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Kris H. Green
St. John Fisher College, kgreen@sjfc.edu

W. John Cocke
University of Arizona

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A Solution of Einstein's Field Equations for a Tachyonic Gas: Possible Astrophysical Applications

W. J. Cocke (cocke@as.arizona.edu)
Steward Observatory, University of Arizona, Tucson, AZ 85721

K. H. Green (green@sjfc.edu)
St. John Fisher College, Rochester, NY 14618

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Keywords: tachyon, gravitation, general relativity, galaxy formation

AMS Subject Classification: 83, 85

1. Introduction

The literature on tachyons in special and general relativity is quite extensive. Recami and Mignani (Recami et al., 1974) have discussed the properties of classical tachyons in special relativity, and Davidson (Davidson, 1987) has treated tachyonic fields in the context of supergravity. Srivastava (Srivastava, 1984) and others have remarked (using calculations similar to those in section 5) that the tachyon density in an expanding Robertson-Walker universe should decrease with time so strongly that few tachyons would exist today. However, Molski (Molski, 1993) has demonstrated that certain types of primordial tachyons could survive to the present day. In any case, if tachyons are capable of aggregating into extended objects, as suggested by (Shiu and Wasserman, 2002) for example, they could play a role in the formation and evolution of the large-scale structure of the universe. Unlike the smaller aggregates in Shiu, however, the present paper shows that tachyons can aggregate into very large objects, essentially infinite in two directions, but having finite thickness. Recently there has been a great deal of interest in the behavior of tachyons in the context of string theory. This is largely due to the work of Sen, who has described a "rolling tachyon" (Sen, 2002a) and the tachyon matter that results (Sen, 2002b). Follow up work has shown that tachyons could help to generate the inflation necessary for driving the expansion of the universe (Gibbons, 2002) and how tachyons could act as a form of dark matter in the present universe, behaving much like a cosmological constant (Padmanabhan, 2002). Generally it is difficult for non-string theorists (like the authors) to interpret the physics in these papers, and no simple solutions of Einstein's field equations for classical tachyonic matter seem to have been discussed in the literature.

In the present paper, however, we show that the simple cosmic string solution may easily be converted to a time-dependent solution for aggregates of tachyonic matter that we model by using the Boltzmann distribution function for a gas composed of collisionless tachyons. These aggregates may be described as walls of tachyons. It is worthwhile to note that the tachyonic matter here is not identical to that of (Sen, 2002a) or (Sen, 2002b). The present work results in a "gas" of tachyons that has zero energy density, and non-zero, non-isotropic pressure. Thus, it is not an ideal gas, as it is in Sen's work. However, Sen does describe the tachyon potential as having a minimum where the energy density vanishes. Although derived from completely different methods in very different contexts, the present solution also bears further resemblance to the rolling tachyon solution, in that the solution to the field equations here results in either circular or hyperbolic cosines, much like the rolling tachyon.

We take the point of view that tachyons may be treated in general relativity as having a rest mass that is positive and real, according to the conventions of (Recami, 1986), and that as a class of particles the only difference between tachyons and bradyons is that tachyon trajectories are space-like. (Bradyons are particles whose world-lines are time-like and are sometimes called *tardyons.*) Thus there is no need to use the terminology "imaginary mass" or "meta-mass," as was common in much of the literature in the 1960's through the early 1980's.

We note that tachyons have never been observed in the laboratory and that there are causality paradoxes that arise when one considers their interaction with bradyonic (*i. e.,* ordinary) matter (Recami et al., 1974). It may be, however, that the only interaction between tachyons and bradyons is gravitational. This would mitigate both of these problems.

In any case, the wall solutions discussed here are interesting from the cosmological point of view, since they tend to compress extended material objects or focus particle trajectories, as we show in section 3. Thus these walls could act as a type of dark matter that would in some cases stabilize galaxies or galaxy clusters. If present in the early universe, these same tachyon walls could stimulate the formation of galaxies and large-scale structure. Also, we present preliminary calculations showing that an encounter between a galaxy disk and a wall seems to stimulate the formation of spiral arms. These topics are discussed in section 5.

2. Tachyon Walls as Mutations of the Cosmic String Solutions

In this section we show that the solutions to the field equations for cosmic strings may be easily changed to solutions representing walls composed of tachyons. The cosmic string metric may be written as (Peebles, 1993)

$$ds^2 = -dr^2 - B(r)d\theta^2 - dz^2 + dt^2, \tag{1}$$

where the field equations $R^\mu_\nu - \frac{1}{2}R\delta^\mu_\nu = -8\pi G T^\mu_\nu$ imply that (see Tolman's version of Dingle's formulae (Tolman, 1987)) $T^{tt} = -T^{zz}$. For positive energy density $T^{tt} > 0$, meaning that the string is under tension.

Changing this metric to represent solutions for walls of tachyons is simple, and there are two possibilities, which we call Case I and Case II. In Dingle's notation (Tolman, 1987),

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dx^2 - dy^2 - du^2 + D(u)dv^2 \quad \text{Case I} \tag{2}$$

and

$$ds^2 = -dx^2 - dy^2 - C(u)dv^2 + du^2 \quad \text{Case II.} \tag{3}$$

In both cases the field equations (Tolman, 1987) imply $T^{xx} = T^{yy}$. Using the notation $\xi(u)^2 \equiv C(u)$ and $\xi(u)^2 \equiv D(u)$, we find (Tolman, 1987)

$$-8\pi G T^x_x = +8\pi G T^{xx} = \frac{\xi''}{\xi} \quad \text{Case I} \tag{4}$$

and

$$-8\pi G T^x_x = +8\pi G T^{xx} = -\frac{\xi''}{\xi} \quad \text{Case II} \tag{5}$$

For empty space, $T^{\mu\nu} = 0$, and hence $C(u) = D(u) = (c_0 u + c_1)^2$, with c_j constants, so that the metrics (2) and (3) are just flat spacetime in Rindler coordinates.

For non-vanishing $T^{\mu\nu}$, the metrics (2) and (3) are known as "hyper-surface-homogeneous" spacetimes, and their mathematical properties are discussed, for example, by Kramer *et al.* (Kramer et al., 1980).

To interpret the sign of $T^{xx} = T^{yy}$, consider as a preliminary model a collisionless gas composed of non-interacting tachyons. We take the rest mass m_0 to be real and positive. We define the 4-momentum $P^\mu \equiv m_0 dx^\mu/ds$ to be real and of course change the signs in the metrics (2) and (3) so that ds^2 is negative for space-like dx^μ. Then if the tachyons have a phase-space distribution function $f(x,P)$, the corresponding energy-momentum tensor is (Cocke, 1996)

$$T^{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \int d^4P \, P^\mu P^\nu \frac{f(x,P)}{m_0(x,P)}, \tag{6}$$

where the relation $P^\mu \equiv m_0 dx^\mu/ds$ defines $m_0^2(x,P) \equiv -g_{\mu\nu}P^\mu P^\nu$, the minus sign holding for tachyons.

