4-2002

Technology and Student Understanding in Mathematics

Sage Miller
St. John Fisher College

Follow this and additional works at: https://fisherpub.sjfc.edu/education_ETD_masters

Part of the Education Commons

How has open access to Fisher Digital Publications benefited you?

Recommended Citation

Please note that the Recommended Citation provides general citation information and may not be appropriate for your discipline. To receive help in creating a citation based on your discipline, please visit http://libguides.sjfc.edu/citations.

This document is posted at https://fisherpub.sjfc.edu/education_ETD_masters/2 and is brought to you for free and open access by Fisher Digital Publications at St. John Fisher College. For more information, please contact fisherpub@sjfc.edu.
Technology and Student Understanding in Mathematics

Abstract
Advances in technology have made graphing calculators more readily available to high school students and reformed standards in New York State encourage the integration of technology as a tool to improve student comprehension. This is a case study of three students (a typical A student, a special education C/D student and an unmotivated D/F student) as they are introduced to graphing relations (functions and non-functions) with the assistance of the graphing calculator. The results of the study were ambiguous, neither fully supporting the use of technology nor refuting the use of technology. However, the study did provide a great deal of insight into how important the methods of introducing technology are to a successful implementation of technology.

Document Type
Thesis

Department
Education

First Supervisor
Lucia Guarino

Subject Categories
Education

This thesis is available at Fisher Digital Publications: https://fisherpub.sjfc.edu/education_ETD_masters/2
Technology and Student Understanding in Mathematics

Sage Miller
St. John Fisher College

Follow this and additional works at: http://fisherpub.sjfc.edu/education_ETD_masters
Part of the Education Commons

Recommended Citation

This Thesis is brought to you for free and open access by the Ralph C. Wilson, Jr. School of Education at Fisher Digital Publications. It has been accepted for inclusion in Education Masters by an authorized administrator of Fisher Digital Publications.
Technology and Student Understanding in Mathematics

Abstract
Advances in technology have made graphing calculators more readily available to high school students and reformed standards in New York State encourage the integration of technology as a tool to improve student comprehension. This is a case study of three students (a typical A student, a special education C/D student and an unmotivated D/F student) as they are introduced to graphing relations (functions and non-functions) with the assistance of the graphing calculator. The results of the study were ambiguous, neither fully supporting the use of technology nor refuting the use of technology. However, the study did provide a great deal of insight into how important the methods of introducing technology are to a successful implementation of technology.

Degree Type
Thesis

Degree Name
MS in Mathematics, Science, and Technology Education

Department
Education

First Supervisor
Lucia Guarino

Subject Categories
Education

This thesis is available at Fisher Digital Publications: http://fisherpub.sjfc.edu/education_ETD_masters/2
Technology and Student Understanding in Mathematics

Sage E. Miller
St. John Fisher College
April 2002
Advisor: Dr. Lucia Guarino
Abstract

Advances in technology have made graphing calculators more readily available to high school students and reformed standards in New York State encourage the integration of technology as a tool to improve student comprehension. This is a case study of three students (a typical A student, a special education C/D student and an unmotivated D/F student) as they are introduced to graphing relations (functions and non-functions) with the assistance of the graphing calculator. The results of the study were ambiguous, neither fully supporting the use of technology nor refuting the use of technology. However, the study did provide a great deal of insight into how important the methods of introducing technology are to a successful implementation of technology.
Introduction

As New York State changes its methods of assessment in high school mathematics, the focus of the curriculum is being altered. One such change, causing a great deal of discussion among mathematics teachers, is the increased use of technology and more specifically the use of the graphing calculator. The National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards for School Mathematics (1989) states that the new technology that is available for use in the classroom has “changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them” (p.8). There appear to be two schools of thought about the result of this increased availability of graphing calculators.

On one hand, there are teachers who believe that using the graphing calculators will be very beneficial. These teachers think that the calculators will allow students to deepen their understanding of concepts because the increased speed allows for the exploration of many more examples. In the past, every example had to be tediously calculated and graphed by hand, making it possible to fully explore only a few examples. NCTM (1991) states:

Technology changes the nature and emphasis of the content of mathematics as well as the pedagogical strategies used to teach mathematics. With the introduction of technology, it is possible to de-emphasize algorithmic skills; the resulting void may be filled by an increased emphasis on the development of mathematical concepts. (p. 134)
Another problem that arises without calculators is that students often get bogged down in the arithmetic, making errors in multiplying or adding. With the use of calculators, this problem can be reduced allowing many more students access to higher learning. NCTM (1989) states:

Contrary to the fears of many, the availability of calculators and computers has expanded students’ capability of performing calculations. There is no evidence to suggest that the availability of calculators makes students dependent on them for simple calculations. ... Students should have a balanced approach to calculation, be able to choose appropriate procedures, find answers, and judge the validity of those answers. (p. 8)

Despite these reassurances from NCTM, there are mathematics teachers who feel that students are becoming too dependent on calculators and not spending time thinking for themselves. These teachers feel that students are not fully developing the concepts behind the mathematics but instead are relying on calculators to supply answers without understanding any of the process. Students are memorizing what order to press the keys and would not be able to replicate or explain any of the mathematics that the calculator is performing. If this is the case, then these students would certainly not be able to judge when it is appropriate to use a calculator and when it is not, nor would these students be able to judge the reasonableness of an answer found using the calculator.
My interest in this subject stems from my ability to see both sides of this issue and finding myself agreeing with each side in different situations. Over my five years of teaching at Athena High School, I have encountered many students that cannot get past the basic arithmetic of math in order to explore the higher ideas. However, I have also encountered students who cannot (or will not) perform basic math, like $5 + 3$, without using a calculator. Therefore, I ask the question, “Does the use of calculators in the mathematics classroom help students to develop a better understanding of mathematical concepts?”

One of the units in which I incorporate the use of the graphing calculator is a unit on graphing systems of linear, parabolic and circular equations. I would like to explore how the use of the graphing calculator affects students’ understanding of the “big picture.” This leads me to narrow my question to, “Do students in my classroom develop a better understanding of the connections between multiple representations of equations (algebraic, tabular and graphical) through the use of the graphing calculator?” My study will follow a few students’ thought processes as we move through the unit. I will be concentrating on how dependent the student becomes on the calculator, probing for student understanding of the concepts involved in graphing lines, parabolas and circles, students’ ability to recognize the different representations (tables, graphs and algebraic expressions) of lines, parabolas and circles, and how the student felt about his/her accomplishments through the unit.

I am not the only one interested in student comprehension of multiple representations of equations. Others feel strongly that this is a critical area to explore.
Romberg, Fennema, and Carpenter (1993) argued, “Research in this area is sparse, but with the increasing use of computers and graphing calculators, a coherent body of knowledge about how the connections are developed among tables, graphs, and the algebraic expressions related to functions is desperately needed” (p. ix). Knuth (2000) firmly supported these beliefs, executing his own research in this area at the high school level. Hollar and Norwood (1999) also explored this issue, although their work was performed at a college level. They concluded, “A continued research focus is needed to help find ways to facilitate the transition from operational to structural conceptions in students. Research on the reification of functions and other concepts should be expanded” (p.225). These are just a few out of many examples of researchers interested in this topic and performing research in the classroom in recent years.
Literature Review

There has already been a tremendous amount of research done related to calculators (or technology in general) and their effect on mathematics education. Most of these findings support the inclusion of technology in the mathematics classroom. The key, however, to successful inclusion seems to lie in the method of inclusion and usage, especially regarding methods of assessment.

General Calculator Use

There is a great deal of research supporting the belief that incorporating calculators into mathematics lessons is beneficial to students. Tarr, Uekawa, Mittag and Lennex (2000) state:

Campbell and Stewart (1993) found that calculators aid in problem solving, number sense, and understanding of arithmetic operations. Student confidence, enthusiasm and self concept at all levels were increased by calculator usage (Campbell & Stewart, 1993; Hembree & Dessart, 1986). According to Suydam (1987), over 100 studies reported that the use of calculators can improve the average student's paper-and-pencil skills, both in basic operations and in problem solving. (p.3)

Tarr, et al. (2000) explored the results of the Third International Mathematics and Science Study (TIMSS) as related to calculator usage at the middle school level. They focused on the United States, Portugal and Japan for various reasons. Tarr, et al. found that approximately 70% of students in the United States and Portugal reported frequent use of calculators during math lessons, while close to 75% of students in
Japan reported never using calculators during math lessons and less than 4% reported frequent use of calculators.

Tarr, et al. (2000) found that students in the US most often used calculators to promote higher order thinking skills. They concluded:

In the US, the effect of calculator use on student achievement in the TIMMS was particularly salient when the technology was used for nonroutine purposes, such as exploring number concepts and solving complex problems. This key finding suggests that the greatest attribute of handheld calculators lies in their ability to foster students' learning of mathematics concepts. Moreover, the calculator – when used in nontrivial ways – is a valuable tool for problem-solving and for the discovery of mathematical relationships.

It becomes clear that how technology is used in the classroom will affect how beneficial the technology is to the student. At a recent National Technology Leadership Retreat, representatives from national organizations in the core curricular subjects (math, science, social studies, English and technology education) and teacher education associations discussed appropriate integration of technology in the classroom. Lederman and Niess (2000) report on the results pertaining to the mathematics classroom. There are five areas of emphasis in the mathematics guidelines for integrating technology: introduce technology in context, address worthwhile mathematics, take advantage of technology, connect mathematics topics and incorporate multiple representations.
In all of these areas, it is stressed that teachers should not incorporate technology usage simply because it is possible, but rather to extend student concepts (Lederman and Niess, 2000). Under the heading of introduce technology in context, it is stated that "the use of technology in mathematics teaching is not for the purpose of teaching about technology, but for the purpose of enhancing mathematics teaching and learning with technology" (p. 347). Under the heading of addressing worthwhile mathematics, it is stated that "activities should support sound mathematical curricular goals and should not be developed merely because technology makes them possible. Indeed the use of technology in mathematics teaching should support and facilitate conceptual development, exploration, reasoning, and problem solving, as described by the NCTM" (p. 347). The technology should be used to investigate areas that were not previously possible due to tedious and lengthy computations, extending students concepts beyond the level which was previously possible. Lederman and Niess (2000) are careful to point out that using technology to approach curriculum in the traditional method not only does not benefit students but also negates the benefits of the technology.

Most importantly, Lederman and Niess (2000), under the heading of incorporating multiple representations, report that:

Activities should incorporate multiple representations of mathematical topics. Research shows that many students have difficulty connecting the verbal, graphical, numerical and algebraic representations of mathematical functions.
the high school level are algebraic expressions (e.g., \( Y = 2x + 3 \)), tables (e.g., \( x=-1, y=1; x=0, y=3; x=1, y=5; \) etc.) and graphs (e.g., a straight line crossing the y-axis at +3 with a slope of 2/1). The question is how to assess student understanding of these representations and ability to maneuver between them appropriately. Several researchers have broken student understanding of functions into categories in order to create a framework for interpreting the nature of student comprehension.

