, all three of the students began the project using the same strategies. However, as the task increased in difficulty and as each student discovered a personal pattern for number sense and a problem-solving strategy. There strategies were individualized and there was an increase in the number of strategies used.

For problem solving, the students started the project using the “Stop, Think, and Plan” strategy for the first two sessions, before establishing a pattern of taking turns. However, after the first few tasks were completed, the students no longer needed the “Stop, Think, and Plan” strategy.

As I reviewed the videotapes of the students immersed in their tasks, I also realized that the guess-and-check strategy is built into the curriculum. Student C and Student E clearly had discovered their own individual pattern, whereas Student D continued to work towards finding a pattern.

One of the most interesting outcomes of this project is the difference in Number Sense strategies displayed by each student. Student C made best use of estimates by first using doubling or skip counting before offering a multiplication sentence that was equivalent. Student D used sequential-order counting, counting-on, and doubling or skip counting if the number was six or below. Student E completed series of quick mental multiplication operations. It is clear that differential math skills were used.

Results for Individual Students (Transcripts of Activities). As should be clear from the data presented, there were wide variations in spatial and mathematical skills among students who were within the same grade level and classroom. This is common in a teaching situation, and it further implies that there is a need for self-directed learning and the use of manipulatives within a structured environment.
Student A. It was obvious after the first task that Student A needed additional time on a task prior to moving to another activity. Moving the student to the more complex activities the acquisition of a full understanding of a basic step confused the student and kept the student behind throughout the unit. Student A was very dependent on a partner.

Student B. Student B was a very observant student, always looking ahead. Most of the mathematical operations in the study involved skip counting; however, this student could give a multiplication-based equal when doing the task. When asked to demonstrate specific observations, or the problem-solving technique to support a position, the student was always very confident and frequently answering, “I just knew.” This indicated that the completion of a quick mental calculation.

Student C. Student C started the project by setting up the cubes in an aerial view until Student D suggested constructing the buildings upright. With that simple prompt, Student C excelled and experienced success beyond my expectation.

Student D. Student D’s high level number sense did allow not for the level of success that I had anticipated. This was because of the spatial reasoning problems that which the student encountered. Because the student had always excelled in mathematics, and as a result accepting help from a partner was difficult. Frequently the student rejected the partner’s assistance and temporarily their friendly relationship became hostile.

Student E. Student E worked at a low level of mathematics, counting sequentially, adding on and doubling or skip counting when the number of cubes in a row or layer did not exceed five. Nevertheless, this student experienced success throughout the project because she scaffolded throughout each task.

Student F. Student F was dependent on a partner throughout the project, requiring additional practice in the beginning tasks. However, this student was also the most imaginative student I studied, as was evident, for example, when she suggested that we create boxes that were three-feet wide by taping papers together to make lunch boxes and pocketbooks.
Summary of Findings For Students Arranged by Skills. In this section, the results of the tasks and other information on students (Mid-year Math Assessment, Table 8) are summarized and students are placed into three groups by using the following criteria:

- Criteria 1 (Spatial Reasoning Ability)
- Criteria 2 (Problem-solving Skills)
- Criteria 3 (Number Sense Skills)

As is shown in Table 8, the students had varying mathematical reasoning skills. These skill levels are linked to the NYS Mathematic Standards (Appendix A).

Table 8. Students Categorized by School District Mid-year Math Assessment

<table>
<thead>
<tr>
<th>High</th>
<th>Middle</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student C</td>
<td>Student B</td>
<td>Student E</td>
</tr>
<tr>
<td>Student D</td>
<td>Student A</td>
<td>Student F</td>
</tr>
</tbody>
</table>

Based upon performance on their spatial reasoning skills, we categorized students into the three groups (explorers, practitioners, and proficient, Table 9).