Equation (6) of the paper by Cocke (Cocke, 1996) is not written correctly, but the conclusions deduced from it are correct. Taking into account the fact that the variables x^μ and P^μ are considered formally independent, one should write Cocke's (Cocke, 1996) equation (6) as

$$P^\mu \frac{\partial}{\partial x^\mu}\left(\frac{f}{m_0}\right) = -\frac{\partial}{\partial P^\mu}\left(\frac{dP^\mu}{ds} f\right). \tag{7}$$

Our equation (6) shows that $T^{\mu\nu}$ is positive for $\mu = \nu$, and therefore we choose $T^{xx} = T^{yy} > 0$ in equations (4) and (5). Since $T^{uu} = T^{vv} = 0$, the tachyons composing the gas would have no motion in the u, v plane and would thus have infinite speed. The distribution function $f(x,P)$ would be isotropic in the P^x- and P^y-directions and of course would not depend on x, y, or v.

It would be possible to choose $T^{xx} = T^{yy} < 0$, but this would mean that the tachyon material would be under tension, requiring a model more elaborate than a simple collisionless gas.

To study the essential features of the walls, we investigate the simple case $T^{xx} = T^{yy} = P_0 = \text{constant} > 0$ for $u_1 < u < u_2$. Equations (4) and (5) for Case I and Case II then show that the $\xi(u)$ are, respectively, hyperbolic or circular functions.

The geometry of the walls thereby obtained is taken from the geometry of the two types of Rindler coordinates: In Case I, the surfaces of constant u are time-like, and the boundaries are given by the equations

$$z = u \cosh v \qquad t = u \sinh v. \tag{8}$$

For Case II, the surfaces are space-like, and the boundaries are given by

$$z = u \sinh v \qquad t = u \cosh v. \tag{9}$$

See figure 1 for a graphical representation of the geometry of the two cases. In Case I the boundary curves $u = u_1$ and $u = u_2$ are those of a massive bradyon undergoing constant acceleration. Case II is similar, except that the particles involved would be tachyons.

We must now discuss the continuity conditions of the metric across the boundaries between the interior and exterior (vacuum) solutions. The fact that the exterior spaces (regions 1 and 3) are flat means that the solution as a whole may be unphysical. We show below, however, that all the relevant continuity conditions are satisfied.

First of all, note that the flux of energy and momentum across the boundaries vanishes: In cases I and II, the equation of the boundaries is $f(x^\mu) = u - u_j = 0$, and thus the normal 1-form to the boundaries is just $n_\mu = \partial_\mu f = \delta^u_\mu$. The flux across a boundary is $T^{\nu\mu} n_\mu = T^{\mu u}$. This vanishes, since the only non-zero components of $T^{\mu\nu}$ are T^{xx} and T^{yy}.

The remaining issue is that of the continuity of $g_{\mu\nu}$ and its derivatives, and we consider Case I and Case II in parallel, with $T^{xx} = T^{yy} = P_0 = \text{constant} > 0$ for $u_1 < u < u_2$. Equations (4) and (5) are satisfied if we use the forms

$$\begin{aligned} \xi(u) &= A_1 \cosh[q(u - \varphi_1)] \quad \text{(Case I)} \\ \xi(u) &= A_2 \cos[q(u - \varphi_2)] \quad \text{(Case II)}, \end{aligned} \tag{10}$$

where $q \equiv \sqrt{8\pi G P_0}$. In region 1, we have $\xi(u) = u$, and therefore continuity of g_{vv} and $\partial_u g_{vv}$ requires

$$(A_1)^2 = (q^2 u_1^2 - 1)/q^2, \qquad \varphi_1 = u_1 - \frac{1}{q} \tanh^{-1}\left(\frac{1}{q u_1}\right), \tag{11}$$

$$(A_2)^2 = (q^2 u_1^2 + 1)/q^2, \qquad \varphi_2 = u_2 + \frac{1}{q} \tan^{-1}\left(\frac{1}{q u_1}\right). \tag{12}$$

Extension to region 3 is similar: We write, in region 3, $\xi(u) = B_0(u - u_2) + B_1$ and define the constant $\mu \equiv q(u_2 - u_1)$. We then find that continuity gives

$$B_0 = \cosh\mu + q u_1 \sinh\mu, \quad B_1 = u_1 \cosh\mu + \frac{1}{q}\sinh\mu \quad \text{Case I} \tag{13}$$

and

$$B_0 = \cos\mu - q u_1 \sin\mu, \quad B_1 = u_1 \cos\mu + \frac{1}{q}\sin\mu \quad \text{Case II.} \tag{14}$$

In these coordinates, the metric in region 3 is $ds^2 = -dx^2 - dy^2 - du^2 + (u - u_2 + B_1/B_0)^2 B_0^2 dv^2$ for Case I, with an analogous expression for Case II [see equations (2) and (3)]. Transforming to new variables $\bar{u} \equiv u - u_2 + B_1/B_0$ and $\bar{v} \equiv B_0 v$ obviously recovers Rindler coordinates in flat spacetime.

The behavior of the walls for observers in the spacetime is interesting: For Case I, observers at rest in region 1 see an infinite wall approaching from $z = +\infty$. The wall decelerates, stops at $z = u_1$, and accelerates back up the z-axis. For Case II, the wall first appears at $t = u_1$ as a thin sheet located at $z = 0$. It increases in thickness until $t = u_2$, whereupon it bifurcates, creating two walls, which accelerate in opposite directions down the z-axis. The velocities of the walls in this case are superluminal.

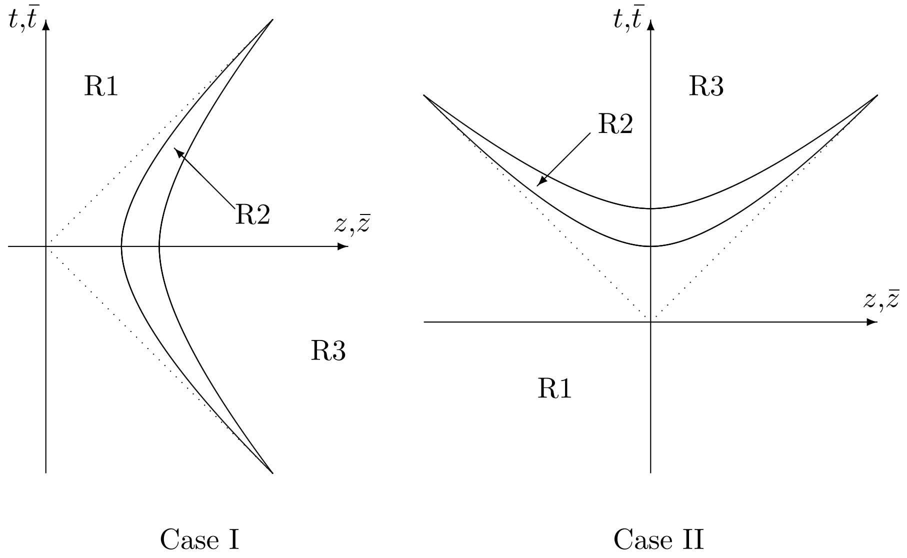

Figure 1. Schematic of the two cases of tachyon walls. In both cases the wall is infinite in the x and y directions, which have been supressed in the illustration. The dotted lines mark the limiting boundaries of the wall. R1 and R3 mark regions 1 and 3 (outside the wall), while region 2 marks the wall interior.