Moschkovich, Schoenfeld, and Arcavi (as cited in Knuth, 2000) broke functions into two dimensions: (1) the means available for representing the functions and (2) the perspective from which a function is viewed. The first dimension includes the three common representations discussed previously (algebraic, tabular and graphical.) Within the second dimension, how a function is thought of, there are two distinct possibilities.

A function can be viewed as a process or an object (Moschkovich et al. as cited by Knuth, 2000). Hollar and Norwood (1999) chose different terms to describe these same two perspectives: operationally (as an object) or structurally (as a process). A similar division of perspective was identified by Schwarz and Hershkowitz (1999) when they described two ways to approach teaching functions: the concept approach and the prototype approach. The concept approach is similar to the idea of viewing functions as a process, while the prototype approach is like viewing a function as an object.

All of these researchers describe the process approach as the lower end of Bloom’s taxonomy (Arends, 1994). The process perspective involves memorizing definitions (knowledge level of Bloom’s taxonomy) and matching \( x \) values to corresponding \( y \) values (comprehension, application and analysis levels of Bloom’s
taxonomy.) The object perspective, on the other hand, involves the higher levels of Bloom’s taxonomy: synthesis and evaluation. When a function is viewed as an object, the student looks at the function in a holistic manner, recognizing a function as the same basic object before and after transformations (Knuth, 2000).

Knuth (2000) studied students enrolled in various high school level math courses being taught through traditional methods. His study explored student understanding of the Cartesian Connection, which he describes, at the process level, as the ability to make equation-to-graph and graph-to-equation connections. He focused his study on “students’ abilities to use a particular aspect of the Cartesian Connection – the connection that the coordinates of any point on a line will satisfy the equation of the line – when responding to problems that require its use.” Knuth’s results were disappointing. He found that seventy-five percent of students used an algebraic solution to a given question even when a graphical solution would be much more straightforward. In addition, many students did not even acknowledge the possibility of using a graphical method to find a solution. Knuth concluded, “…students do not develop ability to flexibly employ, select, and move between algebraic and graphical representations. In fact, many students either perceived the graphical representation as unnecessary or used it as a means to support their algebraic-solution methods rather than as a means to a solution in and of itself” (p. 506). Knuth found that while most students had mastered the equation-to-graph connection, many students lacked the graph-to-equation connection. The graphing calculator provides students with a tool
that allows students to quickly switch between the graph and equation, making it easier to investigate the connection between algebra and graph in both directions.

*Functions, the Graphing Calculator and Assessment*

Beckmann, Thompson and Senk (1999) report “graphing calculators allow investigation of functions through tables, graphs and equations in ways that were not possible before their proliferation” (p. 2). However, by the very virtue of the accessibility of graphing calculators, the nature of assessment must be altered in order to truly examine student understanding. Typical questions about functions probe for student understanding in four areas: finding domain and range, evaluating a value, solving systems of functions and analyzing periodic functions. Beckmann et al. state “the desired information [with these types of questions] can often be obtained through use of button-pushing procedures on a graphing calculator... [providing] a limited view of students’ ability to work with functions or the depth of their understanding of functions” (p. 2).

Beckmann, et al. (1999) focus on how to alter typical questions related to functions to better assess student understanding when access to a graphing calculator is unlimited. Greenes and Rigol and Harvey (as cited by Beckmann, et al., 1999) characterized three levels of calculator use:

- **Calculator-inactive problems** are those for which there is no advantage (perhaps even a disadvantage) to using a calculator. **Calculator-neutral problems** are those that can be solved without a calculator, although a calculator might be useful.
Calculator-active problems are those requiring the use of calculators for their solutions (p. 2).

Beckman et al. (1999) suggest several ways of modifying typical function questions to make them more useful for assessment with a graphing calculator. One of the modifications requiring the least amount of change is simply to add the requirement that an answer be explained or justified. This change also promotes NCTM Standard 2: Mathematics as Communication (1989). A second alteration, which forces students to understand connections between graphs, tables and equations, is to provide the student with only one perspective, forcing them to use that perspective to answer the question. This approach offers a solution to Knuth’s (2000) problem where students were not displaying an ability to make the connection from graph-to-equation.

*Functions, Technology and Level of Understanding*

According to Hollar and Norwood (1999), “reification refers to the transition from the operational [process] to the structural [object] phase of concept development” (p. 221). In a study performed by O’Callaghan (1998), college students were found to have a better understanding of functions at the operational level after a Computer Intensive Algebra (CIA) course than students who experienced a traditional college algebra course. He used a function test that probed for knowledge of functions at four levels: modeling, interpreting, translating and reifying. Modeling, interpreting and translating are all categorized as operational skills. Reification is categorized as a structural skill. He found improvement in each of the first three levels in the CIA students, however, he did not find a significant improvement in structural (reifying) understanding.
Hollar and Norwood (1999) further explored these four areas of function understanding at the college level, using the TI-82 graphing calculator instead of computers. They concluded that although reification was the lowest scoring area of the four areas, both in the traditional and the technology classes, all areas showed significant improvement with the use of the TI-82 graphing calculator. They concluded that the reason for the improvement in reification, which was not seen in O'Callaghan's study, was due to the increased access to the TI-82. Students were able to use the calculators during all work time – inside and outside of class – whereas O'Callaghan's students only had access to the computers during lab time.
Methodology

Participants

The focus of this study was an Applied Geometry class at Greece Athena High School located just outside of Rochester, New York in the Town of Greece. Athena High School is a suburban school with a population of approximately thirteen hundred students. The Greece School District allows students residing in Greece to choose which high school to attend, so the student population is drawn from throughout Greece rather than only from the immediate surrounding area.

The Applied Geometry class is made up of mostly juniors and seniors with a few sophomores. At the time of this study, there were twenty-four students enrolled in the class, with an average of two to three students absent per day. The class provides a place for students who are struggling in Course II to opt down to a lower level math class, so students are added to the class throughout the school year. Seven of the current students are classified as needing modifications (such as extended time, simplifying language or minimal distractions) in testing situations. These modifications are provided within the classroom setting whenever possible. The class is an even split of female versus male students. Applied Geometry is typically a course offered for students who struggled in (but passed) Regents Course I and are not quite ready to plunge into Regents Course II. However, due to the changes in the math curriculum, this is the transition year and Course II will not be available next year. Students can gain the minimal regents math requirement by passing the Math A Exam and then continue in the new curriculum by taking Math 2. Thus, this course has been
redesigned, for this year, to prepare students for the Math A Exam and help adjust them to the non-traditional methods used in the new curriculum.

My research focused on three students – a high achiever, a mediocre achiever and a low achiever. The first student is a junior girl who is always present for class and participates well in every activity. She has maintained a high A average throughout the school year so far. The second student is a sophomore boy with testing modifications because he is a slow processor. He joined the class four weeks into the semester, after first trying to succeed in a Course II Regents class. He misses class no more than once a week but is often late to class by a few minutes, missing the introductory material. He is intelligent and works hard but is often playing catch-up because of his weaker processing skills. He has maintained a C/C+ average so far this year. The third student is a junior boy who has struggled with the material all year. He is often unmotivated, doing little work outside of class, but glad to volunteer in class when he knows an answer. His attendance is fairly good, although he has missed a few classes and is also a student that added in the fourth week of school, after finding Course II too difficult. His average is borderline D/F for the school year thus far.

I chose students at different levels and with different histories because I was interested in comparing how different types of students react to the graphing calculator and this method of introducing concepts.
The Unit

This study was conducted during a sixteen day unit on graphing lines, parabolas and circles, with parabolas being the main focus. The objectives for this unit are taken directly from the NCTM Standards:

- represent and analyze relationships using tables, verbal rules, equations and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- analyze the effects of parameter changes on the graphs of functions;

The graphing of lines should be a review, but the calculator was used to refresh students’ concepts of slope and y-intercept as related to a line. The entire concept of a parabola was new to most of these students and is heavily dependant on the graphing calculator. Students explored for themselves how the various parameters affected the appearance of the graph of a parabola. These skills were then applied to graphing circles in the last part of the unit. The unit concludes with using graphs to find solutions to systems of equations that can involve any combinations of the equations discussed (e.g. a parabola and a circle, a line and a parabola, two circles, etc.). The graphing calculator was available as a tool during all in class activities, although students were not able to take them home to use on homework. Most of the homework was designed to be calculator neutral. The reader should see the Appendix for a complete outline of the unit.

Data Collection and Analysis Methods

The data used in this study has been collected from a variety of sources. Throughout the unit, I constantly reviewed student work on homework, quizzes and the
unit test. As I reviewed each student’s work, I looked both for misconceptions and clear conceptions of the mathematics being addressed. I compared and contrasted the work of the three students, looking for common strengths and weaknesses. Some of the homework questions were used to address the student’s feelings about the unit and the use of the graphing calculator as we progressed through the unit.

Interviews with each student were conducted as a follow-up activity to further explore any misconceptions that were found in the student work. I also conducted interviews to probe more deeply into each student’s understanding of the concepts involved in this unit. The graphing calculator was available to the student for answering my questions during these interviews and I took field notes assessing how often and when the student chose to use the technology. These interviews were also used to get a feel for the student’s confidence in his/her abilities and whether he/she felt that the graphing calculator had increased that confidence. Each of these interviews was audio-taped to ensure that I accurately recorded the discussion. When analyzing the interviews, I looked for similarities and differences between the three students. I also focused on individual misconceptions, determining if the calculator increased, decreased or had no effect on their occurrence.

Finally, I spent time observing student behavior during independent practice, quiz and test situations, taking field notes regarding my observations. During these observations, I watched the students’ classroom participation and behavior to try to determine if motivation was affected (positively or negatively). Second, I tried to determine how often the calculator was being accessed during activities, in order to
gain insight into the student’s degree of dependence and the student’s ability to judge when/how to use the calculator.