Table 9. Students Categorized by Rating on 3-D Cube Tasks

<table>
<thead>
<tr>
<th>Proficient</th>
<th>Practitioner</th>
<th>Explorer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student C</td>
<td>Student D</td>
<td>Student F</td>
</tr>
<tr>
<td>Student B</td>
<td>Student E</td>
<td>Student A</td>
</tr>
</tbody>
</table>

Explorers/Low Group. Two students (A and F) were categorized as members of the low group or Explorers based upon their spatial reasoning skills. Student A's and Student F's spatial reasoning skills were not sufficient for them to discover the pattern picture. They definitely did not have a strategy. Both students seemed to do better when working with papers that had visual cues (gray areas). Overall, explorer-level students used number strategy that included doubling, skip counting, adding on, and counting by ones.
Parishioners/Middle Group. Two students (D and E) were categorized as members of the middle group on spatial abilities. Student D and Student E demonstrated difficulty with spatial reasoning, noting sides and layers. Student D used doubling or multiplication in his calculations. Student D and Student E both worked with a partial strategy that sometimes offered success, sometimes they were reasonably close to the correct answer, and they also made unreasonable predictions as they worked through various strategies that might bring them success. Student E frequently counted by ones, doubled and calculated by adding-on. She also experienced some difficulty in spatial reasoning. Overall, practitioner-level students used number strategy that included multiplying, doubling, skip counting, adding on, and counting by ones.

Proficient/High Group. The remaining two students (C and B) were categorized as members of the high group on spatial abilities. Student C and Student B demonstrated their understanding of spatial reasoning and number sense in most activities by making calculations through doubling or multiplying by the number rows. They both definitely had a strategy. Student C and Student B showed increasing understanding with each task they performed. Over all, proficient students used number strategies that included multiplying, doubling, skip counting, and adding on.

Next, in Table 10, we looked at the relationships between spatial reasoning skills (3D Cube) and Mid-year Math Assessment scores, and it is immediately clear that there is not a one-to-one relationship.

Table 10. Students by 3-D Cube and Mid-year Math Assessment

<table>
<thead>
<tr>
<th>Student</th>
<th>3-D Cube</th>
<th>Mid-year Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Explorer</td>
<td>Middle</td>
</tr>
<tr>
<td>B</td>
<td>Proficient</td>
<td>Middle</td>
</tr>
<tr>
<td>C</td>
<td>Proficient</td>
<td>High</td>
</tr>
<tr>
<td>D</td>
<td>Practitioner</td>
<td>High</td>
</tr>
<tr>
<td>E</td>
<td>Practitioner</td>
<td>Low</td>
</tr>
<tr>
<td>F</td>
<td>Explorer</td>
<td>Low</td>
</tr>
</tbody>
</table>
These results indicate that the mid-year math assessment score is not a good indicator of a student’s ability to succeed or fail in spatial relation tasks. Further, it appears that changes are needed in the Developmental Levels for using colored, three-dimensional, one-inch cubes should include a curriculum that is developmentally appropriate, which would be one in which students are presented with lessons that promote spatial language and activities and with materials that demonstrate manipulating objects in various areas of space (see pages 2-5).

As is shown in Table 11, there were clear qualitative differences (correct, reasonable, unreasonable) demonstrated within and among pairs in the students’ levels of spatial reasoning strategies, number strategies, and problem-solving skills.

**Table 11:** Students by Scores on Spatial Reason Strategies

<table>
<thead>
<tr>
<th>Student</th>
<th>5a</th>
<th>5b</th>
<th>5c</th>
<th>6a</th>
<th>6b</th>
<th>6c</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>CR</td>
<td>RS</td>
<td>CR</td>
<td>CR</td>
<td>RS</td>
<td>UR</td>
</tr>
<tr>
<td>C</td>
<td>CR</td>
<td>RS</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
</tr>
<tr>
<td>F</td>
<td>RS</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>RS</td>
</tr>
<tr>
<td>B</td>
<td>RS</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
</tr>
<tr>
<td>E</td>
<td>CR</td>
<td>RS</td>
<td>CR</td>
<td>CR</td>
<td>RS</td>
<td>RS</td>
</tr>
<tr>
<td>A</td>
<td>CR</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
<td>UR</td>
</tr>
</tbody>
</table>

*CR (Correct), RS (Reasonable), UR (Unreasonable)*

Students’ assessments in the development in number sense were apparent in their ability to ‘guess’ in a close proximity. It is clear that by the end of the study Student C and Student B were prepared to move on to more advances conceptual possibilities. The remaining students required review activities at various levels, and they needed more time to experience the use of the blocks, understand the mathematical language that the tasks required, and participate in discourse with their partner.

These factors would allow students who lack understanding to invent their own solution methods or with collaboration with their peers, or with the assistance from their teacher, and progress to the next level. The students who gradually improved, as I posed questions designed to help them, engaged in mathematical discourse, thought about their estimations, and used evidence in their reasoning.
I encouraged students to talk with one another about what they were thinking and doing, and I made an effort to maintain a balance between probing for student understanding and not telling them anything that they could discover for themselves.