We note that these solutions can be written in the general form

$$ds^2 = \epsilon[du^2 - A^2(u, \epsilon)dv^2] + dx^2 + dy^2 \tag{15}$$

where $\epsilon = \pm 1$. In this form, the solution appears in (Kramer et al., 1980) as equation (11.3) with $k = 0$, in MacCallum (MacCallum et al., 1980) as (A1a), in Ellis (Ellis, 1967) as class II, and in Petrov (Petrov, 1969) as (32.11). These solutions were derived in a different fashion however and represent a general class of solutions with local rotational symmetry. Further, the general solution admits a $G_3 I$ or $G_3 VII_0$ for $\epsilon = -1$ and $\epsilon = +1$.

Our thesis is that these walls can be thought of as being composed of tachyons, and thus it is difficult to find a rationale for their behavior. It should, however, suffice to say that the boundaries of the walls are just the loci of certain tachyon geodesics that are "instantaneous" in the frames of reference appropriate to equations (2) and (3). The fact that the relevant continuity conditions are satisfied assures us that it is physically correct to imbed the metrics in flat spacetime.

3. Geodesics

3.1. The Geodesic Equations for Bradyons and Photons

The geodesic equations for bradyons and photons

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0 \tag{16}$$

reduce to an innocent-looking set of equations in both Case I and Case II. The only difference is the presence of the factor $D(u)$ in Case I and $C(u)$ in Case II. The equations are

$$\frac{d^2x}{d\lambda^2} = 0, \qquad \frac{d^2y}{d\lambda^2} = 0, \tag{17}$$

$$\frac{d^2u}{d\lambda^2} + \Gamma^u_{vv}\left(\frac{dv}{d\lambda}\right)^2 = 0, \quad \text{and} \quad \frac{d^2v}{d\lambda^2} + 2\Gamma^v_{uv}\frac{du}{d\lambda}\frac{dv}{d\lambda} = 0, \tag{18}$$

where λ is an affine parameter. For bradyons, $\lambda = \tau$, the proper time; and for photons, λ is such that the momentum is $P^\mu = dx^\mu/d\lambda$.

We find in both cases that (letting R_i stand for region i)

$$\Gamma^u_{vv} = \begin{cases} u & R_1 \\ \xi(u)\xi'(u) & R_2 \\ B_0(B_0(u-u_2)+B_1) & R_3 \end{cases} \qquad \Gamma^v_{uv} = \begin{cases} \frac{1}{u} & R_1 \\ \frac{\xi'(u)}{\xi(u)} & R_2 \\ \frac{B_0}{B_0(u-u_2)+B_1} & R_3 \end{cases}. \tag{19}$$

The right-hand equation in (18) has the solution

$$\frac{dv}{d\lambda} = \frac{c_0}{D(u)} \quad \text{[Case I]} \quad \text{or} \quad \frac{c_0}{C(u)} \quad \text{[Case II]}. \tag{20}$$

One can now use the line element to find that (Case I)

$$\frac{du}{d\tau} = \sqrt{\frac{c_0^2}{D(u)} - k^2} \quad \text{[photons]} \quad \text{or} \quad \sqrt{\frac{c_0^2}{D(u)} - (k^2+1)} \quad \text{[bradyons]} \tag{21}$$

and (Case II)

$$\frac{du}{d\tau} = \sqrt{\frac{c_0^2}{C(u)} + k^2} \quad \text{[photons]} \quad \text{or} \quad \sqrt{\frac{c_0^2}{C(u)} + (k^2+1)} \quad \text{[bradyons]} \tag{22}$$

where we have defined $k^2 \propto (P^x)^2 + (P^y)^2$ to be constant in accord with equation (17). For simplicity, throughout this paper we consider only the planar case, in which $k = 0$. For details on the cases where $k \neq 0$ see (Green, 1999).

In regions 1 and 3, since these are Minkowski spaces, the geodesics for photons and particles are simply straight lines. The effect of the tachyon wall occurs only in region 2. Then, whatever shift in position or energy the wall induces will carry over into region 3. Thus, we only need to solve the equations of geodesic motion in region 2. This proves to be difficult but it is possible, and has been done in most cases (see reference (Green, 1999) for details.) The complicated form of the geodesic solutions hampers the interpretation of the results. For the purpose of interpreting the results, approximate solutions are perfectly adequate and much less complicated.

3.2. The Sudden Approximation

Throughout the rest of this paper, we use the so-called "sudden" approximation, in which the thickness of the wall is taken to be so small that the change in the momentum of a particle can be evaluated by assuming that its position suffers a small change during its interaction with the wall. Thus we take the thickness of the wall $\Delta u \equiv u_2 - u_1$ to be first order in some small parameter, say $\delta > 0$. We then find that Δv, the change in v as the particle or photon passes through the wall, is also first order in δ.

It is consistent with this approximation to let q be zeroth order, so that $\mu \equiv q\Delta u$ is a 1st-order quantitiy. Then particles which are initially nonrelativistic will remain so after passing through a wall. This gives us a Newtonian sudden approximation, or NSA. This approximation is appropriate for discussing the possible application to the dark-matter problem, since we find that large values of q and Δu lead to relativistic momentum boosts for particles which pass through such walls. Also, we may use this approximation for studying photon geodesics, provided that the relative change in photon energy is small.

The main effect of this approximation is to simplify the expressions for the constants B_0 and B_1. For Case I, the constants are given by (13). A straight-forward Taylor series expansion of these expressions around $\mu = 0$ yields

$$B_0 \approx 1 + q^2 u_1 \Delta u, \qquad \text{and} \qquad B_1 \approx u_1 + \Delta u = u_1 \left(1 + \frac{\Delta u}{u_1}\right). \quad (23)$$

The expressions for the constants in Case II (see (14)) can be approximated by

$$B_0 \approx 1 - q^2 u_1 \Delta u, \qquad \text{and} \qquad B_1 \approx u_1 + \Delta u. \tag{24}$$

To get concise expressions for the changes in the momenta and energies of bradyons and photons, we use the line element to get an approximate relationship between Δv and Δu. For photons, we have either $du = \sqrt{D(u)}dv$ or $du = \text{sgn}(c_0)\sqrt{C(u)}dv$ depending on whether we are considering Case I or Case II. Thus, to get Δv in either case, we simply integrate the line element from u_1 to u_2. However, the NSA described above simplifies these results. We find, for photons,

$$\Delta v \approx \text{sgn}(c_0)\frac{\Delta u}{u_1} \tag{25}$$

for both cases (note that $c_0 > 0$ for Case I).