Results and Analysis

I am astonished by the range of results in this study. My A student has once again performed well, developing the skills and knowledge that I identified as objectives for this unit. However, my special education student got so overwhelmed with trying to use the graphing calculator that he never became proficient with it, nor did he meet even the basic objectives of the unit. He got stuck in the mode of trying to answer every question by first turning to the graphing calculator and “forgot” that he had other skills and resources for answering questions. At the beginning of the unit, I had high hopes for my third student. He was motivated and involved in lessons, he was completing homework assignments. I thought, “Wow, the technology really did it for him!” However, he did not maintain this change in behavior throughout the unit. When I questioned him about this, his response was that it was easy at first, but then it got hard. This student also became very dependent on the calculator to provide answers, not wanting to do any of the thinking for himself. As I take a step back, looking at the overall results from the entire class, it does seem that the students fit into two categories – those that use the calculator as a tool to check answers and help with complex calculations and those who became completely dependent on the calculator and unable to answer questions designed to be calculator-inactive.
Case Study #1

Sue\(^1\) began this unit with no background experience with the graphing calculator. She performed very well on the pre-test, making only a simple graphing error. After the introductory lesson on the use of the graphing calculator, Sue's response was very positive. She states, "I enjoyed working with the graphing calculator. I think it will make the work easier and it will be fun to use them." Sue immediately caught on to the idea that the equation determines the shape of the graph. On her first quiz, in response to the request to explain why the graph of \(y = -x^2\) is shaped the way it is, Sue responds, "Since the exponent is squared [...] the graph will be a U-shape."

After several days of exploring the effect of parameters on the graph of a parabola, Sue seemed to have a complete understanding. On a warm-up in class where students were encouraged to use their notes to describe the effect on the graph of each underlined parameter in the equation \(y = 2(x - 3)^2 - 1\), she accurately described each one. "The -3 moves the graph to the right, the -1 moves the graph down and the 2 changes how wide the graph is." However, on the second quiz, when asked to describe the difference in the graphs of \(y = x^2\) and \(y = 2(x + 3)^2 - 6\), she focused only on the change in width, not mentioning anything about the position. In a follow-up interview, Sue explained that she didn't realize that she was supposed to tell about the change in position. She was able to tell me, in the interview, the effect of each of the parameters on the graph. She did use the graphing calculator to remind herself which

---

\(^1\) Names of students in case study have been changed to maintain anonymity.
parameter was causing the up/down movement and which parameter was causing the right/left movement.

The third quiz in this unit was designed to allow me to determine whether students could graph with and without the calculator. There were 3 equations to graph – a line, a parabola and a circle. A line and a parabola can both be entered in the calculator and the table of values can be used to plot points. However, since a circle is not a function, its equation can not be entered into the calculator in the standard way and students were not taught how to use the calculator as a tool to help with graphing circles. Sue was able to graph the line and the parabola without any problems. However, she did have some trouble with the circle. She remembered that the equation 

$$(x -1)^2 + (y - 4)^2 = 9$$

represents a circle and how to find and graph the center of the circle, but forgot that the radius is squared in the equation and thus graphed a radius of 9 rather than 3. However, on follow-up homework assignments, Sue correctly graphed circles and on the unit test she had no problems graphing circles (see figure 1).
On the unit test, Sue showed a superb comprehension of the unit overall. She made a few errors on the test but they were mostly computational in nature, not misconceptions. The only misconception that she displayed on the test was in comparing graphs of the parabolas $y = x^2$ and $y = x^2 - 3$. She described the graph of
y = x^2 - 3 as being “longer and thicker” than y = x^2. This misconception was caused by viewing the two graphs on the graphing calculator screen on a 10x10 window. This causes y = x^2 - 3 to appear longer because more of the graph shows in the window. Sue apparently forgot her prior learning on this question and depended on the calculator to provide an answer.

Sue’s attendance did not show any significant change during this unit. She was absent one day for a school field trip and she spent her lunch period with me the same day to catch up on the material missed. Her classroom behavior also stayed consistent with past behavior – attentive and involved in lessons, contributing when called on but being shy about volunteering answers.

The final interview was conducted one week after Sue took the unit test. The focus of the interview was on how she perceived her use of the graphing calculator. I had noticed in classroom observations that Sue did not turn to the calculator right away on any given question. She would first do as much work as she could without the calculator and then use the calculator to check her answer. When I questioned Sue about how she decides when to use the calculator, her response was, “I use the calculator mostly to check my answers.” I wanted to determine whether she could judge the reasonableness of an answer given by the calculator so I told her I was going to enter the equation y = x^2 + 2 and I showed her the graph of a straight line to see how she responded. She immediately wanted to see what equation I had actually typed into the calculator, but I wanted her to problem solve without seeing what I typed. Following is an excerpt from this point in the interview.
Teacher: Why do you want to see the equation I entered?
Sue: Because I know that can’t be the graph of \( y = x^2 + 2 \).
Teacher: Why not?
Sue: Because \( y = x^2 + 2 \) has to be a u-shape.
Teacher: How do you know?
Sue: Because the x is squared.
Teacher: What would make the graph a line?
Sue: the x would not have any exponent
Teacher: So what do you think I might have done wrong?
Sue: You forgot to square it!
Teacher: Right! (Shows the equation entered.)

This portion of the interview certainly demonstrates Sue’s ability to judge the reasonableness of an answer supplied by the calculator. She also shows an ability to problem-solve to determine where the calculator (or more likely the user) went wrong.

Sue met and exceeded my expectations for this unit. She became proficient with the technology available and learned to use it as a resource rather than become dependent on it to supply all of the answers.

*Case Study #2*

Bob began this unit with some background familiarity with the graphing calculator. He was introduced to graphing lines in Course I through the use of the graphing calculator. One would hope that this would have given Bob an advantage in
mastering the technology, but it did not seem to. As previously mentioned, Bob has slower processing skills, requiring that he have more time to assimilate material. However, the very nature of the graphing calculator allows lessons to move quickly between the various representations of a relation – table, graph and equation. Almost as soon as we began working with the graphing calculator, Bob seemed to always be several steps behind the rest of the class. There were other students that struggled with various parts of the graphing calculator, but after repeatedly performing the same operations, most students slowly gained proficiency.

Bob was a different story. Despite after school sessions, where we focused on manipulating the graphing calculator, by the end of the unit Bob had gained only the bare minimum of proficiency on the graphing calculator. He did not make the connection between the table of values and the graph of the function until the follow-up interview which took place nearly two weeks after the unit test. On the second quiz, Bob correctly used the calculator to fill in a table of values for \( y = -x^2 + 2x - 3 \), but in graphing the parabola only plotted the vertex point (see figure 2). On the unit test, Bob did not look at the table of values at all, instead depending on the graphing screen to estimate where he should plot points on his paper.

Figure 2. Bob’s response to question #4 on Quiz 2.
However, in the exit interview, Bob seemed to be able to explain what the table of values shows us. It seemed as if he had never made the connection between knowing what the table represents and using the table to complete a graph. Here is the discussion:

Teacher: One of the things that I noticed on your test was that you mostly had the basic shapes but you weren’t sure how to get the exact points to graph. I noticed on the quizzes that you had really long decimals for points. I think you might have been doing this (demonstrates using the cursor on the graphing calculator to try to estimate a point on the graph).

Student: Yeah, I did.

Teacher: Can you tell when it’s right on the parabola?

Student: I usually tried to get it right here (demonstrates moving the cursor to approximately where the vertex is). You have to look and when it gets clear, (pointing to the x-value shown by the cursor position) you can see which point is on the graph.

Teacher: Watch the difference between these two buttons. (Demonstrates how the cursor can be moved anywhere on the screen. Then uses TRACE to move the cursor around and shows how the cursor is “stuck” to the graph.) Which way do you think gives points on the graph?

Student: The second way.
Teacher: Do you remember any other way we learned of finding points on the graph? (Points to the button choices.) Do you think any of these would be helpful?

Student: Table?

Teacher: Okay (pushes the Table key), what does the Table tell us?

Student: Where the parabola is located.

Teacher: Yes. So, the chart says that at (-1, 5) there’s a point. If I go back to my graph, and I use the cursor can I get to the point (-1, 5)

Student: No.

Teacher: What if I use the TRACE button?

Student: Oh! Now you can get to (-1, 5)!

Bob immediately answered the question of what the table of values tells us about the graph, but he had trouble applying it at first. We continued our discussion by having Bob plot points for several of the equations from the test and then compare his original sketches to the graphs when he plotted actual points. By the end of the interview, Bob understood how to graph using the table of values. However, he was still having problems graphing, which leads me to discuss the next issue he faced.

On top of Bob’s problems with the technology, he seems to lack the basic skills required of seventh grade students in graphing. He cannot keep straight which is the x-axis and which is the y-axis. This became apparent over several interviews and on the unit test. Unfortunately, it was not obvious at the beginning of the unit because when Bob took the pre-test he was able to accurately label the axes and plot points. On the
table of values that he was asked to generate, he did make several computational errors, leading me to believe that the graphing calculator would be a great asset for him, allowing him to bypass the computational frustrations and gain an understanding of the material. Thus, in the first few after school sessions, I did not focus on the basics of graphing, assuming that Bob already had those concepts mastered. However, Bob apparently needed much more reinforcement in these areas – on the test he labeled the axes wrong on all but the first graph. I am unsure how to analyze this. Most students who get confused labeling the axes will decide on a way to label and at least stay consistent in their labeling. Bob seemed to alter his ideas about the axes from one problem to the next, within a time span of 5 minutes.

As the unit progressed, it became more obvious that Bob was struggling with the basic concepts being presented. On the first quiz, Bob’s response to the request to explain why \( y = -x^2 \) is shaped the way it is showed his lack of understanding regarding the idea that the equation of a relation determines the shape of the graph. He was able to use the calculator to get an accurate sketch of the graph, but his explanation of the shape was, “because it starts as a positive then ends as a negative.” Upon interviewing Bob about this response, it was discovered that by positive, Bob meant that the graph was increasing and by negative he meant that the graph was decreasing. In the interview, he did not remember why the graph was u-shaped. In the exit interview, I questioned Bob about this idea again to see if his understanding had developed.

Teacher: Do you remember what made a graph be this shape (points to a parabola) instead of that shape (points to a line)?
Student: Oooh ... ummm ... it was like all in the formula. I don't remember too well because obviously I didn't do too well on the test. I know that if it's like $2x$ ... the 2 depends ... wait, am I allowed to use the calculator?

Teacher: Yeah, go ahead!

Student: I think it's a lot easier with the calculator because you can just type in the formula and see if it works. I know that you add numbers somewhere, I don't know where to add them, but when you add numbers it changes the shape of the graph. Like it could make it wider.

Teacher: But can adding a number make it become a u-shape instead of a straight line?

Student: Umm ... if you add numbers to it?

Teacher: Yes.

Student: Well, for the circles it's a different formula.

Teacher: Okay, but we have a u-shape here (points to the calculator screen). How could we make that a line instead?

Student: The larger that number is (points to coefficient of $x^2$) the thinner the u will be and the smaller the number, it will be a lot fatter.

Teacher: Okay, but what I'm trying to get at is what makes it u-shaped as opposed to a line?

Student: A line? The squared?

Teacher: So if $x$ is squared you get a u-shape. How do you get a line?