In the area of number sense, the students who reached a proficiency level in the beginning counted by ones. As they became more comfortable – or as Student C explains it, “I found a pattern” – they started using more advanced strategies, such as doubling or offering an equal multiplication operation for their answer. Obviously without the appropriate level of spatial reasoning, the student’s level of mathematics does not matter much. For example, Student D had very good math skills; however, do to a lack of spatial reasoning ability, the student was only able to answer the majority of questions with a reasonable, though not a correct answer.

“Stop, Think, and Plan” was an issue at the beginning of a task; however, after the students became comfortable in a task, they did not discuss a plan unless a partner forgot that it was his or her turn, forgot a responsibility, or re-negotiated a responsibility.

The students seemed to enjoy the change from their traditional curriculum because the material was interesting and challenging. It was clear that connecting geometry to real life experiences provided students with a physical and a visual sense of mathematics concepts. These activities can add enrichment to a weak spatial reasoning program by helping to support a first grade student’s spatial structuring ability and administrator’s requirements to meet state standards. The tasks can also provide teachers with a sequential curriculum that will prepare students for middle school geometry.

In a regular classroom situation, be cautious about moving students to a more advanced level of activity. Make certain they have understood and completed a previous level satisfactorily so that they are not frustrated.

**Summary of the Results.** The activities is this study were designed to encourage active constructive exploration of the cubes and developing concepts within a supportive context that included student-student interaction, sensory involvement, and learning through doing.

The activities devised in this study can serve as tools to improve the identification of students who are experiencing difficulty in the area of spatial reasoning, number sense,
and problem solving, even among students who typically score high on traditional measures of math ability. Further, these activities can enable teachers to provide immediate, interesting, and quality intervention.

Additional Observations. The assessment forms for this study were not devised for statistical analysis, however, the sequential design of the activities with first graders offered very useful insights into student’s spatial reasoning skills; insights that very few elementary teachers are able to explore.

When students were working with cubes, students at all levels began by counting by ones, and it appeared that they did so because they needed to verify that any hidden cubes were actually under another layer of cubes. This seems to imply that some students might not have a strong sense of object permanence, which is commonly developed in infancy. Battista also found this checking behavior in his third and fourth graders, which caused his students difficulties because they could not visually and spatially structuring the cubes (Battista and Clements, 1996).

I recommend that teachers develop portfolios of children’s’ work in order to build a profile to help measure student progress validly. Close observation of students and student working in pairs can provide a keen awareness of abilities, strengths, and diversity. These observations will also provide valuable information that will help a teacher to individualize the curriculum accordingly.

This project is one that can be readily completed within a classroom setting. As a study, this project took a month to complete, but in a regular classroom setting at the appropriate time of year, it would take about two weeks, or students could work on the activities independently in centers. If the students have had the appropriate materials offered to them in preschool and kindergarten, the students would be more prepared to do these activities in first grade because they would have had experience with the cubes and the language to discuss and understand the activities. This study took longer because I began the tasks in the middle of May, and at the end of the school year there were mandated district assessments, routine end of the year assemblies, activities related to dismantling of the classroom, and a general student perception that “school work is over and summer is here.
In addition, these tasks can be implemented in a classroom's Math Lab or activity center, as they were for this study. Also, these activities can be done at times when a group of students leaves the classroom for art and library, or more assistance is available, such as an E.S.O.L. teacher, paraprofessional, or parent. Within a lab or center setting, students who are able to work independently can move right from seatwork into center activities.
SECTION IV: CONCLUSION

It was eye-opening to work closely with small groups and pairs of students using the developmental curriculum and observational procedures that I designed because the activities helped me understand how my students think through solving problems, as well as why they chose a particular algorithm when answering questions. The activities also helped me gain insights about my student’s spatial reasoning processes. In addition, the activities were more interesting and exciting for the students and myself than traditional, paper and pencil approaches, and the activities I developed were more challenging, developmental, and in-depth than what is presently offered by the textbooks that I reviewed.

The project design made it easy to attend to individual differences. Throughout the project you could hear the students say, “This is fun.” There was laughter and the students were actively engaged. Several students volunteered suggestions as to how to make the tasks more difficult and how to connect the tasks to real world applications.