The relationship between Δu and Δv is a bit more complicated for bradyon geodesics (e.g. observers.) We find that

$$dv/du \;=\; c_0/\sqrt{D(u)[c_0^2 - D(u)]} \qquad \text{Case I} \tag{26}$$

$$dv/du \;=\; c_0/\sqrt{C(u)[c_0^2 + C(u)]} \qquad \text{Case II.} \tag{27}$$

Integrating these expressions exactly is difficult. However, the NSA reduces the expressions to

$$\Delta v_I \approx \frac{c_0 \Delta u}{u_1\sqrt{c_0^2 - u_1^2}}, \qquad \text{and} \qquad \Delta v_{II} \approx \frac{c_0 \Delta u}{u_1\sqrt{c_0^2 + u_1^2}}. \tag{28}$$

3.3. Bradyon Geodesics

For a bradyon or observer passing through a wall, we want to know whether or not it has been accelerated in region 2 and what the resulting direction of motion is in region 3. As is usual, we define $U^\alpha = dx^\alpha/d\tau$ as the four-velocity of the bradyon. For a bradyon at rest in region 1 with $k = 0$ we have $U^t = 1, U^z = 0$. To examine the resulting motion in Minkowski coordinates $(\bar{t}, \bar{z})$ in region 3, we compute $U^{\bar{z}}$ using the chain rule.

$$\frac{d\bar{z}}{d\tau} = \frac{d\bar{z}}{d\bar{u}}\frac{d\bar{u}}{d\tau} + \frac{d\bar{z}}{d\bar{v}}\frac{d\bar{v}}{d\tau} = \frac{\partial \bar{z}}{\partial \bar{u}}\frac{du}{d\tau} + B_0\frac{\partial \bar{z}}{\partial \bar{v}}\frac{dv}{d\tau}. \tag{29}$$

A bradyon interacts with a Case I wall at (u_1, v_1) with $P^v = c_0/u_1^2$. However, the chain rule and the definition of v in region 1 tell us that

$P^v = dv/d\tau = dv/dt = z_0/u_1^2 > 0$ for a bradyon at rest at $z = z_0$. Thus, in Case I, $c_0 = z_0 > 0$. Further, $\bar{u} = B_1/B_0$ at $u = u_2$ in region 3 and $D(\bar{u}) = B_0^2\bar{u}^2 = B_1^2$ so that (29) becomes

$$\begin{aligned} \frac{d\bar{z}}{d\tau} &= \cosh(B_0 v_2)\sqrt{\frac{c_0^2}{D(\bar{u})} - 1} + B_0\frac{B_1}{B_0}\sinh(B_0 v_2)\frac{c_0}{D(\bar{u})} \\ &= \frac{c_0}{B_1}\left[\cosh(B_0 v_2)\sqrt{1 - \frac{B_1^2}{c_0^2}} + \sinh(B_0 v_2)\right] \end{aligned} \tag{30}$$

Now in the NSA we have

$$B_0 v_2 \approx (1 + q^2 u_1 \Delta u)(v_1 + \Delta v) \approx v_1 + \Delta v + q^2 u_1 v_1 \Delta u \tag{31}$$

to first order. Defining Δ by the relationship $\Delta = \Delta v + q^2 u_1 v_1 \Delta u$ allows the simplification $B_0 v_2 \approx v_1 + \Delta$ so that

$$\begin{aligned} \cosh(B_0 v_2) &\approx \cosh v_1 \cosh\Delta + \sinh v_1 \sinh\Delta \\ &\approx \cosh v_1 + \Delta \sinh v_1, \quad \text{and} \end{aligned} \tag{32}$$

$$\sinh(B_0 v_2) \approx \sinh v_1 + \Delta\cosh v_1. \tag{33}$$

To first order in $\Delta u, \Delta v$ and μ we have

$$\left(\frac{d\bar{z}}{d\tau}\right)_I \approx \frac{c_0}{u_1}\left[\frac{1}{\sinh v_1}\frac{\Delta u}{u_1} + \frac{1}{\cosh v_1}(\Delta v + q^2 u_1 v_1 \Delta u)\right] \approx q^2 u_1 v_1 \Delta u. \tag{34}$$

Since bradyons at rest in region 1 make contact with the wall at $v_1 < 0$, we see that $d\bar{z}/d\tau < 0$ so that the bradyons are forced into a second interaction with the wall, this time passing from region 3 back into region 1.

In Case II we allow for the possibility that c_0 and v_1 take either sign. In fact, here $P^v = dv/d\tau = dv/dt = -z_0/u_1^2$ for bradyons at rest in region 1. Again $\bar{u} = B_1/B_0$ and $C(\bar{u}) = B_0^2\bar{u}^2$ at (u_2, v_2) in region 3 so that (29) can be written as

$$\frac{d\bar{z}}{d\tau} = \frac{|c_0|}{B_1}\left[\sinh(B_0 v_2)\sqrt{1 + \frac{B_1^2}{c_0^2}} + \operatorname{sgn}(c_0)\cosh(B_0 v_2)\right]. \tag{35}$$

The NSA then gives us

$$B_0 v_2 = (1 - q^2 u_1 \Delta u)(v_1 + \Delta v) \approx v_1 + \Delta v - q^2 u_1 v_1 \Delta u \equiv v_1 + \Delta \tag{36}$$

so that, to first order, we again have (32) and (33) which reduces (35) to

$$\left(\frac{d\bar{z}}{d\tau}\right)_{II} \approx -q^2 u_1 v_1 \Delta u \tag{37}$$

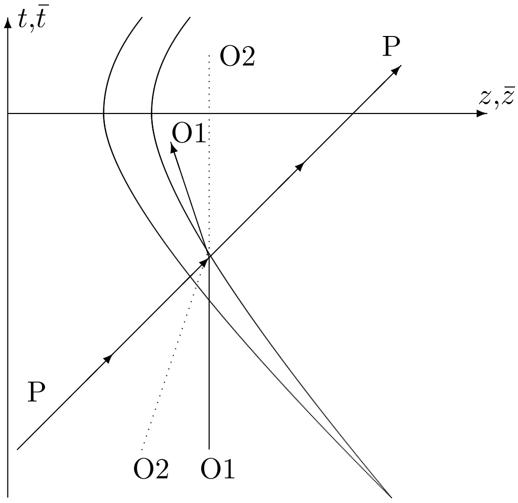

Figure 2. Close up view of the motion of bradyons and photons encountering a Case I wall. This is an illustration of the focusing effect of such walls. The boundaries of the wall are represented by the two curved lines. The labels correspond to the different paths: P=Photon, O1=Observer 1 (initially at rest), O2=Observer 2 (emerges at rest).

which takes the sign opposite v_1 and hence the same sign as c_0 for a bradyon initially at rest. This indicates that the wall "focuses" material toward its center, where $v = 0$.