Student: I take the squared away – just have a regular $x$. 
This excerpt from the exit interview shows a few key points. First, Bob can eventually answer the question. He tells me that the exponent changes the shape of the graph from a line to a parabola. But this interview also shows me that Bob’s thought processes need quite a bit of directing in order to retrieve the appropriate response. Bob’s thoughts were focused on what effect the parameters of a parabola have on the shape of the parabola and he had a lot of difficulty refocusing his thoughts in order to answer the question, “What makes a graph u-shaped instead of a line?” Later in the interview, Bob again tried to steer the conversation back to the effect of parameters on the shape of a parabola when we were on a different topic. It was as if Bob knew that the parameters were an important piece and wanted to share what he knew in that area rather than be questioned in areas he was unsure about.

During several interviews with Bob, he made errors in entering equations in the graphing calculator and was unable to recognize that something was wrong by looking at the shape of the graph. In one interview, we were discussing the effects of the various parameters on the graph of the parabola. We spent time looking at shifts up or down and shifts left or right. After spending about fifteen minutes discussing these shifts, using the graphing calculator to view the graphs, I asked Bob to tell me about the graph of $y = x^2 + 3$. He entered $y = x + 3$ in the calculator and then proceeded to describe the position of the line without realizing that $y = x^2 + 3$ could not be a line and that something must be wrong.

In classroom observations during independent practice, quizzes and tests, what stood out most was that Bob spent an inordinate amount of time on the calculator.
Questions that did not require any use of the calculator had Bob trying and trying to "remember" how he was supposed to use the calculator to get the answer. His assumption, I found through interviews, was that he had forgotten how to perform the task with the calculator rather than that it was not intended that he use the calculator. Even for questions that were intended as calculator-dependent questions, Bob spent a long time figuring out which keys to press, etc. rather than already being prepared to use the calculator.

Finally, after reviewing Bob's performance on the unit test and discussing the ideas in the unit test during the exit interview, it is apparent that Bob never gained a complete and firm understanding of the "big idea" that the equation of a relation determines the shape of the relation. During the exit interview, he sketches the different shapes that we learned to graph during the unit but only after a great deal of discussion and prompting does he determine equations that would cause a line, parabola and cubic. He does not ever come up with an equation for a circle. On the unit test, he graphs two parabolas when the question shows the equations of two circles right after graphing a circle and parabola in the previous question.

I find it very difficult to analyze the causes of this situation. If technology had not been available, it is possible that Bob would have gotten caught up in the computational process and been just as lost as he was with the technology. The technology did allow him to see an accurate representation the graphs of various relations, even though he was not able to transfer that knowledge to paper. Therefore, the technology may have had benefits that are not immediately obvious in a test.
situation. In the exit interview, Bob was able to reconstruct his knowledge about the parameters of a parabola although he had no retention without the use of a calculator.

Bob did not maintain high motivation throughout the unit despite starting off with good intentions. He came for extra help (and interviews) several times in the beginning of the unit, but stopped these visits towards the middle of the unit, in spite of my strongly suggesting that he stay for help. After the unit was complete, we scheduled four different exit interviews before Bob remembered to show up.

Case Study #3

Greg began this unit on a very strong note. He participated in class, completed each homework assignment and seemed highly motivated. Greg did an excellent job on his pre-test, confusing only the direction of a line when slope is negative. He also felt very confident about the pre-test. In response to the question of how well he thought he performed, he said he probably earned an A and commented that he found the pre-test easy except for the question about slope.

Greg had not had any prior experience with the graphing calculator. After the introductory lesson on using the graphing calculator, I asked students whether they enjoyed working with it. Greg responded, “No. There are too many keys to use and remember and we can't take them home.” Despite Greg’s uneasiness with the graphing calculator, as the unit progressed, he was often the one to remember which key to use and answer the questions about how to use the calculator during class lessons.
On the first quiz, Greg came through with flying colors! His explanation of why the graph of $y = -x^2$ is shaped the way it is shows a complete understanding at this point in the unit (see figure 3).

Figure 3. Greg’s response on Quiz 1.

However, about five days into the unit Greg lost his motivation. He did not turn in homework on the sixth or seventh days and was absent from class on the seventh day. Despite prompting, Greg does not come in for help on the material he missed during his absence. He was absent again on the eleventh day and came late to class on the twelfth day. Greg did not do any of the review assignments at the end of the unit in order to prepare for the test (days 11 – 13) and did not come in for help on any of the material missed during his absences. So, despite Greg’s strong start, he seemed to be on a downward spiral through the rest of the unit.

On the first quiz, Greg showed a strong understanding of the shape of a graph and its connection to the equation. However, on the third quiz, when asked to graph three equations (a line, parabola and circle), Greg graphed three diagonal lines, of
which the line was not even correctly graphed. On the unit test, he was able to accurately match equations to a sketch of their graphs, but he was unable to graph on his own (see figures 4 and 5).

Figure 4. Greg's response to Question #3 on test – shows understanding of equations.

3) Match each equation with its sketch:
   a. \( y = -x \)
   b. \( y = x^2 \)
   c. \( y = -x^2 \)
   d. \( y = x \)

Figure 5. Greg's response to Question #16 and 17 – shows lack of ability to apply understanding.

16) Graph and state the solutions:
   \[
y = 2 \quad y = -3x^2 + 5
   \]

17) Graph and state the solutions:
   \[
y = x + 3 \quad x^2 + y^2 = 9
   \]

All of this leads me to believe that Greg had some of the basic concepts that I had identified as objectives, but did not have a strong understanding of the material. However, the exit interview changed many of my ideas. After spending approximately an hour going over material from the unit with Greg in an interview type of setting, I developed the opinion that Greg had a fairly strong base of knowledge but did not want to take the time to think. For example, the interview began as follows:
Teacher: Do you remember what shape that would be if you graphed it? (Points to \( y = x \))

Student: A circle.

Teacher: Okay, let's try it. Do you remember how to graph with the calculator?

Student: Yeah. (Uses calculator to graph equation.) It's a line.

Teacher: Just a line. Okay ... and how about the next one (\( y = -x \)).

Student: That's still going to be a line.

Teacher: Okay, what's going to change about it?

Student: the slant

Teacher: Okay, let's double check.

Student: (graphs \( y = -x \) with calculator) It's going the opposite way.

When asked what shape the equation would have, Greg gives an immediate response without pausing to think. However, when asked to check his answer with the calculator, Greg sees his mistake and then can continue answering questions without using the calculator. Through continued questioning, I established that Greg knew a parabola need to have \( x^2 \), that a v-shaped graph contains the absolute value function and that a cubic needs to have \( x^3 \). With some prompting, we also discussed what is necessary in an equation to make a circle.

Teacher: Now here's where you had some trouble on your test. (points to the equation \( x^2 + y^2 = 9 \)) We looked at lines, parabolas, v-shapes and squiggles.

Do you remember any other shapes we learned how to graph?

Student: That's the only thing that I really didn't know ...
Teacher: Okay, do you remember what you thought this was at the beginning?

(Referring back to \( y = x \) where student originally guessed a circle.)

Student: A line.

Teacher: Before you said it was a line?

Student: A parabola.

Teacher: You don’t remember?

Student: No, I don’t

Teacher: It’s a circle.

Student: Oh yeah ... yeah.

Teacher: How do you tell what shape? I mean if you look at these (pointing out the various equations on the page) how can you decide what the graph will look like?

Student: The circle has \( x \) and \( y \).

Teacher: But each of these had \( x \) and \( y \). (Points out \( x \) and \( y \) in several of the other equations.) What’s different about the circle equation?

Student: It has \( x \) plus \( y \).

Teacher: The \( x \) and \( y \) are on the same side? Anything else different?

Student: They are both squared.

Teacher: And with the line? (Points back to the equation \( y = x \)) What was true about the exponents there?

Student: In a line neither one is squared.

Teacher: And in a parabola?
Student: Only one is squared.

Teacher: Which one?

Student: the x.

The interview continued in a similar manner, with me posing questions and Greg getting to the correct answer eventually, sometimes with more prompting than others. When we spent some time using the table of value to graph, it became apparent once again that Greg’s biggest problem was taking his time to do things correctly. He didn’t get confused about the axes or directions for plotting points but he would try to get done so quickly that he would make careless errors plotting points.
Discussion

The conclusion that I have come to regarding the use of technology in the classroom is not quite as clear cut as I had hoped it would be. First, as previous studies have found, I think students are much more successful with technology if it is available both inside and outside of the classroom. Although I attempted to make my homework assignments calculator neutral, it was very difficult to give the students enough practice without having some calculator active problems. Therefore, students would certainly have benefited from having the use of a graphing calculator when completing homework assignments. In addition, one of the major impediments to Bob’s understanding was in being overwhelmed by the technology. Had the graphing calculator been available to him outside of class, he would probably have been much more adept at working with it.

However, this does not mean that using the graphing calculator for this unit was a complete failure. As we saw with Sue, students who were fairly strong in math adapted quickly to the use of the calculator and were able to meet the objectives stated for this unit. Sue became proficient not only with using the graphing calculator but also with knowing when it was appropriate to use the graphing calculator. For the most part this held true for the entire class, with some weaker students surprising me with how easily they adapted to the technology. For these students, integrating technology with new material provided an interesting change in the classroom. Many students showed an improvement in attitude, showing great enthusiasm for learning to use the technology. These same students would probably have been fairly successful without
the graphing calculator component, but it added an element of enjoyment and diversity to the classroom.

The greatest benefit for me, as a teacher, in conducting this study is the insight into how the unit needs to be altered to make it more successful. I cannot control whether students have access to a graphing calculator outside of the classroom, but I now know that it is vital that I provide them with more in-class time to work with the calculators. It would be very helpful if I had the students start working with the calculators on a daily basis at the beginning of the year, so that they are already comfortable with the basic operations by the time we begin this unit. I would also spend at least twice the amount of time introducing the graphing components on the calculator, with most of that time being hands-on experience for the students.

One of the major errors that I saw on the unit test was students who simply sketched on approximate graph instead of plotting exact points. I believe this was because when we began investigating the shapes of various functions, we sketched the function so as not to be bogged down in plotting points. However, students found that first habit difficult to break as we continued with the unit. In reworking the unit, this is definitely something that I would change, making students plot exact points from the beginning so students have a better feel for the connection between the points and the graph. I think this would also work to reinforce the student's grasp of the connection between the table and the graph of an equation.

Another weak area in this unit was the development of the pattern in the y-values of a parabola. We spent a class period looking at the pattern in the y-values of
various parabolas, but I neglected to take it to the next level where students could apply that pattern to make plotting the points much easier. I did not fully realize the impact of this until after the unit test. I felt so strongly about the significance of this missed opportunity that I spent some follow-up time in class exploring the meaning of the patterns and applying them to graphing functions.

The last criticism I would offer for my approach to this unit is that it was too full. In the future, I would separate the idea of solving a system of functions into a separate unit. I believe this would allow students to become more proficient with graphing a single function before taking the next step. It has been my experience that students sometimes perform better when their focus is narrowed, so that they can concentrate on fewer key ideas at one time.