The results also imply that working with three-dimensional cube manipulative, can challenge the students to think spatially if a teacher begins with concrete experiences and links those experiences to the abstract.

In conjunction with the developmental stages that I had anticipated might prevail in children’s spatial reasoning and that are listed in the Three-Dimensional One-inch Cube Developmental Levels Form, I made other observations while the students worked in their tasks, which I would also consider to be developmental.

Spatial Reasoning. In the area of spatial reasoning, all of the students experienced some success visualizing one layer of cubes with double rows (a row of cubes with an additional row behind it) with no more that five cubes in each row. All of the students experienced some difficulty constructing buildings that had cubes angled in the center of the building.

Number Sense. In the area of number sense, all of the students used counting by ones in the beginning stages of the tasks. Students B, C, and D, who were skilled at skip counting to a hundred, found it necessary to take apart a building of two layers that contained four cubes in each layer, and then count each cube by ones. I believe that for a first grader this situation is comparable to object permanence to an infant. The students
needed ample experience to understand that, if there are four cubes in a layer on the top, and two layers are visible, than in all likelihood there are four cubes holding up the top layer.

Each of the students used doubling and skip counting with success up to the number ten, and the proficient students successfully skip counted and or multiplied up to forty. In the 1996 Battista and Clements study, only 7% of the third graders and 29% of the fifth graders correctly used a layering strategy for the problems in their investigations. In contrast, 33% of my first grade students always experienced success and an additional 33% gained partial success. Battista and Clements (1996) also indicated that for problems with larger numbers of cubes, the students scored slightly better when the problem was presented concretely. In my investigation, proficient students enjoyed success, whether the activity was presented pictorially or concretely. Practicing students enjoyed success slightly higher if the activity was presented concretely. Exploring students only experienced success with concrete activities.

**Assessment Options and Techniques.** This project provides evidence that when assessing young students, it is more useful to gather information through interviewing and questioning when they are immediately involved in thinking through their tasks. The teacher must observe the “who, what, when, and how” during observations of pairs and their interactions, as well as individual’s actions, reactions, and statements. Such informal assessment is valuable and can support and enhance more formal assessments. It can also help prepare students for those more formal assessments and present a more individualized curriculum and instructional methods.

**Next Steps for the Classroom.** Each year I select an area in the curriculum and I extend old concepts and prepare new applications and materials using the new knowledge that I have gained through observing student behavior. My goal is to eliminate developmental gaps in the curriculum and design new creative presentations and hands-on activities to support lessons, enhance student development, and improve as a teacher.

Next year, I plan to develop a teacher resource packet on Spatial Reasoning to improve teacher understanding of spatial structuring and enhance the development of elementary students’ spatial structuring skills. I plan to submit a proposal to the (RTC)
Rochester Teacher Center-Teacher Research Program and eventually have my project highlighted on The Rochester Teacher Center web site.

Through this in-depth research and investigation into spatial structuring, I have put together sequentially developed activities that are appropriate for students from preschool through fifth grade. The results of this study indicate that these concrete tasks in collaborative settings promote the growth of student abilities in spatial reasoning, number sense, problem solving and mathematical language.

Perhaps the most significant finding of this study, however, pertains to curriculum and instruction. The results of this study suggest that first-grade students can solve spatial structuring tasks that are usually thought to be appropriate only for students in the fourth grade and above, provided the students have worked through a sequence of developmentally appropriate spatial-reasoning tasks, such as those described in this paper, which involve number sense and problem-solving strategies. Further, as was mentioned in the literature review, another consequence of this paper could very well be the appearance of spatial-structuring activities in the early elementary mathematics curriculum. Such a development would be consistent with the National Council of Teacher’s of Mathematics (NCTM) renewed emphasis on geometry at all levels of school mathematics.
Appendix A

New York State Mathematics Standards

Elementary Grade One
New York State Mathematics Standards – Elementary Grade One

The Rochester City School District grade level 1 content standards
Aligned curriculum promotes the following learning standards
constructed by New York State

New York State Standard
Key 1 – Mathematical Reasoning: Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument. Throughout the curriculum.
1.1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their reasoning.
1.1B. Justify their answers and solution processes.
1.1C. Use logical reasoning to reach simple conclusions.