3.4. Photon Geodesics

We know that photons do not change their direction when they enter region 3. This is due to two facts. First, they are photons, and region 3 is simply a scaled version of Rindler coordinates for both types of walls. Second, c_0 is constant throughout all regions, indicating that if P^v is positive in region 1 then it will remain positive throughout. Thus, only two things can happen to a photon. It could shift to a path which is displaced from its original path after passing through region 2. The photon could also undergo a red- or blue-shift.

The first possibility simply results in the photon being displaced in the v-direction at $u = u_2$ in region 3, the displacement occurring toward increasing or decreasing v according to the signs of P^v and v_1. This effect is not as interesting as the effect of the wall on the energy of the photon. Changes in intrinsic energy are computed here, while section 3.5 deals with redshifts measured by observers on time-like geodesics.

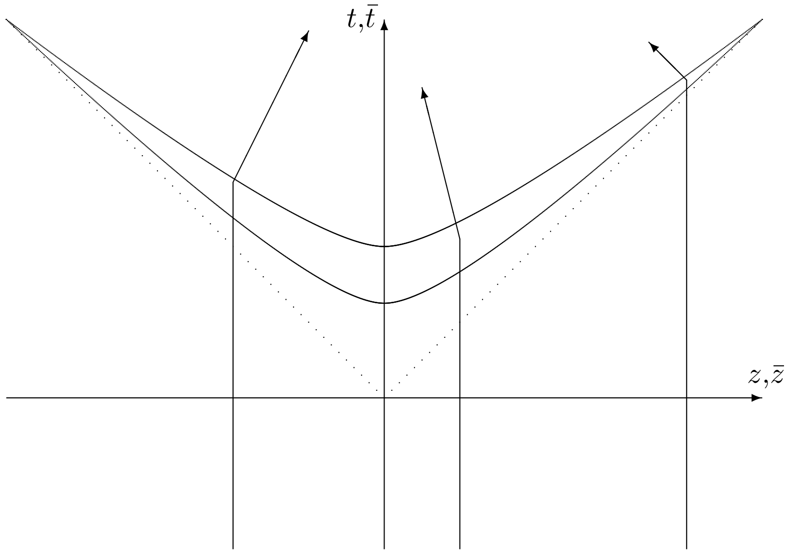

Figure 3. Illustration of the focusing effect csae II walls have on different bradyons which were initially at rest in region 1. Notice that the wall has a greater effect on bradyons further from $z = 0$.

For a photon, we measure the intrinsic energy by the quantity

$$P^t = \frac{dt}{d\lambda} = \frac{\partial t}{\partial u}P^u + \frac{\partial t}{\partial v}P^v \tag{38}$$

in region 1 and by

$$P^{\bar{t}} = \frac{d\bar{t}}{d\lambda} = \frac{\partial \bar{t}}{\partial \bar{u}}P^{\bar{u}} + \frac{\partial \bar{t}}{\partial \bar{v}}P^{\bar{v}} = \frac{\partial \bar{t}}{\partial \bar{u}}P^u + B_0\frac{\partial \bar{t}}{\partial \bar{v}}P^v. \tag{39}$$

in region 3. To compute P^u and P^v we use the appropriate formulae from (20), (21), and (22). However, to distinguish photons from observers in subsection 3.5 we will use d_0 rather than c_0 for the constants in each of the formulae.

In Case I, we see that

$$P^t = \sinh v_1\sqrt{\frac{d_0^2}{u_1^2}} + u_1\cosh v_1\frac{d_0}{u_1^2} = \frac{d_0}{u_1}(\sinh v_1 + \cosh v_1) = \frac{d_0}{u_1}e^{v_1}. \tag{40}$$

Note that all photons which interact with a Case I wall have $d_0 > 0$ so that it is unnecessary to introduce the sign of d_0 into the calculations. We can then calculate

$$P^{\bar{t}} = \sinh B_0 v_2\sqrt{\frac{d_0^2}{D(\bar{u})}} + \bar{u}B_0\frac{d_0}{D(\bar{u})}\cosh B_0 v_2 = \frac{d_0}{B_1}e^{B_0 v_2}. \tag{41}$$

Introducing the NSA expressions for B_0 and B_1 we see that

$$P^{\bar{t}} \approx P^t(1 + q^2 u_1 v_1 \Delta u). \tag{42}$$

This shows us that there are two categories of photons, depending on the sign of v_1. The first category of photons enters region 3 with d_0 and v_1 both positive. They undergo an increase in energy. Category two photons, on the other hand, have d_0 positive and v_1 negative. They lose energy.

In Case II, we find

$$P^t = \cosh v_1 \sqrt{\frac{d_0^2}{u_1^2}} + u_1 \frac{d_0}{u_1^2} \sinh v_1 = \frac{|d_0|}{u_1} e^{\mathrm{sgn}(d_0) v_1}. \tag{43}$$

We again have two categories of photons since d_0 and v_1 can take either sign. Category one photons enter the wall with $d_0 v_1 < 0$ (tranverse to the wall) while category two photons enter the wall with $d_0 v_1 > 0$ (oblique to the wall.)

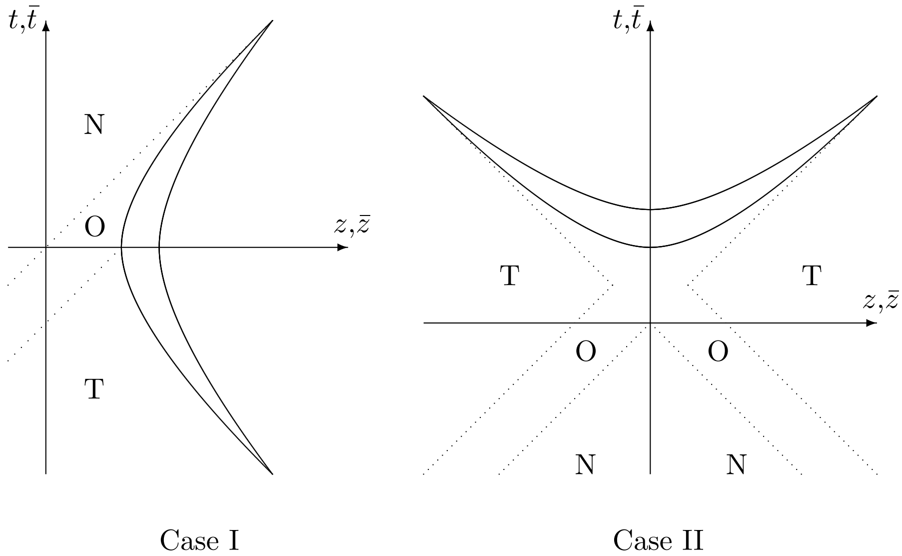

Figure 4. Illustration of the different photon-wall interactions. The regions are separated by dotted lines. The regions of space-time labeled T show the regions from which photons headed toward the wall will enter the wall transversely and loose intrinsic energy. Photons originating in the regions labeled O will enter the wall obliquely and gain instrinsic energy. The regions labeled N mark those regions from which photons are prohibited from interacting with the wall.