Having offered so many criticisms of this unit, I do want to emphasize that both myself and my students enjoyed working with the graphing calculators in this unit. I do think that it was very helpful in investigating many graphs in a shorter amount of time. I will definitely continue to use technology in my classroom to encourage student understanding of content. I am simply much more aware of how important it is to introduce the use of the technology carefully. I am also aware of some pitfalls for this particular material that have to be accounted for in the approach to teaching the material. Hopefully, this experience will make it easier for me to predict trouble spots in other units when I am integrating the use of technology. For example, I now know how important it is to start out slowly with any new technology. Students will always need some time to simply familiarize themselves with the newness of any technology. In
addition, it is easier to move more quickly after students are comfortable with a tool than trying to move too quickly before they are comfortable and then having to backtrack.
References


# Appendix

Complete Outline of Unit

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Pre-test</td>
<td>Worksheet A</td>
</tr>
<tr>
<td>1</td>
<td>Using Graphing Calculators</td>
<td>Worksheet B</td>
</tr>
<tr>
<td>2</td>
<td>Calculator lab</td>
<td>Worksheet C</td>
</tr>
<tr>
<td>3</td>
<td>Looking for Patterns</td>
<td>Worksheet D</td>
</tr>
<tr>
<td>4</td>
<td>Parent function &amp; Constants</td>
<td>Worksheet E</td>
</tr>
<tr>
<td>5</td>
<td>Parent function, Coefficients &amp; Constants</td>
<td>Finish Questions from Notes</td>
</tr>
<tr>
<td>6</td>
<td>Algebra &amp; the Vertex</td>
<td>Worksheet F</td>
</tr>
<tr>
<td>7</td>
<td>Review Day</td>
<td>Worksheet G</td>
</tr>
<tr>
<td>8</td>
<td>Solving Quadratics by Graphing</td>
<td>Worksheet H</td>
</tr>
<tr>
<td>9</td>
<td>Graphing Circles</td>
<td>Worksheet I</td>
</tr>
<tr>
<td>10</td>
<td>Writing Equations</td>
<td>Worksheet J</td>
</tr>
<tr>
<td>11</td>
<td>Solving Systems by Graphing</td>
<td>Worksheet K</td>
</tr>
<tr>
<td>12</td>
<td>More Solving</td>
<td>Worksheet L</td>
</tr>
<tr>
<td>13</td>
<td>Review</td>
<td>Worksheet M</td>
</tr>
<tr>
<td>14</td>
<td>Review</td>
<td>Worksheet N</td>
</tr>
<tr>
<td>15</td>
<td>Test</td>
<td>Sample Math A Test</td>
</tr>
</tbody>
</table>
1) In your own words, explain what it means when a line has a negative slope:

The line slants down from left to right.

2) In which quadrant is the point (3, -2)? **IV**

3) Given the equation \( y = 2x - 5 \), what x-value corresponds to \( y = 13 \)?

\[ \begin{align*}
13 &= 2x - 5 \\
18 &= 2x \\
9 &= x
\end{align*} \]

4) Complete the following table of values for the equation \( y = 3x + 2 \). Then graph the equation.

<table>
<thead>
<tr>
<th>X</th>
<th>Y = 3x + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3(-2) + 2 = -4</td>
</tr>
<tr>
<td>-1</td>
<td>3(-1) + 2 = -1</td>
</tr>
<tr>
<td>0</td>
<td>3(0) + 2 = 2</td>
</tr>
<tr>
<td>1</td>
<td>3(1) + 2 = 5</td>
</tr>
<tr>
<td>2</td>
<td>3(2) + 2 = 8</td>
</tr>
<tr>
<td>3</td>
<td>3(3) + 2 = 11</td>
</tr>
</tbody>
</table>

5) Given the following table of values, plot and connect the points.

| X  | Y = |X| |
|----|-----|
| -3 | 3   |
| -2 | 2   |
| -1 | 1   |
| 0  | 0   |
| 1  | 1   |
| 2  | 2   |
| 3  | 3   |
1) Have you ever used a calculator that can graph lines for you?__________
   a. If you have, when was it?

2) If you were going to give yourself a letter grade (A through F) on today’s pre-test, how well do you think you did?

3) Which part(s) of today’s pre-test were easiest for you to do?

4) Which part(s) of today’s pre-test were most difficult for you to do?

5) In 7th and 8th grade, you were taught to graph by setting up a table and graphing all of the points. In Course I, you were taught to graph lines by finding the y-intercept and the slope. Which way of graphing a line made more sense to you and why?

6) Think of two different ways that you could tell if the point (2, 1) is on the line y=4x-1.
   a. First way:
      Plug in point and solve: 
      \[ 1 = 4(2) - 1 \]
      \[ 1 = 8 - 1 \]
      \[ 1 \neq 7 \]
   b. Second way:
      Graph the line and see if the point is on it
Home Screen: When calculator is first turned on, can be used like a regular calculator.

<table>
<thead>
<tr>
<th>Button</th>
<th>Color</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=</td>
<td>Blue</td>
<td>Used to enter the equation $y = f(x)$</td>
</tr>
<tr>
<td>GRAPH</td>
<td>Blue</td>
<td>Used to see the graph $y = f(x)$</td>
</tr>
<tr>
<td>WINDOW</td>
<td>Blue</td>
<td>Sets the size of the graph</td>
</tr>
<tr>
<td>TRACE</td>
<td>Blue</td>
<td>Tells points on the function</td>
</tr>
<tr>
<td>X,T,θ,n</td>
<td>Black</td>
<td>To get an $x$</td>
</tr>
<tr>
<td>X²</td>
<td>Black</td>
<td>To put an exponent of 2 on an $x$</td>
</tr>
<tr>
<td>MATH</td>
<td>Black</td>
<td>Where various special keys are, like $x^3$ and $\text{abs}(x)$</td>
</tr>
<tr>
<td>TABLE</td>
<td>Yellow</td>
<td>To see $(x, y)$ pairs for the function</td>
</tr>
<tr>
<td>TBLSET</td>
<td>Yellow</td>
<td>To set the starting # and jump of the table</td>
</tr>
<tr>
<td>Arrows</td>
<td>Blue</td>
<td>To move in almost any screen</td>
</tr>
</tbody>
</table>

"Nice" Window:

- `xmin = -9.4` → Smallest $x$
- `xmax = 9.4` → Largest $x$
- `xsc1 = 1` → Jump in $x$ (each tick mark on graph)
- `ymin = -9.4` → Smallest $y$
- `ymax = 9.4` → Largest $y$
- `ysc1 = 1` → Jump in $y$ (each tick mark on graph)
- `xres = 1`...

TBLSET: What would you want to set TblStart and ΔTbl to for:

1) $-8, -7, -6, 5$
   - TblStart: $-8$
   - ΔTbl: $1$

2) $3, 4, 5, 6$
   - TblStart: $1$
   - ΔTbl: $1$

3) $-2, 0, 2, 4, 6$
   - TblStart: $2$
   - ΔTbl: $2$
1) Did you enjoy working with the graphing calculator today in class? Why or why not?

   opinion:

2) Did using the graphing calculator make it easier to graph functions?

   opinion:

3) When you look at the table of values for a function, what is it really telling you?

   a pair of \((x, y)\) coordinates of the function that is graphed → points on the function

4) Does every graph have to be a straight line? Explain your answer.

   no, we graphed U-shapes and V-shapes in class today.

5) How do you change the "jump" in the table of values for a function?

   by going to TABLESET and giving ATbl a different #.

6) Explain how to get to the abs() function on the calculator.

   press the MATH key, arrow over to NUM and choose 1.
Use your graphing calculator to sketch each function and fill in the table of values.

<table>
<thead>
<tr>
<th>Function</th>
<th>Sketch</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y = 2x - 1$</td>
<td><img src="image1" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$Y = x^2 - 8$</td>
<td><img src="image2" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$Y = \text{abs}(x + 2)$</td>
<td><img src="image3" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$Y = -x^2 + 2x + 1$</td>
<td><img src="image4" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
1) Today we graphed functions that had four distinct shapes. Describe them:
   a. Straight lines
   b. V or U shaped
   c. N or \n shaped
   d. \ or \n shaped

2) What do you notice about the exponent in the equations of the functions that look like this:

   \[ \text{The } x \text{ is squared.} \]

3) Based on today lab, what shape do you think each of the following functions would have? (Use your descriptions from above.)
   a. \[ Y = x + 3 \] straight line up
   b. \[ Y = x^2 - 2x + 5 \] U shape
   c. \[ Y = 3x^2 + 2x \] \ shape
   d. \[ Y = \text{abs}(x - 5) \] \ shape
   e. \[ Y = -2x + 6 \] straight line down
   f. \[ Y = -x^2 - 7 \] \ shape

4) In each of the shapes, a similar change occurred when a negative sign was placed at the beginning of the equation. Describe the change.

   revised graph

   \( \text{U} \) became \( \text{L} \) \( \text{\ became \L} \)

5) Look at the table of values for each equation from today. List each equation in one of the column below. Equations whose \( y \)-value changes by:
   a. the same amount each time (ex. the \( y \)'s go up by 3 each time)
      \[ \begin{align*}
      y &= 2x - 1 \quad \text{(up 2)} \\
      y &= \text{abs}(x+2) \quad \text{(up 1)} \\
      y &= -3x + 1 \quad \text{(down 3)} \\
      y &= -\text{abs}(x-3) \\
      \end{align*} \]
   b. different amounts each time (ex. goes up 3, then up 2, then up 1, then down 2, etc)
      \[ \begin{align*}
      y &= x^2 - 5 \\
      y &= -x^2 + 2x + 1 \\
      y &= x^3 - 2x \\
      y &= -x^3 + x \\
      y &= -x^4 + x^2 \end{align*} \]
Complete the table of values for the function \( y = 2x^2 - 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Change in ( y )</th>
<th>Change in change in ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>27</td>
<td>-14</td>
<td>+4</td>
</tr>
<tr>
<td>-3</td>
<td>13</td>
<td>-10</td>
<td>+4</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
<td>-6</td>
<td>+4</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
<td>-2</td>
<td>+4</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>+3</td>
<td>+4</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
<td>+5</td>
<td>+4</td>
</tr>
<tr>
<td>2</td>
<td>+3</td>
<td>+10</td>
<td>+4</td>
</tr>
<tr>
<td>3</td>
<td>+13</td>
<td>+14</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Complete each of the following tables:

1) \( y = x^2 - x - 6 \)  

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Change in ( y )</th>
<th>Change in change in ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>14</td>
<td>-8</td>
<td>+2</td>
</tr>
<tr>
<td>-3</td>
<td>4</td>
<td>-6</td>
<td>+2</td>
</tr>
<tr>
<td>-2</td>
<td>-4</td>
<td>-2</td>
<td>+2</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
<td></td>
<td>+2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td>+2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) \( y = x^2 - 2x - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Change in ( y )</th>
<th>Change in change in ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>21</td>
<td>-9</td>
<td>+2</td>
</tr>
<tr>
<td>-3</td>
<td>12</td>
<td>-7</td>
<td>+2</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>-5</td>
<td>+2</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-3</td>
<td>+2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>-1</td>
<td>+2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>+1</td>
<td>+2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>+3</td>
<td>+2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>+5</td>
<td>+2</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>+7</td>
<td>+2</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sketch a picture of the graph of \( y = -x^2 \). Explain in words the graph is shaped the way it is.