NYS Standard 6/12
Key Idea 2 – Numbers and Numeration: (Relates to RCSD’s numbers and number sense.) Students understand numbers and how to use them to count, to measure, and to locate positions.
1.2A. Count, read and write numbers to 102.
1.2C. Use more than, less than, and equal to compare numbers to 100.
1.2E. Use ordinal number names first to thirty-first and beyond.
1.2F. Read and identify fractional parts of a whole (1/2, 1/4, 1/3).

NYS Standard 7
Key Idea 3 – Operations: (Relates to RCSD’s Computation and Estimation). Students select and apply mathematical operations (including estimation) in a variety of contexts
1.3B. Use strategies to add and subtract to 18.
1.3C. Memorize doubles (i.e., 1 + 1, 2 + 2, 3 + 3).
NYS Standard 1.3.E (Show understanding of cumulative and associative properties extending beyond standards. Identify when to use equal groups (to multiply and divide) in a story problem.)

Key Idea 4 – Modeling/Multiple Representation: (Relates to RCSD’s Geometry and Spatial Sense.) Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

1.4.A. Use manipulatives to model whole numbers and addition and subtraction sets.
1.4.B. Name and recognize properties of plane and solid figures (corners, sides, faces, circle, square, triangle, rectangle, oval, sphere, cube, rectangular prism, cone, pyramid).

NYS Standard 10/11

Key Idea 5 – Measurement: (Relates to RCSD’s Measurement, Probability, and Statistics.) Students select and use units and tools of measurement; use statistical methods to collect and analyze data; and use methods of probability to make predictions.

1.5.D. Define and describe the attributes of length and weight.
1.5.E. Use standard and non-standard units of measurement.

NYS Standard 5

Key Idea 6 – Uncertainty: (Includes Statistics and Estimation): (Relates to RCSD’s Computation and Estimation.) Students use ideas of uncertainty to illustrate that mathematics involves more than exactness in dealing with everyday situations.

1.6.A. Make estimates and compare to actual measurements.

NYS Standard 13

Key Idea 7 – Patterns/Functions: (Relates to RCSD’s Algebra, Patterns, and Functions.) Students use patterns, functions and algebraic methods to describe situations and solve problems.

1.7.A. Recognize, create, and describe a wide variety of patterns including simple numerical patterns.
Rochester City School District
Content Standards Align Curriculum
for Elementary Mathematics

<table>
<thead>
<tr>
<th>RESD Alignment with NYS Learning Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.A</td>
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<tr>
<td>1.1.B</td>
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<td>1.1.C</td>
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<tr>
<td>1.2.E</td>
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<td>1.6.A</td>
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</tr>
<tr>
<td>1.7.A</td>
</tr>
</tbody>
</table>
Appendix B

Summary of Spatial Structuring Strategy Codes
### Summary of Spatial Structuring Strategy Codes

<table>
<thead>
<tr>
<th>Visual Reasoning Strategies</th>
<th>Number Strategies</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SVR1.</strong> Student takes the cube unit apart, unit by unit</td>
<td>NS1. Estimation. Guess and check. Estimate is a tentative calculation – The guess check is to examine your calculation.</td>
<td>PS1. Stop, think and plan</td>
</tr>
<tr>
<td><strong>SVR2.</strong> Student conceptualizes the cube Faces that are visible and</td>
<td>NS2. Counting – As natural numbers in sequential order 1, 2, 3, 4, etc.</td>
<td>PS2. Look for a pattern.</td>
</tr>
<tr>
<td>a. Does not conceptualize the interior cubes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Does conceptualize the interior cubes of the building.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SVR3.</strong> Student conceptualizes layers, rows and/or columns.</td>
<td>NS3. Counting on – a method of addition in which one number is handled mentally and the remaining numbers are combined through counting on natural numbers 7, + 1 = 8, + 1 = 9.</td>
<td>PS3. Guess and check.</td>
</tr>
<tr>
<td></td>
<td>NS4. Addition - + 1, + 1 – the operation of combining numbers, in which each number represents a separate quantity of measure to produce a number to represent the whole measure of all parts. 3 = 9 = 12.</td>
<td>PS4. Draw a picture.</td>
</tr>
<tr>
<td></td>
<td>NS5. Doubling - A method of combining a number (x2) such as 4 and 4, 5 and 5.</td>
<td>PS5. Act it Out</td>
</tr>
<tr>
<td></td>
<td>NS6. Skip Counting – Counting sets of quantities such as counting by 2's, 3's, 4's, 5's etc.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NS7. Multiplication - The operation of combining several equal measures of sizes to produce a single number 6 x 3 = 18.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix C

Tasks/activities and Lesson Plans
for Visual Reasoning and Number Strategies
During the first two weeks of school, "Free Exploration" sessions were conducted to allow the students to play, explore, discover and become familiar with the various manipulatives, including one inch square cubes and the many center stations within the classroom. The students have used the cubes for building towers, constructing patterns, counting sets and skip counting.