In region 3,

$$P^{\bar{t}} = \cosh B_0 v_2 \sqrt{\frac{d_0^2}{C(\bar{u})}} + \bar{u} B_0 \frac{d_0}{C(\bar{u})} \cosh B_0 v_2 = \frac{|d_0|}{B_1} \exp\{\mathrm{sgn}(d_0) B_0 v_2\}. \quad (44)$$

The NSA expressions give

$$P^{\bar{t}} \approx P^t [1 - \mathrm{sgn}(d_0) q^2 u_1 v_1 \Delta u]. \quad (45)$$

This expression for the relative change in energy is little different from the NSA expression in Case I. In both Case I and Case II, the difference in the two categories of photons is now apparent: Photons which strike the wall transversely gain energy as they pass into region 3, and photons which strike the wall obliquely loose energy.

3.5. Redshift Measurements

To calculate observed redshifts, we must consider an observer traveling through the wall and receiving a photon in region 3. For simplicity, we deal only with observers that are initially at rest in region 1. If we further consider only zt-planar photons (hence, $k = 0$) then the familiar expression for observed energy

$$E_{\text{obs}} = U_\alpha P^\alpha = g_{\alpha\beta} U^\alpha P^\beta \quad (46)$$

where $U^\alpha = dx^\alpha / d\tau$ is the four-velocity of the observer and P^β is the four-momentum of the photon, gives us the energy an observer measures for a photon received in either region 1 or 3.

In region 1, let the observer have four-velocity $(U^t, U^z, U^x, U^y) = (1, 0, 0, 0)$. The photon passes through into region 2 at $(u_1, \tilde{v}_1)$ with $P^v = d_0/u_1^2$ and emerges into region 3 at $(u_2, \tilde{v}_2)$ with $P^{\bar{v}} = d_0/B_1^2$. The observer would then detect a photon in region 1 with energy

$$E_1 = P^t = \frac{|d_0|}{u_1} e^{\mathrm{sgn}(d_0)\tilde{v}_1}. \quad (47)$$

In region 3, $U^{\bar{t}}$ and $U^{\bar{z}}$ for the observer are both non-zero. For the photon, $P^{\bar{z}} = \mathrm{sgn}(d_0) P^{\bar{t}}$. This yields

$$E_3 = P^{\bar{t}} \left[U^{\bar{t}} - \mathrm{sgn}(d_0) U^{\bar{z}} \right]. \quad (48)$$

For observers, the line element implies $g_{\alpha\beta} U^\alpha U^\beta = 1$ so that

$$U^{\bar{t}} = \sqrt{1 + (U^{\bar{z}})^2} \approx 1 + \frac{1}{2} (U^{\bar{z}})^2 \approx 1 \quad (49)$$

if $U^{\bar{z}}$ is small (of order δ) as is true in the NSA. Thus, the observer-photon interaction results in

$$E_3 \approx P^{\bar{t}}[1 - \text{sgn}(d_0)U^{\bar{z}}]. \tag{50}$$

For Case I, we can use the results above (see (34) and (42)) to find

$$E_3 \approx P^t(1+q^2u_1\tilde{v}_1\Delta u)(1-q^2u_1v_1\Delta u) \approx E_1[1+q^2u_1(\tilde{v}_1-v_1)\Delta u]. \tag{51}$$

Since the photon must enter the wall with $\tilde{v}_1 > v_1$ for the observer to detect it in region 3, we see that $E_3 > E_1$ so that the observer detects a blue-shifted photon, regardless of whether the photon is category one or two. Case II produces a similar result: Using (37) and (45) we find

$$\begin{aligned} E_3 &\approx P^t[1 - \text{sgn}(d_0)q^2u_1\tilde{v}_1\Delta u][1 + \text{sgn}(d_0)q^2u_1v_1\Delta u] \\ &\approx E_1[1 - \text{sgn}(d_0)q^2u_1(\tilde{v}_1 - v_1)\Delta u]. \end{aligned} \tag{52}$$

Again, we note that this quantity is always greater than E_1 since $\text{sgn}(d_0)(\tilde{v}_1 - v_1) > 0$ for all detectable interactions. The NSA results then indicate that photons always undergo an apparent blue-shift as seen by observers traveling along geodesics and passing through the wall. This result holds for both Case I and Case II walls, regardless of whether the photon is of category one or two.

4. General Discussion on the Effects of Tachyon Walls

Note that the tidal forces produced by these walls tend to focus material. Also note that this focusing is dependent on the quantity v_1, so that the further the interaction occurs from the center of the wall ($v = 0$) the greater the velocity $U^{\bar{z}}$ which the particle has in region 3. This has an interesting effect on objects of finite extent. Indeed, if we view an object at rest as a number of point masses that are at rest with respect to each other and separated in the z direction, then an encounter with the wall produces two effects. First, the entire object is compressed, because of the fact that parts of the object farther from $v = 0$ receive a greater boost toward $v = 0$ than those parts of the object that are closer. Secondly, unless the object is centered on $v = 0$ when it passes into the wall, the entire object will be in motion in region 3.

As an example of this, we have used numerical simulations in MATLAB to simulate a simple encounter between a Case II wall and a toy model of a galaxy. The simulation utilizes the sudden approximation, and the galaxy is made of a distribution of initially circular rings of point masses (stars) which evolve under simple two-body gravitational

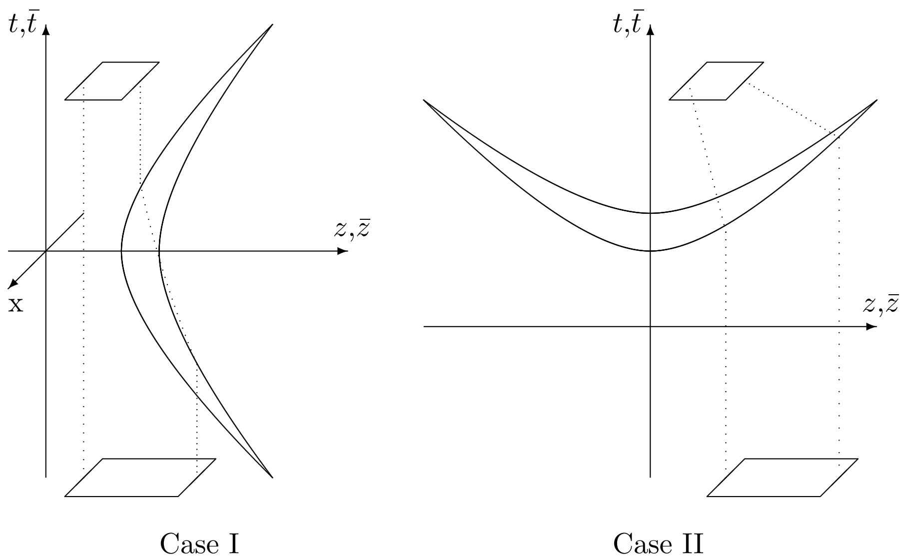

Figure 5. Extended objects which have been compressed (in the z direction) after an interaction with each case of tachyon wall. The x direction is shown in these diagrams to emphasize that only the z-width of the object is affected by the wall.

forces between the stars and the massive central bulge of the galaxy. The parameters for the mass and radius of the galaxy are taken from data for the Milky Way galaxy. Furthermore, the interaction takes place so that the central bulge of the galaxy passes through the wall along the path $v = 0$ and thus receives no momentum boost due to the wall. If the galaxy is not centered on $v = 0$, then the sudden approximation will simply result in the entire galaxy receiving a net translational momentum in region 3.