The \( x \) is squared which gives the graph a u-shape.
There is a negative sign which makes the u point downward.

---

Sketch a picture of the graph of \( y = -x^2 \). Explain in words the graph is shaped the way it is.

---
1) Create a table of values for the function \( y = x^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Change in ( y )</th>
<th>Change in change in ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>16</td>
<td>7</td>
<td>7 + 2</td>
</tr>
<tr>
<td>-3</td>
<td>9</td>
<td>5</td>
<td>5 + 2</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td>3</td>
<td>3 + 2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1 + 2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 + 2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1 + 2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1 + 2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>1</td>
<td>1 + 2</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>1</td>
<td>1 + 2</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>1</td>
<td>1 + 2</td>
</tr>
</tbody>
</table>

2) Create a table of values for the function \( y = 2x^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Change in ( y )</th>
<th>Change in change in ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>32</td>
<td>14</td>
<td>14 + 4</td>
</tr>
<tr>
<td>-3</td>
<td>18</td>
<td>10</td>
<td>10 + 4</td>
</tr>
<tr>
<td>-2</td>
<td>8</td>
<td>6</td>
<td>6 + 4</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>2 + 4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 + 4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2 + 4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>6</td>
<td>6 + 4</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>10</td>
<td>10 + 4</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>14</td>
<td>14 + 4</td>
</tr>
</tbody>
</table>

3) Which of the above functions will be shaped like an upside down "u"?

\[ y = -2x^2 \]

4) What will the other function be shaped like?

5) What are the coordinates of the vertex (highest or lowest point) of each of the below functions?

- **Vertex:** \((-2, 2)\)  
  \( \Delta \text{ in } x: \)  
  \( \Delta \text{ in } y: \)

- **Vertex:** \((3, 5)\)  
  \( \Delta \text{ in } x: \)  
  \( \Delta \text{ in } y: \)

- **Vertex:** \((-1, -4)\)  
  \( \Delta \text{ in } x: \)  
  \( \Delta \text{ in } y: \)
Parent Function of Quadratics: \( y = x^2 \)

Complete the following table by using your graphing calculator to sketch each function. Then answer the questions by comparing each function to the parent function.

<table>
<thead>
<tr>
<th>Function</th>
<th>Constant</th>
<th>Sketch</th>
<th>Vertex (highest or lowest point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 )</td>
<td>( 0 )</td>
<td><img src="image" alt="Sketch of ( y = x^2 )" /></td>
<td>((0, 0))</td>
</tr>
<tr>
<td>( y = x^2 + 6 )</td>
<td>( 6 )</td>
<td><img src="image" alt="Sketch of ( y = x^2 + 6 )" /></td>
<td>((0, 6))</td>
</tr>
<tr>
<td>( y = x^2 + 3 )</td>
<td>( 3 )</td>
<td><img src="image" alt="Sketch of ( y = x^2 + 3 )" /></td>
<td>((0, 3))</td>
</tr>
<tr>
<td>( y = x^2 - 4 )</td>
<td>( -4 )</td>
<td><img src="image" alt="Sketch of ( y = x^2 - 4 )" /></td>
<td>((0, -4))</td>
</tr>
<tr>
<td>( y = x^2 - 2 )</td>
<td>( -2 )</td>
<td><img src="image" alt="Sketch of ( y = x^2 - 2 )" /></td>
<td>((0, -2))</td>
</tr>
</tbody>
</table>

What is the connection between the constant and the vertex? __They are the same.__
1) Write two equations for a quadratic whose vertex is:
   a. (0, 3) \[ y = x^2 + 3 \]
   b. (0, -7) \[ y = x^2 - 7 \]
   c. (0, 10) \[ y = x^2 + 10 \]
   d. (0, -9) \[ y = x^2 - 9 \]

2) Write the vertex of each of the following quadratic equations:
   a. \[ y = x^2 + 15 \] \( (0, 15) \)
   b. \[ y = -x^2 + 1 \] \( (0, 1) \)
   c. \[ y = x^2 - 12 \] \( (0, -12) \)
   d. \[ y = -x^2 - 8 \] \( (0, -8) \)

3) Sketch each of the following quadratics:
   a. \[ y = -x^2 + 5 \]
   b. \[ y = x^2 + 7 \]
   c. \[ y = x^2 - 4 \]
   d. \[ y = -x^2 + 4 \]
Use the graphing calculator to graph each set of equations on the same coordinate plane. Use the table of values to get exact points!

Set 1:
- \( Y = x^2 \) green
- \( Y = 3x^2 \) blue
- \( Y = 5x^2 \) red
- \( Y = \frac{1}{2} x^2 \) (remember \( \frac{1}{2} = 0.5 \))

Compare the graphs:
The larger the number that you multiply \( x^2 \) by, the skinnier the \( U \) shape is.
All have vertex \((0,0)\)

Set 2:
- \( Y = -x^2 \) green
- \( Y = -2x^2 \) blue
- \( Y = -4x^2 \) pink
- \( Y = -\frac{1}{4} x^2 \) (remember \( \frac{1}{4} = 0.25 \))

Compare the graphs:
Negative flips graph upside down but again large \# makes skinny, small \# makes wider.
All have vertex \((0,0)\)

Set 3:
- \( Y = x^2 \) green
- \( Y = (x - 2)^2 \) blue
- \( Y = (x + 3)^2 \) pink
- \( Y = (x - \frac{3}{2})^2 \)

Compare the graphs:
All graphs are the same width but have different vertex.
The number inside parentheses moves the graph side to side.
Definitions:

- **quadratic function**: a function where the highest exponent is 2.
- **standard form**: \( y = ax^2 + bx + c \) where \( a \neq 0 \)
- **parabola**: the U-shape that is the graph of a quadratic.
- **vertex**: the point where the graph turns around, changes direction.
- **maximum**: when the vertex is the highest point.
- **minimum**: when the vertex is the lowest point.
- **axis of symmetry**: line over which parabola can be folded. The 2 sides match, always vertical, always goes through vertex.
- **formula**: \( x = \frac{-b}{2a} \) in front of \( x^2 ? \) when in standard form.

Complete the table:

<table>
<thead>
<tr>
<th>Function</th>
<th>Standard Form</th>
<th>Axis of Symmetry from formula</th>
<th>Vertex</th>
<th>Is the vertex a max or min?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y = (x + 2)^2 - 5 )</td>
<td>( y = (x + 2)(x + 2) - 5 ) ( y = x^2 + 4x + 4 - 5 ) ( y = x^2 + 4x - 1 )</td>
<td>( x = \frac{-b}{2a} ) ( x = -2 ) ( x = -2 )</td>
<td>( y = (-2)^2 + 4(-2) - 5 ) ( y = 4 + 8 - 5 ) ( y = 9 - 8 )</td>
<td>min</td>
</tr>
</tbody>
</table>
| \( Y = -2(x + 1)^2 + 3 \) | \( y = -2(x + 2)(x + 1) + 3 \) \( y = -2x^2 - 4x - 2 + 3 \) \( y = -2x^2 + 4x + 1 \) | \( x = \frac{-b}{2a} \) \( x = \frac{-4}{-4} \) \( x = -1 \) | \( y = -2(-4)^2 - 4(-4) + 3 \) \( y = -2(16) + 16 + 3 \) \( y = -32 + 16 + 3 \) | max
| \( Y = 3(x - 3)^2 + 1 \) | \( y = 3(x - 3)(x - 3) + 1 \) \( y = 3x^2 - 18x + 9 + 1 \) \( y = 3x^2 - 18x + 28 + 1 \) \( y = 3x^2 - 18x + 28 \) | \( x = \frac{-b}{2a} \) \( x = \frac{18}{6} \) \( x = -3 \) | \( y = 3(-3)^2 - 18(3) + 28 \) \( y = 27 - 54 + 28 \) \( y = 1 \) | min
| \( Y = (x - 4)^2 - 2 \) | \( y = (x - 4)(x - 4) - 2 \) \( y = x^2 - 8x + 16 - 2 \) \( y = x^2 - 8x + 14 \) | \( x = \frac{-b}{2a} \) \( x = \frac{8}{2} \) \( x = 4 \) | \( y = (4)^2 - 8(4) + 14 \) \( y = 16 - 32 + 14 \) \( y = -2 \) | min
Applied Geometry
Unit 10 (Day 6)

1) Which of the following tables could represent a quadratic (or parabola)?
   a.  
      \[
      \begin{array}{|c|c|}
      \hline
      x & y \\
      \hline
      5 & 5 \\
      6 & 7 \\
      7 & 9 \\
      8 & 11 \\
      \hline
      \end{array}
      \]
   b.  
      \[
      \begin{array}{|c|c|}
      \hline
      x & y \\
      \hline
      5 & 8 \\
      6 & 5 \\
      7 & 2 \\
      8 & -1 \\
      \hline
      \end{array}
      \]
   c.  
      \[
      \begin{array}{|c|c|}
      \hline
      x & y \\
      \hline
      5 & 17 \\
      6 & 28 \\
      7 & 41 \\
      8 & 56 \\
      \hline
      \end{array}
      \]
   d.  
      \[
      \begin{array}{|c|c|}
      \hline
      x & y \\
      \hline
      5 & -5 \\
      6 & -6 \\
      7 & -7 \\
      8 & -8 \\
      \hline
      \end{array}
      \]

2) Without graphing a quadratic, how can you tell if the vertex is going to be a maximum or minimum?
   Put in standard form.
   Look at the coefficient of \(x^2\):
   - Negative = max
   - Positive = min

3) Does the below graph have a maximum or a minimum? \(\text{max}\)  What is it? \(\text{(3, 4)}\)

4) Match each equation with the general shape of its graph:
   a. \(Y = 2x + 5\)
   b. \(Y = x^2 - 2x + 1\)
   c. \(Y = x^3 + 3x\)
   d. \(Y = \text{abs}(x + 3)\)

5) Without creating a table of values or graphing, how can you tell whether a parabola is going to open up (\(\uparrow\)) or open down (\(\downarrow\))? Look at the coefficient of \(x^2\) when
   the equation is in standard form. \(a\) means \(\uparrow\), \(-a\) means \(\downarrow\).