To investigate the developmental stages of first grade students when counting cubes in 3D arrays, I will first introduce a packet of task cards for the students to replicate, with 3D cubes.

**Activity 1A**

**Aims:** To develop spatial structuring skills.

To replicate buildings on cube pattern task cards with one inch square cubes.

To discover and describe buildings involving even and odd numbers of cubes.

**Materials:** One-inch cubes, task card, post-it-note paper, and pencils

**Activities:**

1. The students will work in pairs to review the odd and even numbers by writing them on the black board. The even numbers will be written in red and the odd numbers in blue.

2. Explain that for this activity the children will be architects and will construct buildings out of the cubes.

3. Pass out the task cards, one-inch blocks, and post-it-paper.

4. Explain that some buildings will have a flat roof-flat roof buildings will have an even number of cubes. Buildings with a chimney will have an odd number of cubes.

5. Demonstrate by building an even building, printing "even" on the post-it, and placing the post-it on the table close to the building.

6. When the groups have completed their tasks, ask the students if they have noticed any patterns.
**Activities 2A, 2B and 2C**

**Aims:** To promote spatial structuring, number strategies and problem solving skills.

Spatial reasoning and number sense.

**Materials:** Task sheet 2A, one-inch square cubes.

**Activities:**

1. Explain to the students that you will give them a work sheet that will have two buildings made of cubes on it. Next, tell the students to look at the details in the building and write on line the letter x for building X or Y for building y which building has the most cubes.

2. Next, tell students to make buildings X and Y with one-inch cubes to check their answers.

3. Observe students guess and observe students construction of buildings X and Y and complete special reasoning and number sense observation form.

4. Ask the students to make a comparison of their building to the buildings on the 2A task sheet. Do they look the same and or how are they different. How many blocks are in each row, how many rows.

Repeat this process with activity sheets 2B and 2C.
Activities 3A and 3B

Aims: To familiarize students with one-inch grid paper

Introduction to one inch-grid paper, patterns, counting, and visual thinking

Materials: One-inch block grid paper, one-inch blocks, pencils, and crayons

Activities: This activity introduces students to one application of one-inch grid paper.

1. The students will work in pairs and complete their individual tasks.
2. Distribute grid paper with worm pattern.
3. Ask the students to describe the worm on their paper.
4. Next, have the students build the worm pattern with one-inch cubes.
5. When the students have completed their worm, ask what patterns they have noticed. How many orange cubes did you use? How many green cubes did you use? How many cubes did you use altogether?
6. Ask the students to draw a worm on one-inch grid paper.
7. Observe to see how the students plan where to begin drawing or coloring.

When they have completed their drawing interview students and ask each student to explain how they made their worm by telling what they did 1st, 2nd, 3rd etc.

This process is repeated for activity 3B the one-inch grid paper turtle.
Activities 4A, 4B and 4C

Aims: To promote spatial structuring, number strategies and problem solving skills.

Constructing rectangular 3D arrays with gray pattern pictures and one-inch block cubes, spatial/visual reasoning, mathematical strategies, and estimation.

Materials: Estimating the number of one-inch cubes needed to complete pattern picture 4A. Pattern pictures, one-inch blocks, guess-and-check forms, pencils, and removable tape.

Activities:

1. Explain to the students that the gray area represents the bottom of the box. The white sections represent the sides of the box.
2. The students will work in pairs to collaborate on predicting the number of cubes that will fill each box.
3. Students construct box 4A from grid paper, and then fill the box with cubes to check their predictions.
4. Each student will complete their individual guess check form. The teacher will function as facilitator and will remain close by to observe, to scaffold student generated problems, to encourage alternative solutions when needed and encourage mathematical reflection.

This process will be repeated for activities 4B and 4C.
Activities 5A, 5B and 5C

Aims: To promote spatial structuring, number strategies and problem solving skills.

Estimating the number of one-inch cubes needed to complete pattern picture 5A, and guess what shade will emerge from the pattern. Constructing rectangular arrays with pattern pictures and one-inch block cubes.