In the series of plots below, the wall encounter takes place instantaneously at $t = 0$. The galaxy is initially rotating so that the angular momentum is in the y direction, parallel to the wall, and the x axis (also parallel to the wall surface) is the vertical axis. The plots clearly show that the momentum boost imparted by the wall is sufficient to generate spiral structures in the galaxy. These structures are transient, but they do reform during longer simulations. In general, they tend to survive for a time equivalent to slightly more than ten rotations of the initially circular galaxy.

Notice that the simulation does not take into account the N-body forces that would actually be present in real galaxies. To accurately and efficiently include these forces, one would have to make use of an order $N \log N$ code (such as the tree code developed by Joshua Barnes (Barnes, 1997)). On the other hand, our MATLAB simulations run

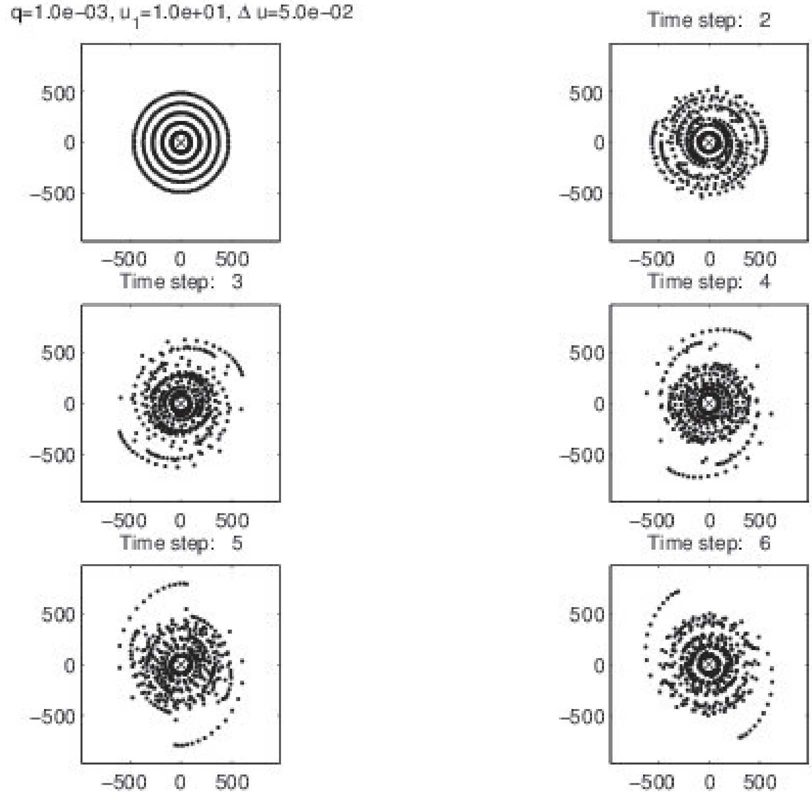

Figure 6. Simulation results of the interaction of a case II wall with a toy galaxy. The units on the axis are in hundreds of parsecs. The wall parameters (q, u_1 and μ) are given above the initially circular distribution. The time interval between the plots is about one-tenth the time it would take this initial distribution to complete one complete rotation.

essentially in real time and mimic the formation of spiral structures that such tree codes are capable of producing. It is hoped that taking into account both N-body forces and possible encounters with tachyon walls will provide more stable and less transient spiral structures. This combination of effects would be equivalent to simply changing the initial conditions for an N-body simulation in such a way as to capture the momentum boosts due to the passage of a Case II tachyon wall through the initial distribution of stars.

We note that the mechanisms for forming and maintaining spiral arms are poorly understood, and it seems that several different mechanisms are necessary to explain the observational data. See for example the article by Seigar and James (Seigar et al., 1998), where recent observations are compared with the predictions of density-wave theories (simple Shu-Linn theory and the more complex modal theories), bar-driven models, and large-scale shock theories.

The question of how tachyon walls might be generated has not yet been explored by the authors. However, Srivastava (Srivastava, 1984) has discussed the possibility that Schwarzschild black holes could emit a plane wave of tachyons. If the center of a galaxy contains massive black holes, then this would be one possibility for generating a tachyon wall which could induce the development of spiral structure in the surrounding galaxy.

It should be clear that more complicated interactions between a galaxy and our tachyon walls would be possible. For example, the subsequent evolution of a galaxy would obviously depend on the orientation of the galaxy relative to the wall. If a wall as described above were to interact with a galaxy disk whose angular momentum vector was aligned with the z axis, then effects of the wall on the galaxy would be minimal, since the galaxy's z extent would be small, and velocity components of the the stars in the z direction would also be small. However, this interaction would tend to compactify the galaxy in the z direction, because of the focusing effects of the wall. Other orientations of the galaxy and wall would produce more complicated effects. Another scenario would involve a galaxy encountering multiple walls with different orientations at different times during its evolution.

The effects can be generally characterized by the following statements: Velocity components in the direction that is spatially normal to the wall (in this paper, the z direction) will be altered so that objects moving toward the center of the wall ($z = 0$) receive a boost toward the center, and objects moving away from the center are slowed down or even turned back toward the center. Velocity components along the wall are unaffected.

The overall conclusion seems to be that collections of objects (stars in a galaxy or galaxies in a cluster) which are are collapsing are made to collapse yet faster; and conversely, collections which are expanding (and perhaps gravitationally unbound) may become more stable.

It is therefore tempting to speculate on the possible role of tachyon walls as dark matter. One may invoke continuing encounters with walls as stabilizing agents for galaxies or galaxy clusters. Thus there might be no need for the continuous presence of dark matter; the occasional encounter with a wall might be sufficient to stabilize a galaxy or cluster which would otherwise seem to be breaking up.

The effect of a Case-I wall on an extended object might be more drastic. If only part of a galaxy or cluster were to encounter such a wall, then that part would be compressed toward the unaffected part.

The cumulative effect of a number of such encounters would be quite complicated.

It would be very interesting to do numerical simulations on the evolutionary effects of a number of encounters on collections of stars or galaxies, as the presence of such walls during the early universe might provide some of the coarseness needed for the formation of large-scale structure. See the next section for remarks on the behavior of walls embedded in an expanding universe.

5. Cosmological Behavior and Effects of Tachyon Walls

In this section, we investigate the behavior of tachyon walls as components of Robertson-Walker universes. For our purposes it suffices to look at individual tachyon geodesics in a Robertson-Walker universe. The results here are consistent with the ideas in (Gibbons, 2002) where tachyons are shown to accelerate the expansion of the universe.