6) Without putting the equation in standard form, what is the vertex of \(y = (x - 3)^2 + 2\)? \(\text{(3, 4)}\)
1) Does the graph of \( y = -(x + 1)^2 + 3 \) open downward or upward? **Explain why!**

   Open downward because there is a negative sign in front of \( (x+1)^2 \).

2) Find the axis of symmetry and the vertex:
   a. \( y = 2x^2 + 8x + 1 \)
      
      \[
      x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2
      \]
      
      \( y = 2(-2)^2 + 8(-2) + 1 \)
      
      \( y = 2(4) + 16 + 1 = 8 + 16 + 1 \)
      
      Vertex: \( (-2, -7) \)
   
   b. \( y = (x + 2)^2 + 5 \)
      
      
      Vertex: \( (-2, 5) \)

3) Describe how the graph of \( y = 2(x + 3)^2 - 6 \) is different from the graph of \( y = x^2 \). You may want to use a sketch to help explain.

   The graph of \( y = 2(x+3)^2 - 6 \) has the vertex at \( (-3, -6) \) and is much skinnier than \( y = x^2 \) which has vertex \( (0, 0) \) because of the 2 before \( (x+3)^2 \).

4) Use the graphing calculator to complete the table of values and graph the equation \( y = -x^2 + 2x - 3 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>-11</td>
</tr>
</tbody>
</table>
Does the graph of the equation open downward or upward? Explain why.

1) \( y = -2x^2 + 10 \)  
   up, \( x^2 \) is positive  

2) \( y = 0.1x^2 - 6x + 9 \)  
   up, \( x^2 \) is positive  

3) \( y = 4(x - 3)^2 - 15 \)  
   up, \( x^2 \) is positive

Find the axis of symmetry and the vertex for the graph of each equation.

1) \( y = 5x^2 \)
   \( \chi = \frac{-b}{2a} = \frac{0}{10} = 0 \)
   \( y - 5(0)^2 = 0 \)
   Axis: \( x = 0 \)
   Vertex: \( (0, 0) \)

2) \( y = 4x^2 - 2x + 15 \)
   \( \chi = \frac{-b}{2a} = \frac{2}{8} = \frac{1}{4} \)
   \( y = 4(\frac{1}{4})^2 - 2(\frac{1}{4}) + 15 = 4\cdot\frac{1}{16} - \frac{1}{2} + 15 = \frac{1}{4} - \frac{1}{2} + 15 = 14.75 \)
   Axis: \( x = \frac{1}{4} \)
   Vertex: \( (0.25, 14.75) \)

3) \( y = -3x^2 + 15 \)
   \( \chi = \frac{-b}{2a} = \frac{0}{-6} = 0 \)
   \( y = -3(0)^2 + 15 = 15 \)
   Axis: \( x = 0 \)
   Vertex: \( (0, 15) \)

Describe how each of the following graphs is different from the parent graph \( y=x^2 \). Explain why.

1) \( y = x^2 - 7 \)
   The vertex is 7 units below the origin, \( -7 \) units.

2) \( y = 4x^2 \)
   The parabola is much skinnier.

3) \( y = (x + 8)^2 \)
   8 units.
   The vertex is a left of the origin.

Make a table of values and graph each equation.

1) \( y = x^2 - 2x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>Work</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>((-1)^2 - 2(-1) + 3 = 1 + 2 + 3)</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>(0^2 - 2(0) + 3 = 0 + 0 + 3)</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>(1^2 - 2(1) + 3 = 1 - 2 + 3)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(2^2 - 2(2) + 3 = 4 - 4 + 3)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(3^2 - 2(3) + 3 = 9 - 6 + 3)</td>
<td>6</td>
</tr>
</tbody>
</table>
Solving Quadratics by Graphing

1) \( x^2 - 2x - 4 = 0 \)
   a. Written as two equations:
      \[
      \begin{array}{c|c|c|c|c}
      x & y & \sqrt{x^2 - 2x - 4} & y = x^2 - 2x - 4 \\hline
      3 & 4 & 5 & 3 \\hline
      1 & 0 & 1 & 1 \\hline
      0 & -4 & 2 & -4 \\hline
      \end{array}
      \]
   b. Answer is where the two graphs ________
   c. Solutions:
      \((-1, -1)\) \((-3, -1)\)
      \(x = -1\) \(x = 3\)
   d. Checks:
      \[
      \begin{array}{c|c|c|c|c|c|c|c|c}
      x & y & \sqrt{x^2 - 2x - 4} & y = x^2 - 2x - 4 \\hline
      -1 & -2.82 & 1 & -4.82 \\hline
      3 & 7.82 & 1 & 4.82 \\hline
      \end{array}
      \]

2) \( x^2 + 125 = 0 \)
   a. Two equations:
      \[
      \begin{array}{c|c|c|c|c|c|c|c|c|c}
      x & y & \sqrt{x^2 + 125} & y = x^2 + 125 \\hline
      -10 & -12.25 & 10 & -22.25 \\hline
      10 & 12.25 & 10 & 22.25 \\hline
      \end{array}
      \]
   b. Solutions:
      none - graphs don't cross
   c. Checks:

3) \( x^2 - 2x - 2 = -3 \)
   a. Two equations:
      \[
      \begin{array}{c|c|c|c|c|c|c|c|c|c}
      x & y & \sqrt{x^2 - 2x - 2} & y = x^2 - 2x - 2 \\hline
      1 & -3 & 1 & -3 \\hline
      -1 & 3 & 1 & 3 \\hline
      \end{array}
      \]
   b. Solutions:
      \((-1, -3)\) \((-3, -3)\)
      \(x = -1\)
   c. Checks:
      \[
      \begin{array}{c|c|c|c|c|c|c|c|c|c}
      x & y & \sqrt{x^2 - 2x - 2} & y = x^2 - 2x - 2 \\hline
      -1 & 2 & 1 & 2 \\hline
      3 & -1 & 1 & -1 \\hline
      \end{array}
      \]
Solve the equation by graphing:

\[ x^2 + 4x + 1 = x + 1 \]

**Equation #1:**
\[ y = x^2 - 4x + 1 \]

**Table for #1:**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>6</td>
</tr>
<tr>
<td>-4</td>
<td>1</td>
</tr>
<tr>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

**Equation #2:**
\[ y = x + 1 \]

**Table for #2:**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Solution(s):**

\[ (-3, -1) \]
\[ (0, 1) \]
\[ x = 3 \]
\[ x = -1 \]

**Check(s):**

\[ x^2 + 4x + 1 = x + 1 \]
\[ (3)^2 + 4(3) + 1 = 3 + 1 \]
\[ -3 + 1 = -3 + 1 \]
\[ 0 = 0 \]
\[ 4 + 4(0) + 1 = 0 + 1 \]
\[ 5 = 1 \]
Graphing Circles

List the differences between the following equations:

1) \( y = x + 1 \)  \( \text{no exponents} \)
2) \( y = x^2 + 1 \)  \( x \) is squared
3) \( y = x^3 + 1 \)  \( x \) is cubed
4) \( y^2 = x^2 + 1 \)  \( y \) and \( x \) are squared

Shapes of graphs:

1) When neither \( y \) nor \( x \) have exponents, the graph is a \underline{straight line}.
2) When \( y \) does not have an exponent and \( x \) is squared, the graph is a \underline{parabola}.
3) When \( y \) does not have an exponent and \( x \) is cubed, the graph is a \underline{cubic function}.
4) When both \( y \) and \( x \) are squared, the graph is a \underline{circle}.

Circles:

\( (x - h)^2 + (y - k)^2 = r^2 \)

\((h, k)\) is the \underline{center}.
\(r\) is the \underline{radius}.

Graph each circle:

1) \( x^2 + y^2 = 16 \)
   - Center: \((0, 0)\)
   - Radius: 4

2) \( x^2 + y^2 = 36 \)
   - Center: \((6, 0)\)
   - Radius: 6
Graph each of the following circles.

1) \((x + 1)^2 + (y + 0)^2 = 16\)
   Center: \((-1, 0)\)
   Radius: 4

2) \((x - 2)^2 + (y + 1)^2 = 4\)
   Center: \((2, -1)\)
   Radius: 2

3) \((x - 3)^2 + (y - 2)^2 = 9\)
   Center: \((3, 2)\)
   Radius: 3
Find the axis of symmetry and the vertex:

\[ y = 3x^2 - 12x + 3 \]

\[ x = \frac{-(-12)}{2 \cdot 3} = \frac{12}{6} = 2 \]

\[ y = 3(2)^2 - 12(2) + 3 \]
\[ y = 12 - 24 + 3 \]
\[ y = -9 \]

Axis of symmetry: \( x = 2 \)
Vertex: \((+2, -9)\)

Find the axis of symmetry and the vertex:

\[ y = 3x^2 - 18x + 3 \]

\[ x = \frac{-(-18)}{2 \cdot 3} = \frac{18}{6} = 3 \]

\[ y = 3(3)^2 - 18(3) + 3 \]
\[ y = 27 - 54 + 3 \]
\[ y = -24 \]

Axis of symmetry: \( x = 3 \)
Vertex: \((3, -24)\)

Find the axis of symmetry and the vertex:

\[ y = 2x^2 - 12x + 3 \]

\[ x = \frac{-(-12)}{2 \cdot 2} = \frac{12}{4} = 3 \]

\[ y = 2(3)^2 - 12(3) + 3 \]
\[ y = 18 - 36 + 3 \]
\[ y = -15 \]

Axis of symmetry: \( x = 3 \)
Vertex: \((3, -15)\)

Find the axis of symmetry and the vertex:

\[ y = 4x^2 - 16x + 3 \]

\[ x = \frac{(-(-16))}{2 \cdot 4} = \frac{16}{8} = 2 \]

\[ y = 4(2)^2 - 16(2) + 3 \]
\[ y = 16 - 32 + 3 \]
\[ y = -13 \]

Axis of symmetry: \( x = 2 \)
Vertex: \((2, -13)\)
Writing the Equation from the Graph

- **Line**: $y = mx + b$ (cross $y$-axis)
- **Parabola**: $y = (x - h)^2 + k$ ($h, k$ is vertex)
- **Circle**: $(x - h)^2 + (y - k)^2 = r^2$ ($h, k$ is center, $r$ is radius)

Write the equation for each graph shown:

1. $(x-1)^2 + (y-2)^2 = 4$
2. $y = -x^2$
3. $(x+1)^2 + (y+1)^2 = 16$
4. $y = -\frac{3}{5}x + 3$
5. $y = (x+2)^2 + 1$
6. $y = 1x - 1$
7. $y = (x-3)^2 = 3$
8. $y = -(x+4)^2 + 3$
9. $(x-4)^2 + (y-3)^2 = 4$
Write the equation for each graph shown:

1. \( y = -(x-1)^2 + 4 \)
2. \( (x+2)^2 + (y+4)^2 = 4 \)
3. \( y = 2x + 5 \)
4. \( y = (x+3)^2 - 2 \)
5. \( (x+1)^2 + (y-3)^2 = 16 \)
6. \( y = (x-4)^2 + 1 \)
7. \( y = -x - 3 \)
8. \( y = -(x+3)^2 + 4 \)
9. \( (x-1)^2 + (y+2)^2 = 9 \)
Graph each of the following equations.