Activities:

1. Explain to students that they will work in pairs and will fill out their individual guess-and-check forms.
3. Instruct students to carefully observe the details in the pattern shown on the grid paper.
4. Have students record their guess of the number of one-inch blocks that will fill the box.
5. Have students explain how they decided on their guess response.
6. Ask students to build the box, using the grid paper and removable tape.
7. Fill the box with blocks.
8. Check your answer (process will vary). Discuss differences and/or similarities in the guess-and-check answers. Elicit student reasoning about their mathematics strategies and visual reasoning.

This process will be repeated for activities 5B and 5C.
Activities 6A, 6B and 6C

Aims: To foster spatial structuring, number strategies and problem solving activities. Estimating and checking predictions on the number of one-inch cubes that will fill a rectangular box.

Materials: Grid pattern, Box 6A, guess-and-check forms, pencils, and removable tape.

Activities:

1. The students will work in pairs to encourage collaboration and mathematical language.
2. Show grid pattern for Box 4A and construct box with removable tape in student's view.
3. Place box 6A in front of the pair of students.
4. Next hold up a one-inch cube square and ask the students to guess and record the number of cubes that will fill the box.
5. Interview the students on how they decided upon their estimation.
6. Asks the students if they have noticed any patterns.
7. Have the students fill the box with cubes, count the cubes, record their (check) answer.
8. Compare their guess answer to their count answer. Ask students to describe why the guess-and-check answers are different. Have a student empty the box, and to carefully unfold the box so that the students can see the pattern picture structure.

Activities 6B and 6C should be repeated with this same process.
Three-dimensional One-inch Cube
Developmental Levels

3. **Proficient**
   - Strategy leads to correct solution
   - Conceptualize the rectangular array as in rows or layered
   - Number strategy used include, multiplying layers, skip counting layers
   - Repeats pattern and extends correctly

2. **Practitioner**
   - A partially effective strategy, student will reflect and self correct
   - Conceptualizes sub-units of rows and layers
   - Estimator/guess in close proximity
   - Number strategy used included counting by ones, adding or skip counting
   - Describes basic pattern

1. **Explorer**
   - No evidence of a planned strategy, seeks direction
   - Conceptualizes individual faces of each cube
   - Estimate/guess with assistance, lacks reasonableness
   - Number strategy used include counting by ones and/or adding on
   - Describes the pattern with assistance
<table>
<thead>
<tr>
<th>Time</th>
<th>Observation Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Indicates ½ minute</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Appendix E

One-inch Three-dimensional Cube Rubric
# One-inch Three-dimensional Cube Rubric

<table>
<thead>
<tr>
<th>Performance Factor</th>
<th>3 Proficient</th>
<th>2 Practitioner</th>
<th>1 Explorer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Organization of Strategy</strong></td>
<td>Strategy leads to correct solution</td>
<td>A partially effective strategy, student will reflect and self correct</td>
<td>No evidence of a planned strategy, seeks direction</td>
</tr>
<tr>
<td><strong>Visual Reasoning</strong></td>
<td>Conceptualizes sub-units of rows, columns and/or layers</td>
<td>Conceptualizes individual faces of each cube</td>
<td></td>
</tr>
<tr>
<td><strong>Estimation Accuracy</strong></td>
<td>Predicts correctly</td>
<td>Predicts in close proximity</td>
<td>Prediction lacks reasonableness</td>
</tr>
<tr>
<td><strong>Understanding of Number Concepts</strong></td>
<td>Number strategies used include, multiplying layers, skip counting layers</td>
<td>Number strategies used include counting by ones, adding-on, and/or skip counting</td>
<td>Number strategies used include counting by ones and/or adding on</td>
</tr>
<tr>
<td><strong>Usage of Pattern Pictures</strong></td>
<td>Successful usage of pattern pictures</td>
<td>Successful using gray bottom pattern pictures</td>
<td>Unsuccessful usage of pattern pictures</td>
</tr>
<tr>
<td><strong>Identifies a Pattern</strong></td>
<td>Repeats pattern and extends correctly</td>
<td>Describes basic pattern</td>
<td>Describes the pattern with assistance</td>
</tr>
</tbody>
</table>
References


Weikart, David, P.; Rogers, Linda; Adcock, Carolyn; and McClelland, Donna (1971). The Cognitively Oriented Curriculum. Urbana, IL: University of Illinois.