Since spherical symmetry holds, we may simply examine radial geodesics. (See also Narlikar and Sudarshan (Narlikar et al., 1976).) The geodesic equations for tachyons in the Robertson-Walker metric $-ds^2 = -R(t)^2[dr^2/(1-\kappa r^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)] + dt^2$ may be written for $t = t(s)$ and $r = r(s)$, where s is the spacelike interval-length,

$$\frac{d^2t}{ds^2} = -\frac{R\dot{R}}{1-\kappa r^2}(\frac{dr}{ds})^2 \tag{53}$$

$$1 = \frac{R^2}{1-\kappa r^2}(\frac{dr}{ds})^2 - (\frac{dt}{ds})^2. \tag{54}$$

Note that equation (53) implies that in an expanding universe

$$dt^2/ds^2 < 0. \tag{55}$$

Our tachyon walls have $dt/ds = 0$, and it therefore follows that the cosmological trend is to have the tachyons existing in the past but not in the future. Each tachyon trajectory would have two branches taking it into the past. One could take the standpoint that the two branches represent a tachyon-antitachyon pair which meet and annihilate where $dt^2/ds^2 = 0$, but this seems unnecessary. See Narlikar and Sudarshan (Narlikar et al., 1976).

One may easily integrate these two equations by solving equation (54) for dr/ds and substituting into (53). One obtains

$$\frac{dt}{ds} = \pm\sqrt{\frac{c_0^2}{R^2} - 1} \tag{56}$$

$$\frac{dr}{ds} = \frac{c_0}{R^2}\sqrt{1-\kappa r^2}, \tag{57}$$

where c_0 is a constant. For small R, near the Big Bang, $dt/ds \approx \pm c_0/R$, and therefore the tachyons behave like photons.

Such a photon-like component would tend to shorten the age of the universe, but in view of the apparent need for a non-zero cosmological constant (Riess et al., 1998), this would not seem to present any difficulties.

Note that the trajectories of the tachyons making up the Case-I and Case-II walls are geodesics. The fact that tachyon geodesics in a Robertson-Walker universe have two braches going back into the past implies that a wall seen "today" as an aggregate described by equations (2) or (3) would be visible in the past as *two* Lorentz-transformed versions of (2) or (3). This would make the walls even more effective in stimulating the formation of large-scale structure early in the history of the universe.

Narlikar and Sudarshan (Narlikar et al., 1976) and Srivastava (Srivastava, 1984) conclude that any tachyons that have survived to the present epoch must have very small rest masses, since otherwise they would have had unreasonably large energies when they were created in the past. This is due to the fact that any particle traveling on a geodesic in an expanding universe loses energy. This is true for bradyons as well as tachyons, and one can use this argument to show that, for example, any proton created during the quark-hadron transition era should now have a kinetic energy of about 10^{-15} eV. Obviously, this argument neglects any interactions that particles might have undergone since their creation, and it is difficult to say *a priori* what interactions tachyons might have encountered. Certainly, given our current state of ignorance about tachyons, it seems very likely that they would suffer interactions of some sort.

In fact, Recami and Mignani (Recami et al., 1974), Feinberg (Feinberg, 1967), and others have made some progress in describing possible tachyon–bradyon and tachyon–photon interactions. In some cases these interactions lead to the creation of tachyons from a series of bradyon interactions.

A tachyon, moreover, need never have been "created" in the usual sense. It is consistent with its space-like nature for a tachyon to have a *continuous* existence entirely within a domain having a lower limit in time; i. e., for its world line $x^\mu = x^\mu(s)$ to satisfy $0 < t_0 \leq t(s)$.

How aggregates of tachyons might appear as walls is a difficult question. Bradyons tend to collect themselves into sheets, filaments, and disks; and it may be that walls are the tachyonic analogs thereof. Since these walls have no external gravitational fields, they may be some sort of minimum-energy configuration. Since tachyon aggregates show unfamiliar causal behavior, it may be necessary to use statistical boundary conditions in studying their behavior (Cocke, 1967).

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References

Barnes, J.:1997, 'A Guide to Efficient and Stable Treecode', available online at *www.ifa.hawaii.edu/faculty/barnes/treecode/treeguide.html*

Cocke, W.J.:1967, *Phys. Rev.* **Vol. 160**, pp. 1089

Cocke, W.J.:1996, *Astrophys. & Space Sci.* **Vol. 244**, pp. 211

Davidson, A.:1987, *Phys. Rev. D* **Vol. 35**, pp. 1811–1814

Ellis, G.F.R.:1967, *J. Math. Phys.* **Vol. 8**, pp. 1171

Fairbairn, M. and Tytgat, M. H. G.:26 Jun 2002, "Inflation from a Tachyon Fluid", arXiv:hep-th/0204070

Feinberg, G.:1967, *Phys. Rev.* **Vol. 159**, pp. 1089

Gibbons, G. W.:18 Apr 2002, "Cosmological Evolution of the Rolling Tachyon ", arXiv:hep-th/0204008

Green, K.H.:1999, dissertation, University of Arizona

Kramer, D., Stephani, H., Herlt, E., MacCallum, M. and Schmutzer, E.:1980, *Exact Solutions of Einstein's Field Equations*, Cambridge University Press, New York

MacCallum, M.A.H. and Tipler, F.J., editors:1980, *Essays in General Relativity*, chapter 9, Academic Press, New York

Molski:1993 *Hadronic Journal* **Vol. 16** pp. 207–215

Narlikar, J.V. and Sudarshan, E.C.G.:1976, *Mon. Not. R. Astr. Soc.* **Vol. 175**, pp. 105

Padmanabhan, T.:18 Apr 2002, "Accelerated Explansion of the Universe Driven by Tachyonic Matter", arXiv:hep-th/0204150

Peebles, P.J.E.:1993, *Principles of Physical Cosmology*, Princeton University Press, Princeton, NJ

Petrov, A.Z.:1969, *Einstein Spaces*, Pregammon Press, New York

Recami, E. and Mignani, R.:1974, *Rev. Nuovo Cimento* **Vol. 4**, pp. 209–290

Recami, E.:1986, *Riv. Nuovo Cimento(3)* **Vol. 9(6)**, pp. 1–178

Riess, A.G. et al.:1998, *Astron. J.* **Vol. 115**, September

Seigar, M.S. and James, P.A.:1998, *Mon. Not. R. Astr. Soc.* **Vol. 299**, pp. 685

Sen, Ashoke:28 Mar 2002, "Tachyon Matter", arXiv:hep-th/0203265

Sen, Ashoke:13 May 2002, "Rolling Tachyons", arXiv:hep-th/0203211

Shiu, G. and Wasserman, I.:27 Jun 2002, "Cosmological Constraints on Tachyon Matter", arXiv:hep-th/0205003

Srivastava, S.K.:1984, *J. Math. Phys.* **Vol. 24**, pp. 693–697

Tolman, R.C.:1987, *Relativity Thermodynamics and Cosmology*, Dover Publications, Inc., New York