1) \( y = -2x + 5 \)

2) \( y = (x + 3)^2 - 6 \)

3) \( (x - 1)^2 + (y - 4)^2 = 9 \)
Reading a Solution from a Graph

1) What are the solution(s) to the equations whose graphs are shown? Write the equation of each graph.
   a. Sol’n: \((2, 0) \in (-1, 3)\)  
   b. Sol’n: \((-5, 3) \text{ and } (-1, -5)\)  
   c. Sol’n: \((-1, -3)\)

   - **Circle**
     - Equ #1: \((x + 1)^2 + (y + 2)^2 = 4\)  
     - Equ #2: \(y = x - 2\)

   - **Parabola**
     - Equ #1: \(y = (x + 2)^2 - 6\)  
     - Equ #2: \(y = -x^2 - 7\)

   - **Circle**
     - Equ #1: \((x + 1)^2 + (y - 1)^2 = 16\)  
     - Equ #2: \(y = (x + 1)^2 - 3\)

2) How many solutions are there for the equations \(y = (x - 2)^2 - 5\) and \(y = 3\)?
   - **two!**

3) A line contains the points (-2, 3) and (1, 1). In how many points does the line intersect the graph of \(x^2 + y^2 = 4\)?
   - **two!**

4) A circle and a line lie in the same coordinate plane. Which situation is not possible?
   - a. The graphs do not intersect.
   - b. The graphs can intersect in one point.
   - c. The graphs can intersect in two points.
   - **d. The graphs can intersect in three points.**
1) In how many points does the graph of $y = (x + 2)^2 - 7$ intersect the x-axis?

2) A linear and a quadratic function are graphed on the same coordinate plane. What are solution(s) to both equations?

3) A circle and a parabola lie in the same coordinate plane. Draw a sketch of each situation.
   a. The graphs do not intersect.
   b. The graphs intersect in one point.
   c. The graphs intersect in two points.
   d. The graphs intersect in three points.

4) A circle has a radius of 4 and center at the origin. For which equation will the circle and the line not intersect? Sketch each one!
   a. $x = 2$
   b. $y = 2$
   c. $x = -4$
   d. $x = 5$
Graph and solve each pair of equations.

1) \( y = x \) and \( 2y = x^2 \)
   \[ y = \frac{1}{2} x^2 \]

   Solutions:
   (0, 0)
   (2, 2)

4) \( x^2 + y^2 = 25 \) and \( y - 3 = -(x + 4) \)
   \[ y = -x - 1 \]
   \[ y - 3 = -x - 4 \]

   Solutions:
   (4, 3)
   (3, -4)

2) \( x^2 + y^2 = 25 \) and \( x + y = 7 \)
   \[ y = -x + 7 \]

   Solutions:
   (1, 3)

5) \( y = x^2 + 1 \) and \( y = -x^2 - 2 \)

   No Solutions.

3) \( y = -(x + 2)^2 + 9 \) and \( y = \frac{x + 5}{4} \)

   Solutions:
   \((0, 3)\)
   \((-5, 0)\)

6) \( y = \frac{1}{2} x^2 + 2x + 3 \) and \( y = 3 \frac{1}{2} x - 3 \)

   Solutions:
   \[ x = -1.5 \]
   \[ y = -3 \]
   \[ x = -1 \]
   \[ y = 1 \]
   \[ x = 3 \]
   \[ y = 3 \]
   \[ x = 5.5 \]
   \[ y = 5.5 \]
In #1-4, graph and solve each pair of equations.

1) \( y = 2x + 3 \) and \( y \leq x^2 \)

2) \( y = (x - 2)^2 + 4 \) and \( y + x = 1 \)

3) \( y = -(x - 2)^2 + 7 \) and \( y = -2x + 8 \)

4) \( x^2 + y^2 = 9 \) and \( x^2 + y^2 = 4 \)

In #5-6, how many solutions are there?

5) A parabola opens up and has vertex \((-2, 3)\). A line has equation \( y = 3 \).

6) A circle has center at the origin and radius 3. A line has equation \( y = 2x \).
1) Write an equation for a line that is parallel to $y = 3x + 2$.

$$y = 3x + 10$$

2) Describe the slant of the graph of the line $y = -3x + 1$. Why.

Slants downward because the slope (-3) is negative.

3) Match each equation with its sketch:
   a. $y = x$
   b. $y = \text{abs}(x)$
   c. $y = x^2$
   d. $y = x^3$

4) Describe the difference between the graphs of $y = x^2$ and $y = -x^2$.

$y = x^2$ opens up $\cup$ and $y = -x^2$ opens down $\cap$.

5) Describe the difference between the graphs of $y = x^2 + 3$ and $y = x^2$.

$y = x^2 + 3$ has a vertex at $(0, 3)$ and $y = x^2$ has a vertex at $(0, 0)$.

6) Describe the difference between the graphs of $y = \frac{1}{2} x^2$ and $y = x^2$.

$y = \frac{1}{2} x^2$ is wider than $y = x^2$.

7) When does a parabola have a maximum instead of a minimum?

When $x^2$ is negative.

8) Write two equations for a parabola with a vertex of (-2, 3).
   a. $y = (x+2)^2 + 3$
   b. $y = -2(x+2)^2 + 3$

9) Find the vertex of $y = x^2 + 4x - 7$.

$$x = \frac{-b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$$

$$y = (-2)^2 + 4(-2) - 7 = 4 - 8 - 7 = -11$$

10) Write an equation for a quadratic that is skinnier than $y = x^2$ and has a vertex at (3, -1).

$$y = \frac{1}{2} (x - 3)^2 - 1$$

11) Complete the table of values from $x = -2$ to $x = 4$ for $y = 2x^2 - 4x + 1$.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>17</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

12) Find the axis of symmetry of the equation $y = 3x^2 + 6x + 1$.

$$x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1$$

$$x = -1$$
1) John uses the equation \( x^2 + y^2 = 9 \) to represent the shape of a garden on graph paper. Graph the garden.

2) Write an equation for the parabola shown below.

\[ y = (x+3)^2 + 5 \]

3) Solve by graphing \( x^2 - 2x - 16 = 0 \).

4) Write three equations for a parabola whose vertex is \((2, -3)\).
   a. \[ y = (x-2)^2 - 3 \]
   b. \[ y = -(x-2)^2 - 3 \]
   c. \[ y = 2(x-2)^2 - 3 \]

5) Write the equation of a circle whose center is \((3, -1)\) and radius is 6.

\[ (x-3)^2 + (y+1)^2 = 36 \]

6) Find the axis of symmetry and vertex of \( y = x^2 + 6x - 3 \).
   \[ x = \frac{-b}{2a} = \frac{-6}{2} = -3 \]
   \[ y = (-3)^2 + 6(-3) - 3 = 9 - 18 - 3 = -12 \]
   Axis of symmetry: \( x = -3 \)
   Vertex: \((-3, -12)\)

7) The graphs of \( y = x^2 + 4x - 1 \) and \( y + 3 = x \) are drawn on the same set of axes. What is (are) the solution(s)?

8) What is the slope of each of the following lines? Describe the slant of the line.
   a. \( x = 5 \) slope: undefined, vertical
   b. \( y = 2 \) slope: 0, horizontal
   c. \( y = 3x + 1 \) slope: 3, slants up
   d. \( y = -2x - 4 \) slope: -2, slants down

9) Amy tossed a ball in the air in such a way that the path of the ball was modeled by the equation \( y = x^2 + 6x \). In the equation, \( y \) represents the height of the ball in feet and \( x \) is the time in seconds.
   a. Graph the path for \( 0 \leq x \leq 6 \).
   b. At what time, \( x \), is the ball at its highest point? After 3 seconds.
Applied Geometry
Unit 10 (Day 15)

1) Write an equation for a line that is parallel to \( y = -x + 2 \).
   \[ y = -x - 3 \]

2) Describe the slant of the graph of the line \( y = 3x - 5 \) and explain why.
   slants up because slope is positive (3).

3) Match each equation with its sketch:
   a. \( y = -x \)
   b. \( y = x^2 \)
   c. \( y = -x^2 \)
   d. \( y = x \)

4) Describe the difference between the graphs of \( y = 2x^2 \) and \( y = -2x^2 \).
   \( y = 2x^2 \) opens up and \( y = -2x^2 \) open down.

5) Describe the difference between the graphs of \( y = x^2 - 3 \) and \( y = x^2 \).
   \( y = x^2 - 3 \) has vertex at \((0, -3)\)
   and \( y = x^2 \) has vertex at \((0, 0)\).

6) Describe the difference between the graphs of \( y = 5x^2 \) and \( y = x^2 \).
   \( y = 5x^2 \) is much narrower than \( y = x^2 \).

7) Does \( y = x^2 + 3 \) have a maximum instead of a minimum? Explain why.
   It has a minimum because it opens up \( \uparrow \) since \( x^2 \) is positive.

8) Write two equations for a parabola with a vertex of \((5, -2)\).
   a. \( y = (x - 5)^2 - 2 \)
   b. \( y = 2(x - 5)^2 - 2 \)

9) Find the vertex of \( y = 2x^2 + 4x - 1 \).
   \[ x = \frac{-b}{2a} = \frac{-4}{4} = -1 \]
   \[ y = 2(-1)^2 + 4(-1) - 1 = -3 \]
   \( \text{Vertex: } (-1, -3) \)

10) Write an equation for a quadratic that is wider than \( y = x^2 \) and has a vertex at \((-2, 1)\).
    \[ y = \frac{1}{2}(x + 2)^2 + 1 \]

11) Complete the table of values from \( x = -3 \) to \( x = 3 \) for \( y = -3x^2 + 5 \).
    \[
    \begin{array}{c|c}
    x & y \\
    \hline
    -3 & -22 \\
    -2 & -7 \\
    -1 & 2 \\
    0 & 5 \\
    1 & 2 \\
    2 & -7 \\
    3 & -22 \\
    \end{array}
    \]

   a. What is the vertex? \((0, 5)\)
   b. Is it a max or min? \( \text{max} \)

12) Find the axis of symmetry of the equation \( y = x^2 - 2x + 1 \).
    \[ x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1 \]
    \( \text{Axis of symmetry: } x = 1 \)